

Q1.

Question Number	Scheme	Marks
2.(a)	Attempts to substitute $u_2 = 6k + 3$ in $u_3 (= ku_2 + 3)$ $u_3 = k(6k + 3) + 3$	M1 A1 (2)
(b)	Uses $\sum_{n=1}^3 u_n = 117 \Rightarrow 6 + 6k + 3 + k(6k + 3) + 3 = 117$ $6k^2 + 9k - 105 = 0 \Rightarrow k = \dots$ $k = \frac{7}{2}$	M1 dM1 A1 (3) (5 marks)

(a)

M1: Attempts a full method of finding  $u_3$ . Allow if the “+3” is missing once only. E.g. score for an attempt at substituting  $u_2 = 6k + 3$  into  $u_3 = ku_2 (+3)$  or their  $u_2$  into  $u_3 = 6u_2 + 3$   
 May be implied by a correct answer, but an incorrect answer with no substitution is M0.

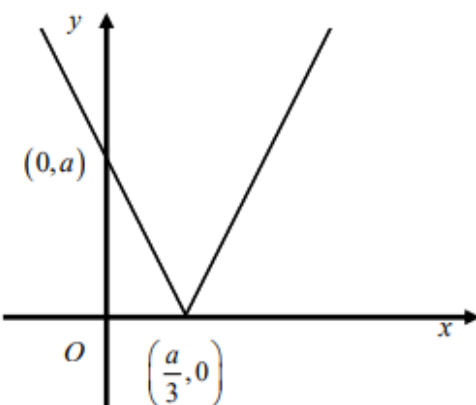
A1:  $u_3 = k(6k + 3) + 3$  OR  $u_3 = 6k^2 + 3k + 3$  but isw after a correct answer is seen

(b)

M1: Sets their  $u_1 + u_2 + u_3 = 117$  to produce an equation in just  $k$ .

dM1: Solves a 3TQ by any valid method to find at least one value for  $k$ .

A1:  $k = \frac{7}{2}$  ONLY

Question	Scheme	Marks
5(a)	 <p>V shape on the +ve <math>x</math> axis</p> <p><math>(0, a)</math> and <math>\left(\frac{a}{3}, 0\right)</math></p>	<p>B1</p> <p>B1</p>
(b) Way 1	<p>Substitutes <math>x = 4</math> into <math> 3x - a  = \frac{1}{2}x + 2 \Rightarrow  3 \times 4 - a  = \frac{1}{2} \times 2 + 2</math></p> <p>Solves <math>12 - a = \pm 4 \Rightarrow a = 8, 16</math></p>	<p>(2)</p> <p>M1</p> <p>dM1 A1</p> <p>(3)</p>
Way 2	<p>Substitutes <math>x = 4</math>, squares both sides and forms a 3TQ in a</p> $(3x - a)^2 = \left(\frac{1}{2}x + 2\right)^2 \Rightarrow 4a^2 - 96a + 512 = 0$ <p>Solves 3TQ to find values for <math>a</math></p> <p><math>a = 8, 16</math></p>	<p>M1</p> <p>dM1</p> <p>A1</p> <p>(3)</p>
	<p>Sets <math>\pm(3x - a) = \frac{1}{2}x + 2</math> and substitutes <math>x = 4</math></p> <p>Rearranges an equation to find a value for <math>a</math></p> <p><math>a = 8, 16</math></p>	<p>M1</p> <p>dM1</p> <p>A1</p> <p>(3)</p>
(c)	<p>Chooses larger value of '<math>a</math>' solves <math>3x - a = \frac{1}{2}x + 2 \Rightarrow x = \dots</math></p> <p><math>x = \frac{36}{5}</math> or 7.2</p>	<p>M1</p> <p>A1</p> <p>(7 marks)</p>
<b>Notes</b>		
(a)		
B1	For a V shape on the positive $x$ - axis in quadrants one and two. It must clearly pass through the $y$ - axis	
B1	Points $(0, a)$ and $\left(\frac{a}{3}, 0\right)$ both lie on the graph. Allow $a$ on the $y$ - axis and $\frac{a}{3}$ on the $x$ - axis	

