Year 13 Mock Pure Paper 1

12^{th} May 2021

This exam has 14 questions, for a total of 100 marks.

- Print in "booklets" will allow all questions to be on the left hand side.
- If instead you print in 2-in-1 settings, print the second page up to the last page first, then print the first page separately.

Question	Marks	Score
1	6	
2	6	
3	2	
4	8	
5	9	
6	5	
7	5	
8	7	
9	10	
10	8	
11	11	
12	10	
13	5	
14	8	
Total:	100	

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1. The binomial series expansion of

$$(1+ax)^{\frac{2}{3}} \qquad |ax|<1$$

up to and including the term in x^2 is

$$1+\frac{1}{2}x+kx^2$$

where a and k are constants.

(a) Find the value of a.

(2)

(b) Find the value of k , giving your answer in its simplest form.

(2)

(c) Hence find the numerical coefficient of x^2 in the series expansion of

$$(4-9x)(1+ax)^{\frac{2}{3}} \qquad |ax|<1$$

(2)

Question 1 continued

(Total for Question 1 is 6 marks)

2. Find the exact solutions, in their simplest form, to the equations

(a) $2\ln(2x+1) - 10 = 0$

(2)

(b) $3^x\, e^{4x} = e^7$

(4)

Question 2 continued

(Total for Question 2 is 6 marks)

3. Prove by contradiction that there is no greatest odd integer.

(2)

Question 3 continued

(Total for Question 3 is 2 marks)

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Given that

$$2\cos(x+50)^\circ = \sin(x+40)^\circ$$

(a) Show that

$$\tan(x)^\circ = \frac{1}{3}\tan 40^\circ$$

(4)

(b) Hence, solve, for $0^\circ \leq \theta \leq 360^\circ$

$$2\cos(2\theta+50)^\circ = \sin(2\theta+40)^\circ$$

giving your answers to 1 decimal place.

(4)

Question 4 continued

(Total for Question 4 is 8 marks)

5. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(a) Express $2\sin\theta - 4\cos\theta$ in the form $R\sin(\theta - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ Give the value of α to 3 decimal places.

(3)

Given that

$$f(\theta) = 4 + 5(2\sin 3\theta - 4\cos 3\theta)^2$$

(b) Find

(i) the maximum value of $f(\theta)$

(ii) the smallest value of θ, for $0 \leq \theta < \pi$, at which this maximum value occurs.

(3)

(c) Find

(i) the minimum value of $f(\theta)$

(ii) the largest value of θ, for $0 \leq \theta < \pi$, at which this minimum value occurs.

(3)

Question 5 continued

(Total for Question 5 is 9 marks)

6. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 7x - 1 \qquad x \in \mathbb{R}$$
$$\mathrm{g} : x \mapsto \frac{4}{x-2} \qquad x \neq 2,\ x \in \mathbb{R}$$

(a) Solve the equation $\mathrm{fg}(x) = x$

(4)

(b) Hence, or otherwise, find the largest value of a such that $\mathrm{g}(a) = \mathrm{f}^{-1}(a)$

(1)

Question 6 continued

(Total for Question 6 is 5 marks)

7. The area A of a circle is increasing at a constant rate of 1.5 $\text{cm}^2\ \text{s}^{-1}$.

Find the rate at which the radius r of the circle is increasing when the area of the circle is 2 cm^2.

Give your answer to 3 significant figures.

(5)

Question 7 continued

(Total for Question 7 is 5 marks)

8. A curve has equation

$$2^x + y^2 = 2xy$$

Find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the point on C with coordinates $(3, 2)$

(7)

Question 8 continued

(Total for Question 8 is 7 marks)

9. (i) Find, using calculus, the x-coordinate of the turning point of the curve with equation

$$y = \mathrm{e}^{3x}\cos(4x), \qquad \frac{\pi}{4} \leq x < \frac{\pi}{2}$$

Give your answer to 4 decimal places.

(5)

(ii) Given $x = \sin^2(2y)$, $0 < y < \frac{\pi}{4}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ as a function of y.

Write your answer in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = p\,\mathrm{cosec}(qy), \qquad 0 < y < \frac{\pi}{4}$$

where p and q are constants to be determined.

(5)

Question 9 continued

(Total for Question 9 is 10 marks)

10. Using the substitution $u = 2 + \sqrt{2x+1}$, or other suitable substitutions, find the exact value of

$$\int_0^4 \frac{1}{2+\sqrt{2x+1}}\,\mathrm{d}x$$

giving your answer in the form $A + 2\ln B$, where A is an integer and B is a positive constant.

(8)

Question 10 continued

(Total for Question 10 is 8 marks)

11.

