Year 13 Mock Set#02
Pure Paper 2

- Advised to print in "A3-booklets", this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

This exam paper has 12 questions, for a total of 100 marks.

Question	Marks	Score
1	7	
2	7	
3	4	
4	9	
5	11	
6	9	
7	6	
8	9	
9	9	
10	12	
11	10	
12	7	
Total:	100	

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1.

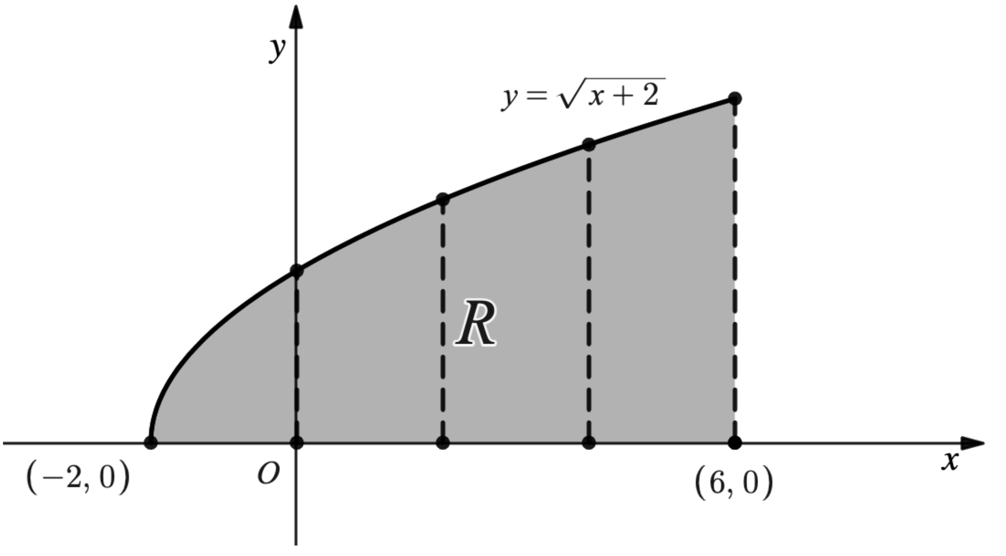

Figure 1: https://www.desmos.com/calculator/2xsfs8acda

Figure 1 shows a sketch of the curve with equation

$$y = \sqrt{x+2} \qquad \{-2 \leq x \leq 6\}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, and the line $x = 6$.

The table below shows corresponding values of x and y for $y = \sqrt{x+2}$, rounded to 4 decimal places.

x	-2	0	2	4	6
y	0	1.4142	2	2.4495	2.8284

(a) Use the trapezium rule, with all of the values of y in the completed table, to find an approximate value of the area of R, giving your answer to 3 decimal places.

(3)

Use your answer to part (a) to find approximate values of

(b) (i) $\displaystyle\int_{-2}^{6} \frac{\sqrt{x+2}}{2}\,\mathrm{d}x$

(ii) $\displaystyle\int_{-2}^{6} \left(2 + \sqrt{x+2}\right)\mathrm{d}x$

(4)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 7 marks)

2. Given that

$$2\log_4(2x+3) = 1 + \log_4 x + \log_4(2x-1) \qquad \{x > \frac{1}{2}\}$$

(a) Show that

$$4x^2 - 16x - 9 = 0$$

(5)

(b) Hence solve the equation

$$2\log_4(2x+3) = 1 + \log_4 x + \log_4(2x-1) \qquad \{x > \frac{1}{2}\}$$

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 7 marks)

3. One of the terms in the binomial expansion of $(3 + ax)^6$, where a is a constant, is $540x^4$

Find the possible values of a.

(4)

Question 3 continued

(Total for Question 3 is 4 marks)

4. (a) Given that

$$\frac{9}{t^2(t-3)} \equiv \frac{A}{t} + \frac{B}{t^2} + \frac{C}{t-3}$$

Find the value of the constants A, B and C.

(3)

(b)

$$I = \int_4^{12} \frac{9}{t^2(t-3)}\,\mathrm{d}t$$

Find the exact value of I, giving your answer in the form $\ln(a) - b$, where a and b are positive constants.

(6)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. **In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.**