Question	Scheme	Marks
(b)		
<b>Way 1</b>		
M1	Scored for setting $ 3x - a  = \frac{1}{2}x + 2$ and substituting in $x = 4$ Implied by $ 12 - a  = 4$ , or $12 - a = 4$ or $12 - a = -4$	
dM1	An acceptable method of finding one value of $a$	
A1	Both $a = 8, 16$	
<b>Way 2</b>		
M1	Substitutes $x = 4$ and squared both sides in either order to form a 3TQ in $a$	
dM1	Solve their 3TQ to find a value for $a$	
A1	Both $a = 8, 16$	
<b>Way 3 (See if Way 1 is more relevant)</b>		
M1	Sets $3x - a = \frac{1}{2}x + 2$ and either $-3x + a = \frac{1}{2}x + 2$ or $3x - a = -\frac{1}{2}x - 2$ and substitutes in $x = 4$	
dM1	Rearranges an equation to find a value for $a$	
<b>Note:</b> If they rearrange to find $a = \dots$ then substitutes in $x = 4$ both M's awarded at this point.		
A1	Both $a = 8, 16$	
(c)		
M1	Chooses the larger value of 'their $a$ ' and solves $3x - a = \frac{1}{2}x + 2 \Rightarrow x = \dots$	
A1	$x = \frac{36}{5}$ or 7.2	
<b>Note:</b> If they use both values of their $a$ then M1 and/or A1 is awarded when the largest value of $x$ following their values of $a$ is selected.		

Q3.

Question		Answer	Marks	AO	Guidance	
6		$(x+h)^3 = x^3 + 3x^2h + 3xh^2 + h^3$	<b>B1</b>	<b>1.1</b>	Correct expansion of $(x+h)^3$	Seen at any point Must have numerical coefficients not ${}^3C_1$ Condone 1 as a coefficient Could use $\delta x$ or instead of $h$ Allow unsimplified ie like terms not collected
		$f(x+h) - f(x) =$ $\{2(x+h)^3 + 3(x+h)\} - \{2x^3 + 3x\}$ $= 2(x^3 + 3x^2h + 3xh^2 + h^3) + 3(x+h)$ $- 2x^3 - 3x$	<b>M1</b>	<b>2.1</b>	Attempt to simplify $f(x+h) - f(x)$	If considering $2x^3$ and $3x$ separately then both must be considered for the M1 Could follow B0 but $f(x+h)$ must be a 4 term cubic Allow BOD for $\dots - 2x^3 + 3x$
		$= 6x^2h + 6xh^2 + 2h^3 + 3h$	<b>A1</b>	<b>2.1</b>	Correct 4 term expression for $f(x+h) - f(x)$ www	Either one expression or two separate expressions
		$\frac{f(x+h) - f(x)}{h} =$ $\frac{6x^2h + 6xh^2 + 2h^3 + 3h}{h}$	<b>M1</b>	<b>2.5</b>	Attempt $\frac{f(x+h) - f(x)}{h}$	$f$ must be in terms of the given function and not just a statement of the general definition $f(x+h)$ does not need to be expanded Allow even if $f(x+h)$ is now incorrect If considering $2x^3$ and $3x$ separately then both must be considered for the M1 Allow BOD for $\dots - 2x^3 + 3x$
		$6x^2 + 6xh + 2h^2 + 3$	<b>A1</b>	<b>1.1</b>	Obtain correct expression www	
		$f'(x) = \lim_{h \rightarrow 0} (6x^2 + 6xh + 2h^2 + 3)$ $= 6x^2 + 3$	<b>A1</b>	<b>2.4</b>	Complete proof by considering limit as $h \rightarrow 0$	Must see 'lim', ' $h \rightarrow 0$ ', and $f'(x)$ Dep on previous 5 marks being awarded
			<b>[6]</b>			<b>NB</b> Starting with $6x^2 + 3$ will get no credit in the entire question as not $f'(x)$

Q4.

Question	Scheme	Marks	AOs
6(a)	$A = 5$	B1	2.2a
	$\left(1 - \frac{3}{4}x\right)^{-\frac{1}{2}} \approx$ $1 + \left(-\frac{1}{2}\right)\left(-\frac{3}{4}x\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}\left(-\frac{3}{4}x\right)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}\left(-\frac{3}{4}x\right)^3$	M1 A1	1.1b 1.1b
	$\frac{10}{\sqrt{4-3x}} \approx 5 + \frac{15}{8}x + \frac{135}{128}x^2 + \frac{675}{1024}x^3$	A1	1.1b
		(4)	
(b)	$k = \frac{4}{3}$	B1	2.2a
		(1)	
(c)	$x = \frac{1}{3} \Rightarrow 5 + \frac{15}{8}\left(\frac{1}{3}\right) + \frac{135}{128}\left(\frac{1}{3}\right)^2 + \frac{675}{1024}\left(\frac{1}{3}\right)^3 = \frac{5905}{1024}$ $x = \frac{1}{3} \Rightarrow \frac{10}{\sqrt{4-3x}} = \frac{10}{\sqrt{3}} \Rightarrow \sqrt{3} \approx 10 \div \frac{5905}{1024} \text{ or } \sqrt{3} \approx \frac{3}{10} \times \frac{5905}{1024} = \dots$	M1	1.1b
	$\Rightarrow \sqrt{3} \approx \frac{2048}{1181} \text{ or } \frac{3543}{2048}$	A1	2.2a
		(2)	
(7 marks)			
Notes			