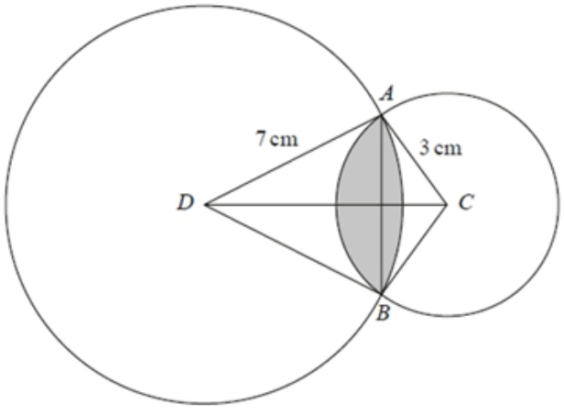

Figure 1

Figure 1 shows two intersecting circles.

The larger circle has its centre at the point D and has radius 7 cm.

The smaller circle has its centre at the point C and has radius 3 cm.

AB is the common chord of the two circles, and the distance CD is 9 cm.

(a) Find, to 3 significant figures, the size of

(i) $\angle ADC$ in radians

(ii) $\angle ACD$ in radians

(4)

The region common to both circles is shown shaded in Figure 1.

(b) Find the perimeter of the shaded region.

(3)

(c) Find the area of the shaded region

(4)

Question 11 continued

(Total for Question 11 is 11 marks)

12. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(a) Prove that

$$2\cot(2x) + \tan x \equiv \cot x \qquad x \neq \frac{n\pi}{2}, n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $-\pi \leq x \leq \pi$,

$$6\cot(2x) + 3\tan x = \operatorname{cosec}^2 x - 2$$

give your answer to 3 decimal places.

(6)

Question 12 continued

(Total for Question 12 is 10 marks)

13.

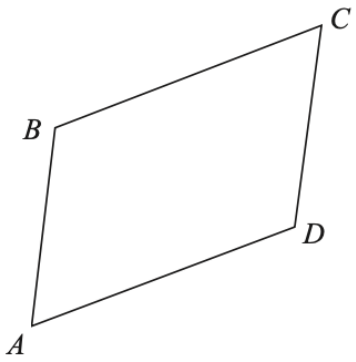

Figure 2

Figure 2 shows a sketch of parallelogram $ABCD$.
Given that $\overrightarrow{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{BC} = 2\mathbf{i} + 5\mathbf{j} + 8\mathbf{k}$

(a) Find the size of $\angle ABC$

(3)

(b) Find the area of the parallelogram $ABCD$, giving your answer to one decimal place.

(2)

Question 13 continued

(Total for Question 13 is 5 marks)

14.

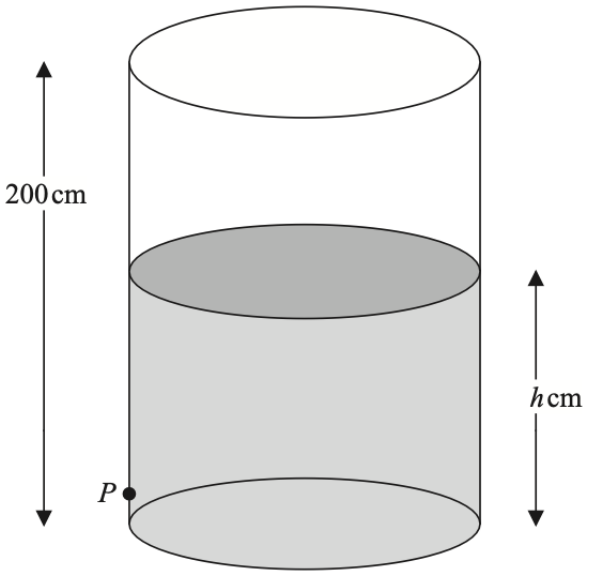

Figure 3

Figure 3 shows a vertical cylindrical tank of height 200 cm containing water. Water is leaking from a hole P on the side of the tank.

At time t minutes after the leaking starts, the height of water in the tank is h cm.

The height h cm of the water in the tank satisfies the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = k(h-9)^{\frac{1}{2}}, \qquad 9 < h \leq 200$$

where k is a constant.

Given that, when $h = 130$, the height of the water is falling at a rate of 1.1 cm per minute.

(a) Find the value of k

(2)

Given that the tank was full of water when the leaking started.

(b) Solve the differential equation with your value of k, to find the value of t when $h = 50$

(6)

Question 14 continued

Question 14 continued

Question 14 continued

Question 14 continued

(Total for Question 14 is 8 marks)

Total for paper is 100 marks