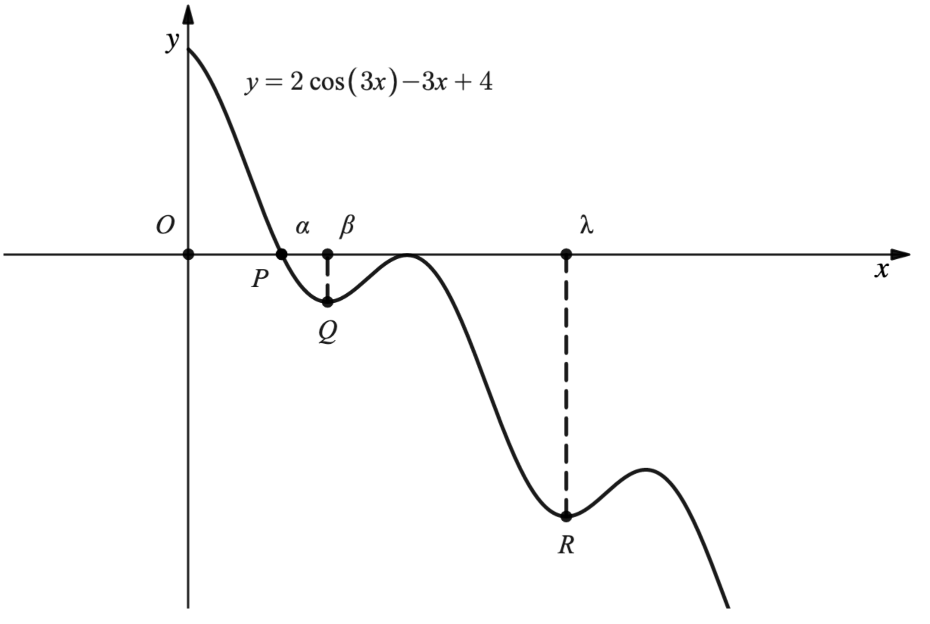

Figure 2: https://www.desmos.com/calculator/g0zwufzqir

Figure 2 shows a sketch the curve with equation

$$y = 2\cos 3x - 3x + 4 \qquad \{x > 0\}$$

where x is measured in radians.
The curve crosses the x-axis at the point P, as shown in Figure 2.
Given that the x-coordinate of P is α,

(a) show that α lies between 0.8 and 0.9

(2)

The iteration formula

$$x_{n+1} = \frac{1}{3}\arccos(1.5x_n - 2)$$

can be used to find an approximate value for α

(b) Using this iteration formula, with $x_1 = 0.8$, find, to 4 decimal places, the value of

(i) x_2

(ii) x_5

(3)

The point Q and the point R are local minimum points on the curve, as shown in Figure 2. Given that the x-coordinates of Q and R are β and λ respectively, and that they are the two smallest values of x at which local minima occur,

(c) find, using calculus, the exact value of β and the exact value of λ

(6)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 11 marks)

6. A curve has equation $y = \mathrm{g}(x)$

Given that

- $\mathrm{g}(x)$ is a cubic expression in which the coefficient of x^3 is equal to the coefficient of x
- the curve with equation $y = \mathrm{g}(x)$ passes through the origin
- the curve with equation $y = \mathrm{g}(x)$ has a stationary point at (2,9)

(a) find $\mathrm{g}(x)$

(7)

(b) prove that the stationary point at (2,9) is a maximum.

(2)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. Relative to a fixed origin O, the point A has position vector $6\mathbf{i} + 5\mathbf{j}$ and the point B has position vector $3\mathbf{i} + 9\mathbf{j}$

(a) Find $\overrightarrow{AB}$ as a simplified vector in terms of $\mathbf{i}$ and $\mathbf{j}$

(2)

The line PQ is parallel to AB. Given that $\overrightarrow{PQ} = 12\mathbf{i} + \lambda\mathbf{j}$

(b) find the value of λ

(2)

(c) Find a unit vector parallel to AB

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 6 marks)

8. (a) show that the equation

$$3\sin(x + \alpha) = 5\sin(x - \alpha)$$

can be written in the form $\tan x = 4\tan\alpha$

(5)

(b) Hence solve, to the nearest integer, the equation

$$3\sin(2y + 30)^\circ = 5\sin(2y - 30)^\circ \qquad \{90 \leq y < 180\}$$

(4)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 9 marks)

9.