(7 marks)			
Notes			
<p>(a)</p> <p>B1: For deducing that <math>A = 5</math>. This may be seen as part of their final answer or as e.g.</p> $\frac{10}{\sqrt{4-3x}} = \frac{10}{2\sqrt{1-\dots}} \text{ or } \frac{10}{\sqrt{4-3x}} = 10 \times \frac{1}{2}(1-\dots)$ <p>M1: Uses a correct binomial expansion of their <math>(1 \pm \dots)^n</math></p> <p>A1: Correct unsimplified expansion</p> <p>A1: All correct</p> <p>Note direct expansion gives:</p> $10(4-3x)^{-\frac{1}{2}} \approx 10\left(4^{\frac{1}{2}} + \left(-\frac{1}{2}\right)\left(4^{-\frac{3}{2}}\right)(-3x) + \left(\frac{-\frac{1}{2} \times -\frac{3}{2}}{2}\right)\left(4^{-\frac{5}{2}}\right)(-3x)^2 + \left(\frac{-\frac{1}{2} \times -\frac{3}{2} \times -\frac{5}{2}}{6}\right)\left(4^{-\frac{7}{2}}\right)(-3x)^3\right)$			

Score B1 for “5”, M1 for correct structure of the expansion, A1 for correct unsimplified terms and A1 as above

(b)

B1: Deduces the correct value

(c)

M1: Fully correct strategy: Substitutes  $x = \frac{1}{3}$  into their expansion and divides into 10 or

multiplies by  $\frac{3}{10}$

A1: Deduces either value (oe)

Q	Marking instructions	AO	Mark	Typical solution
9	Begins proof by contradiction. This may be evidenced by: stating assumption at the start "the sum is rational" Or Sight of "contradiction" later as part of argument.	3.1a	M1	Assume m is rational and n is irrational and their sum is rational.  Then
	Forms an equation of the form rational + irrational = rational with the rationals written algebraically $\frac{a}{b} + n = \frac{c}{d}$ n must clearly be irrational and not written as an algebraic fraction and not a specific value.	2.5	M1	$\frac{a}{b} + n = \frac{c}{d}$ Where a, b, c and d are all integers. $n = \frac{c}{d} - \frac{a}{b}$ $= \frac{bc - ad}{bd}$
	Manipulates their equation to show that n is rational	1.1b	A1	$\therefore n$ is rational, which is a contradiction.
	Explains or demonstrates why there is a contradiction	2.4	E1	So the original statement is false and the sum of a rational and irrational must be irrational.
	Completes rigorous argument to prove the required result including correct initial assumptions Where a, b, c and d are all integers.	2.1	R1	
Total			5	

Q6.

Question number	Scheme	Marks
2.	<p>(a) <math>f(1.6) = \dots</math>    <math>f(1.7) = \dots</math>    (Evaluate both)</p> <p>0.08... (or 0.09), -0.3...    One +ve, one -ve or Sign change, <math>\therefore</math> root</p> <p>(b) <math>f'(x) = -4\sin x - e^{-x}</math></p> <p><math>1.6 - \frac{f(1.6)}{f'(1.6)}</math></p> <p><math>= 1.6 - \frac{4\cos 1.6 + e^{-1.6}}{(-4\sin 1.6 - e^{-1.6})} \quad \left( = 1.6 - \frac{0.085\dots}{-4.2\dots} \right)</math></p> <p><math>= 1.62</math></p>	<p>M1</p> <p>A1    (2)</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1    (4)</p> <p><b>6</b></p>
	<p>(a) Any errors seen in evaluation of <math>f(1.6)</math> or <math>f(1.7)</math> lose A mark so -0.32 is A0 Values are 0.0851 and -0.3327    Need concluding statement also.</p> <p>(b) B1 may be awarded if seen in N-R as <math>-4\sin 1.6 - e^{-1.6}</math> or as -4.2 M1 for statement of Newton Raphson (sign error in rule results in M 0 ) First A1 may be implied by correct work previously followed by correct answer Do not accept 1.620 for final A1. It must be given and correct to 3sf. 1.62 may follow incorrect work and is A0 No working at all in part (b) is zero marks.</p>	



Q7.

Question Number	Scheme		Marks
9(a)	$f(x) = 8x^{-1} + \frac{1}{2}x - 5$ $\Rightarrow f'(x) = -8x^{-2} + \frac{1}{2}$	M1: $-8x^{-2}$ or $\frac{1}{2}$	M1A1
		A1: Fully correct $f'(x) = -8x^{-2} + \frac{1}{2}$ <b>(may be un-simplified)</b>	
	Sets $-8x^{-2} + \frac{1}{2} = 0 \Rightarrow x = 4$	M1: Sets their $f'(x) = 0$ i.e. a “changed” function (may be implied by their work) and proceeds to find $x$ .	M1A1
		A1: $x = 4$ (Allow $x = \pm 4$ )	
	(4, -1)	Correct coordinates (allow $x = 4, y = -1$ ). Ignore their (-4, ...)	A1
			<b>(5)</b>
(b)(i)	$(x =) 2, 8$	$x = 2$ and $x = 8$ <b>only</b> . Do <b>not</b> accept as coordinates here.	B1
(b)(ii)	(4, 1)	(4, 1) or follow through on their solution in (a). Accept $(x, y + 2)$ from their $(x, y)$ . <b>With no other points.</b>	B1ft
(b)(iii)	$(x =) 2, \frac{1}{2}$	Both answers are needed and <b>accept</b> $(2, 0), \left(\frac{1}{2}, 0\right)$ here. Ignore any reference to the image of the turning point.	B1
			<b>(3)</b>
			<b>(8 marks)</b>

Q8.

Question Number	Scheme	Marks
9(a)	$\frac{3x^2-4}{x^2(3x-2)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{3x-2}$ $\frac{2}{x^2}, \frac{-6}{3x-2} \quad (B=2, C=-6)$ $3x^2-4 \equiv Ax(3x-2) + B(3x-2) + Cx^2 \Rightarrow A=..$ $\frac{3}{x} \quad (A=3) \text{ is one of the fractions}$	B1, B1, M1 A1 [4]
(b)	$\int \frac{1}{y} dy = \int \frac{3x^2-4}{x^2(3x-2)} dx$ $\ln y = \int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{3x-2} dx$ $= A \ln x - \frac{B}{x} + \frac{C}{3} \ln(3x-2) \quad (+k)$ $y = e^{A \ln x - \frac{B}{x} + \frac{C}{3} \ln(3x-2) + D} \quad \text{or } y = D e^{A \ln x - \frac{B}{x} + \frac{C}{3} \ln(3x-2)}$ $y = K x^3 (3x-2)^{-2} e^{-\frac{2}{x}} \quad \text{or } \frac{K x^3 e^{-\frac{2}{x}}}{(3x-2)^2} \quad \text{or } \frac{e^k x^3 e^{-\frac{2}{x}}}{(3x-2)^2} \quad \text{oe}$	B1 M1 M1A1ft M1 A1cso [6] <b>(10 marks)</b>

(a)

B1 For either  $+\frac{2}{x^2}$  or  $\frac{-6}{3x-2}$  being one of the "partial" fractions

B1 For two of the partial fractions being  $+\frac{2}{x^2}$  and  $\frac{-6}{3x-2}$

M1 Need three terms in pfs and correct method either compares coefficients or substitutes a value to obtain A  
Look for  $3x^2-4 \equiv Ax(3x-2) + B(3x-2) + Cx^2 \Rightarrow A=..$

A1  $\frac{3}{x}$

(b)

B1: Separates variables correctly. No need for integral signs

M1 Integrates left hand side to give  $\ln y$  and uses their partial fractions from part (a) (may only have two pf's)

M1 Obtains two  $\ln$  terms and one reciprocal term on rhs (need not have constant of integration for this mark) (must have 3 pf's here). Condone a missing bracket on the  $\ln(3x-2)$

A1ft Correct (unsimplified) answer for rhs for their A, B and C (do not need constant of integration at this stage)

M1 For undoing the logs correctly to get  $y = \dots$  now need constant of integration.

Accept  $y = e^{A \ln x - \frac{B}{x} + \frac{C}{3} \ln(3x-2) + D}$  OR  $y = D e^{A \ln x - \frac{B}{x} + \frac{C}{3} \ln(3x-2)}$  BUT NOT  $y = e^{A \ln x - \frac{B}{x} + \frac{C}{3} \ln(3x-2)} + D$

A1 cso One of the forms of the answer given in the scheme o.e.