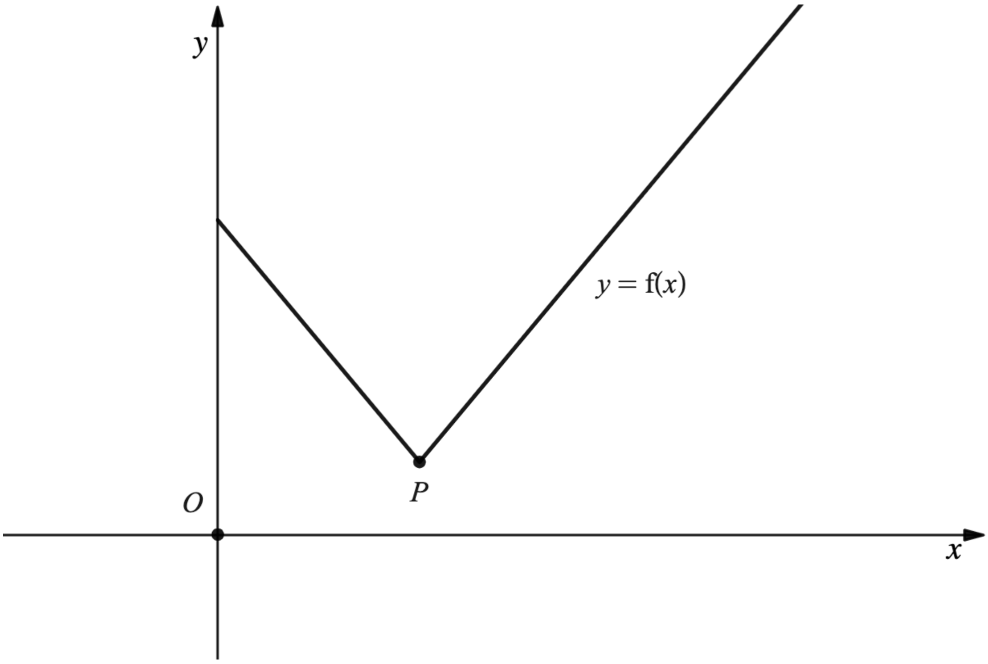

Figure 3

Figure 2 shows part of the curve equation $y = \text{f}(x)$, where

$$\text{f}(x) = 2\,|2x - 5| + 3 \qquad \{x \geq 0\}$$

The vertex of the graph is at P as shown.

(a) State the coordinates of P.

(2)

(b) Solve the equation $\text{f}(x) = 3x - 2$

(4)

Given that the equation

$$\text{f}(x) = kx + 2$$

where k is a constant, has exactly two roots,

(c) find the range of values of k

(3)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 9 marks)

10.

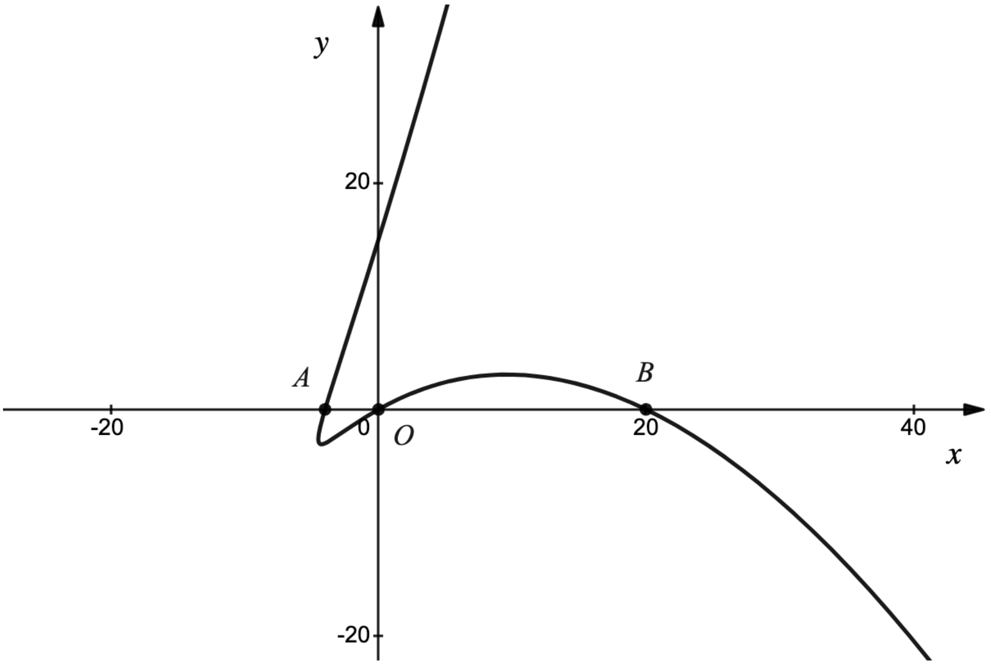

Figure 4

Figure 4 shows a sketch of the curve with parametric equations

$$x = 2t^2 - 6t, \qquad y = t^3 - 4t, \qquad \{t \in \mathbb{R} : -20 \le t \le 20\}$$

(a) Find the coordinates of A and show that B has coordinates (20,0).

(3)

(b) Show that the equation of the tangent to the curve at B is

$$4x + 7y - 80 = 0$$

(5)

The tangent to the curve at B cuts the curve again at the point P.

(c) Find, using algebra, the x-coordinate of P.

(4)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 12 marks)

11. A circle C has equation

$$(x-k)^2 + (y-2k)^2 = k+7$$

where k is a positive constant.

(a) Write down, in terms of k,

(i) the coordinates of the centre of C,

(ii) the radius of C.

(2)

Given that the point $P(2, 3)$ lies on C,

(b) (i) show that $5k^2 - 17k + 6 = 0$

(ii) hence find the possible values of k.

(3)

The tangent to the circle at P intersects the x-axis at the point T

Given that $k < 2$

(c) calculate the exact area of the triangle OPT, where O is the origin.

(5)

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 10 marks)

12. A sequence $a_1, a_2, a_3, \ldots$ is defined by

$$a_1 = p - 3$$
$$a_{n+1} = 2(a_n + 3)^2 - 7$$

where p is a constant.

(a) Find an expression for a_2 in terms of p, giving your answer in simplest form.

(1)

Given that $\sum_{n=1}^{3} a_n = p + 15$

(b) find the possible values of a_2.

(6)

Question 12 continued

Question 12 continued

Question 12 continued

Question 12 continued

(Total for Question 12 is 7 marks)

Total for paper is 100 marks