**Special case: For students who use two partial fractions**

Very common incorrect solutions using two partial fractions are

$$\frac{3x^2-4}{x^2(3x-2)} \equiv \frac{A}{x^2} + \frac{B}{3x-2} = \frac{2}{x^2} + \frac{3}{3x-2} \text{ using substitution and comparing terms in } x^2$$

$$\text{or } \frac{3x^2-4}{x^2(3x-2)} \equiv \frac{A}{x^2} + \frac{B}{3x-2} = \frac{2}{x^2} + \frac{-6}{3x-2} \text{ using substitution}$$

Both of these will scoring B1B1M0A0 in SC in (a)

In part (b) this could score B1, M1 M0 A0 M1 A0 for a total of 5 out of 10.

For the final M1 they must have the correct form  $y = e^{\dots \ln(3x-2) + D}$  or  $y = D e^{\dots \ln(3x-2)}$  or equivalent

Question	Scheme	Marks	AOs
9	$a = \left(\frac{3}{4}\right)^2 \text{ or } a = \frac{9}{16}$ <p>or</p> $r = -\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^{\circ} = \frac{\frac{9}{16}}{1 - \left(-\frac{3}{4}\right)} = \dots$	M1	3.1a
	$= \frac{9}{28} *$	A1*	1.1b
	(3)		
	Alternative 1:		
	$\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^{\circ} = \frac{-\frac{3}{4}}{1 - \left(-\frac{3}{4}\right)} = \dots \text{ or } r = -\frac{3}{4}$	B1	2.2a
	$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^{\circ} = -\frac{3}{7} - \left(-\frac{3}{4}\right)$	M1	3.1a
	$= \frac{9}{28} *$	A1*	1.1b
	Alternative 2:		
	$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^{\circ} = \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^3 + \left(\frac{3}{4}\right)^4 - \dots$	B1	2.2a
	$= \left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^4 + \dots - \left(\frac{3}{4}\right)^3 - \left(\frac{3}{4}\right)^5 - \dots$ $\left(\frac{3}{4}\right)^2 + \left(\frac{3}{4}\right)^4 + \dots = \left(\frac{3}{4}\right)^2 \left(\frac{1}{1 - \left(\frac{3}{4}\right)^2}\right) \text{ or } -\left(\frac{3}{4}\right)^3 - \left(\frac{3}{4}\right)^5 - \dots = -\left(\frac{3}{4}\right)^3 \left(\frac{1}{1 - \left(\frac{3}{4}\right)^2}\right)$ $\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^{\circ} = \left(\frac{3}{4}\right)^2 \left(\frac{1}{1 - \left(\frac{3}{4}\right)^2}\right) - \left(\frac{3}{4}\right)^3 \left(\frac{1}{1 - \left(\frac{3}{4}\right)^2}\right)$	M1	3.1a
	$= \frac{9}{28} *$	A1*	1.1b

<b>Alternative 3:</b>			
$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^{\circ} = S = \left(\frac{3}{4}\right)^2 - \left(\frac{3}{4}\right)^3 + \left(\frac{3}{4}\right)^4 - \dots$	B1	2.2a	
$= \left(\frac{3}{4}\right)^2 \left(1 - \left(\frac{3}{4}\right) + \left(\frac{3}{4}\right)^2 - \dots\right) = \left(\frac{3}{4}\right)^2 \left(\frac{1}{4} + S\right) \Rightarrow \frac{7}{16}S = \frac{9}{64} \Rightarrow S = \dots$	M1	3.1a	
$= \frac{9}{28}^*$	A1*	1.1b	
<b>(3 marks)</b>			
<b>Notes</b>			
B1: Deduces the correct value of the <b>first</b> term or the common ratio. The correct first term can be			

seen as part of them writing down the sequence but must be the **first** term.

M1: Recognises that the series is infinite geometric and applies the sum to infinity GP formula with  $a = \frac{9}{16}$  and  $r = \pm \frac{3}{4}$

A1\*: Correct proof

**Alternative 1:**

B1: Deduces the correct value for the sum to infinity (starting at  $n = 1$ ) or the common ratio

M1: Calculates the required value by subtracting the first term from their sum to infinity

A1\*: Correct proof

**Alternative 2:**

B1: Deduces the correct value of the **first** term or the common ratio.

M1: Splits the series into “odds” and “evens”, attempts the sum of both parts and calculates the required value by adding both sums

A1\*: Correct proof

**Alternative 3:**

B1: Deduces the correct value of the **first** term

M1: A complete method by taking out the first term, expresses the rhs in terms of the original sum and rearranges for “S”

A1\*: Correct proof

**Q10.**

Question Number	Scheme	Marks
<b>5(a)</b>	States or implies that $\log_{10} p = 0.32$ <b>or</b> $\log_{10} q = \frac{0.56 - 0.32}{8}$	M1
	$p = \text{awrt } 2.089$ <b>or</b> $q = \text{awrt } 1.072$	A1
	States or implies that $\log_{10} p = 0.32$ <b>and</b> $\log_{10} q = \frac{0.56 - 0.32}{8}$	M1
	$p = \text{awrt } 2.089$ <b>and</b> $q = \text{awrt } 1.072$	A1
		<b>(4)</b>
<b>(b)</b>	States or implies that $\frac{dA}{dt} = p \ln q \times q'$ with their values for $p$ and $q$	M1 A1
	Rate of increase in pond weed after 6 days is 0.22 (m <sup>2</sup> /day)	A1
		<b>(3)</b>
		<b>(7 marks)</b>

(a)

M1: States or implies that  $\log_{10} p = 0.32$  **or**  $\log_{10} q = \frac{0.56 - 0.32}{8}$  or equivalent equations

A1:  $p = \text{awrt } 2.089$  **or**  $q = \text{awrt } 1.072$

M1: States or implies that  $\log_{10} p = 0.32$  **and**  $\log_{10} q = \frac{0.56 - 0.32}{8}$  or equivalent equations

A1:  $p = \text{awrt } 2.089$  **and**  $q = \text{awrt } 1.072$

(b)

M1: Uses  $\frac{d}{dt}q' \rightarrow kq'$   $k \neq 1$

A1: States or implies that  $\frac{dA}{dt} = p \ln q \times q'$  with their values for  $p$  and  $q$

A1: awrt 0.22 Units are not required

Alt (b) using  $\log_{10} A = 0.03t + 0.32$  as a starting point

M1: Attempts to differentiate and reaches  $\frac{1}{A} \frac{dA}{dt} = k$  or equivalent

A1:  $\frac{1}{A \ln 10} \frac{dA}{dt} = 0.03$

A1: awrt 0.22 Units are not required

Question	Scheme	Marks	AOs
3(a)	$h = 0.1$	B1	1.1a
	$A \approx \frac{0.1}{2} \{1.632 + 1.930 + 2(1.711 + 1.786 + 1.859)\}$	M1	1.1b
	$= 0.714$	A1	1.1b
		(3)	
(b)	$\int_{0.5}^{0.9} (3f(x) + 2) \, dx = 3 \times "0.714" + \dots$	M1	1.1b
	$\int_{0.5}^{0.9} (3f(x) + 2) \, dx = \dots + 2 \times 0.4$	M1	3.1a
	$\int_{0.5}^{0.9} (3f(x) + 2) \, dx = 3 \times "0.714" + 0.8 = 2.942$	A1ft	2.2a
		(3)	
(6 marks)			
Notes			
(a) B1: States or uses $h = 0.1$ M1: Correct attempt at the trapezium rule. Must be an attempt at the correct structure e.g. $\frac{h}{2} \{y_{0.5} + y_{0.9} + 2(y_{0.6} + y_{0.7} + y_{0.8})\}$ with brackets as shown unless they are implied by subsequent work A1: For awrt 0.714 (b) M1: For multiplying their answer to part (a) by 3 M1: For a correct strategy for the "+ 2" part of the integral. May see e.g. $2 \times 0.4$ or $2 \times (0.9 - 0.5)$ or $\int_{0.5}^{0.9} 2 \, dx = [2x]_{0.5}^{0.9} = 2 \times 0.9 - 2 \times 0.5$ A1ft: For awrt 2.94 or follow through $3 \times$ their answer to part (a) + 0.8			

**Q12**

Question Number	Scheme	Marks
6	$u = 3 + 4 \sin x \Rightarrow \frac{du}{dx} = 4 \cos x$ $\int \frac{16 \sin 2x}{(3 + 4 \sin x)^2} dx = \int \frac{32 \sin x \cos x}{(3 + 4 \sin x)^2} dx = \int \frac{2(u-3)}{u^2} du$ $= \int \frac{2}{u} - \frac{6}{u^2} du = 2 \ln u + \frac{6}{u}$ $\Rightarrow 2 \ln 7 + \frac{6}{7} - 2 \ln 5 - \frac{6}{5} = -\frac{12}{35} + \ln \frac{49}{25}$ <p>Uses limits of 5 and 7</p>	<p>B1</p> <p>M1 A1</p> <p>dM1 A1</p> <p>M1 A1</p> <p><b>(7 marks)</b></p>

B1: States or uses  $\frac{du}{dx} = 4 \cos x$  o.e. This may be seen within the integrand.  
M1: Attempts to write all in terms of  $u$ .

Look for 
$$\int \frac{16 \sin 2x}{(3 + 4 \sin x)^2} dx = \int \frac{\dots \sin x \cos x}{(3 + 4 \sin x)^2} dx = \pm \int \frac{\dots(u \pm 3)}{u^2} du$$

It is common to lose the square on the  $u^2$  which would be M0.  
Only the penultimate mark would then be available.

A1: Correct 
$$\int \frac{2(u-3)}{u^2} du$$
 o.e. Allow this to be unsimplified.  
Condone a missing  $du$  this can be implied by further work.

dM1: Integrates 
$$\int \frac{\dots(u \pm 3)}{u^2} du$$
 to  $\dots \ln u \pm \frac{\dots}{u}$  o.e. Condone a missing  $du$  this can be implied by further work.

Note that 
$$\int \frac{\dots(u \pm 3)}{u^2} du \rightarrow \dots \ln u^2 \pm \frac{\dots}{u}$$
 is acceptable

You may see attempts using parts.

$$\int \frac{1}{u^2} \times (2u - 6) du = -\frac{1}{u} \times (2u - 6) - \int -\frac{1}{u} \times 2 du = -\frac{1}{u} \times (2u - 6) + 2 \ln u$$

A1: Correct 
$$2 \ln u + \frac{6}{u}$$
 . Note that  $\ln u^2 + \frac{6}{u}$  or  $-\frac{1}{u} \times (2u - 6) + 2 \ln u$  is also completely correct  
M1: Uses limits 5 and 7 within their attempted integral and subtracts . Condone poor attempts at the integration

Alternatively converts their answer in  $u$  back to  $x$ 's using the correct substitution and uses the given limits

A1: 
$$-\frac{12}{35} + \ln \frac{49}{25}$$
 or equivalent such as  $\ln 1.96 - \frac{12}{35}$





Question Number	Scheme	Marks
5 (a)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{16 \sec^2 t \tan t}{2 \sec^2 t} = 8 \tan t$	M1 A1
	At $x = 3, \tan t = -1 \Rightarrow$ Gradient $= -8$	dM1 A1
		(4)
	(b) Attempts to use $1 + \tan^2 t = \sec^2 t \Rightarrow 1 + \frac{(x-5)^2}{4} = \frac{y}{8}$	M1 A1
	$y = 2(x-5)^2 + 8$	A1
(c)		(3)
	8,, f,, 32	M1 A1
		(2)
		(9 marks)

(a)

M1: Attempts to use the rule  $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$ . Condone incorrect attempts on  $\frac{dy}{dt}$  and  $\frac{dx}{dt}$

A1: A correct expression for  $\frac{dy}{dx} = \frac{16 \sec^2 t \tan t}{2 \sec^2 t}$  or unsimplified equivalent. Note that there may be many

different versions of this including ones that have used double angle formulae. Look carefully

dM1: Dependent upon the previous M. It is for

- either substituting  $\tan t = -1$  into their  $\frac{dy}{dx} = g(t)$  to find the gradient
- or substituting  $t = -\frac{\pi}{4}$  into their  $\frac{dy}{dx} = g(t)$  to find the gradient. Condone  
 $t = \text{awrt } -0.785$

A1: CSO Gradient  $= -8$ . This cannot be awarded from differentiation of  $y = 2(x-5)^2 + 8$

(b)

M1: Attempts to use  $\pm 1 \pm \tan^2 t = \pm \sec^2 t$  with  $\tan t$  being replaced by an expression in  $x$  and  $\sec^2 t$  being replaced by an expression in  $y$

A1: A correct unsimplified equation  $1 + \frac{(x-5)^2}{4} = \frac{y}{8}$  o.e.

A1:  $y = 2(x-5)^2 + 8$ .  $f(x) = 2(x-5)^2 + 8$  is also fine

NB 1: It is possible to use part (a), find  $\frac{dy}{dx} = 4(x-5)$ , and then integrate using a point such as (3, 16) to find "c"

NB2: It is possible to use points to generate  $f(x)$ .

For the M1 you should expect to see  $f(x) = ax^2 + bx + c$ , or equivalent, and the use 3 points to set up 3 simultaneous equations in  $a$ ,  $b$  and  $c$  which must be then solved. At least one of three points used must be correct. Examples of possible points that can be used are  $(5, 8)$ ,  $(3, 16)$ ,  $(5 - 2\sqrt{3}, 32)$  and  $(7, 16)$

For the A1 you need to see a correct equation, e.g.  $y = 2x^2 - 20x + 58$

The final A1 will be  $y = 2(x-5)^2 + 8$ .

If you see a solution worthy of merit and you cannot see how to award the marks then send to review.

(c)

M1: One correct end found, condoning strict inequalities.

Look for the lower value of the range to be 8 (or their  $c$ ) or the higher value to be 32

A1: 8,, f,, 32 or equivalent such as  $[8, 32]$

Question	Scheme	Marks	AOs
5(a)	$\pm \overrightarrow{AB} = \pm (7\mathbf{i} + \mathbf{j} + 2\mathbf{k} - (5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}))$ $\Rightarrow  \overrightarrow{AB}  = \sqrt{2^2 + (-2)^2 + 4^2} \text{ or } \Rightarrow  \overrightarrow{AB} ^2 = 2^2 + (-2)^2 + 4^2$	M1	1.1b
	$ \overrightarrow{AB}  = 2\sqrt{6}$	A1	1.1b
		(2)	
(b)	$\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{BA} = 4\mathbf{i} + 8\mathbf{j} - 3\mathbf{k} - 2\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$	M1	1.1b
	$\overrightarrow{OD} = 2\mathbf{i} + 10\mathbf{j} - 7\mathbf{k}$	A1	1.1b
		(2)	
(c)	$\overrightarrow{OE} = \overrightarrow{OA} + \frac{3}{2}\overrightarrow{AC} \text{ or } \overrightarrow{OE} = \overrightarrow{OC} + \frac{1}{2}\overrightarrow{AC}$	M1	3.1a
	$E \text{ is } (3.5, 10.5, -3.5)$	A1	1.1b
		(2)	
(6 marks)			
Notes			
<p>(a)</p> <p>M1: Subtracts either way round and applies Pythagoras to find <math> \overrightarrow{AB} </math> or <math> \overrightarrow{AB} ^2</math></p> <p>A1: For <math>2\sqrt{6}</math></p> <p>(b)</p> <p>M1: Correct strategy to find the position vector of <math>D</math></p> <p>A1: Correct vector</p> <p>(c)</p> <p>M1: Interprets the given ratio correctly and then adopts a correct approach to find the coordinates of the point <math>E</math></p> <p>A1: Correct coordinates and no other coordinates</p>			

**Q15**

Question Number	Scheme	Marks
<b>9(a)</b>	$R = 13$	B1
	$\tan \alpha = \frac{5}{12} \Rightarrow \alpha = \text{awrt } 0.395$	M1A1
		<b>(3)</b>
<b>(b)</b>	$g(\theta) = 10 + 13 \sin\left(2\theta - \frac{\pi}{6} - 0.395\right)$	
<b>(i)</b>	(i) Minimum value is $-3$	B1 ft
<b>(ii)</b>	$2\theta - \frac{\pi}{6} - 0.395 = \frac{3\pi}{2} \Rightarrow \theta = \text{awrt } 2.82$	M1 A1
		<b>(3)</b>
<b>(c)</b>	$h(\beta) = 10 - 169 \sin^2(\beta - 0.395)$	
	$-159 \leq h \leq 10$	M1 A1
		<b>(2)</b>
		<b>(8 marks)</b>

(a)

B1:  $R = 13$  ( $R = \pm 13$  is B0)

M1:  $\tan \alpha = \pm \frac{5}{12}$ ,  $\tan \alpha = \pm \frac{12}{5} \Rightarrow \alpha = \dots$

If  $R$  is used to find  $\alpha$  accept  $\sin \alpha = \pm \frac{5}{R}$  or  $\cos \alpha = \pm \frac{12}{R} \Rightarrow \alpha = \dots$

A1:  $\alpha = \text{awrt } 0.395$  Note that the degree equivalent  $\alpha = \text{awrt } 22.6^\circ$  is A0

(b)(i)

B1ft: States the value of  $10 - R$  following through their  $R$ .

(b)(ii)

M1: Attempts to solve  $2\theta - \frac{\pi}{6} \pm 0.395 = \frac{3\pi}{2} \Rightarrow \theta = \dots$

A1:  $\theta = \text{awrt } 2.82$ . No other values should be given

(c)

M1: Achieves one of the end values, either  $-159$  (or  $10 - (\text{their } R)^2$  evaluated) or  $10$

A1: Fully correct range  $-159 \leq h \leq 10$ ,  $-159 \leq h(\beta) \leq 10$ ,  $-159 \leq \text{range} \leq 10$ ,  $-159 \leq h(x) \leq 10$ ,  $[-159, 10]$  or equivalent correct ranges.