Question 1:

Find, giving your answer to 3sf where appropriate, the value of x for which

(a) **[2 marks]** $5^x = 10$

(b) **[2 marks]** $\log_3(x-2) = -1$

Question 2:

[5 marks] Find the values of x such that

$$2\log_3(x) - \log_3(x-2) = 2$$

Question 3:

(a) **[3 marks]** Given that $y = 3x^2$, show that $\log_3(y) = 1 + 2\log_3(x)$

(b) **[3 marks]** Hence, or otherwise, solve the equation

$$1 + 2\log_3(x) = \log_3(28x - 9)$$

Question 4:

[6 marks] Given that $0 < x < 4$, and

$$\log_5(4-x) - 2\log_5(x) = 1$$

find the value of x.

Question 5:

(a) **[3 marks]** Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}$$

(b) **[4 marks]** Given that

$$\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15) \qquad x \neq -5$$

find x in terms of e.

Question 6:

(a) **[2 marks]** Find the positive value of x such that

$$\log_x(64) = 2$$

(b) **[6 marks]** Solve for x

$$\log_2(11 - 6x) = 2\log_2(x-1) + 3$$

Question 7:

$$f(x) = 2x^3 - 5x^2 + ax + 18$$

where a is a constant.

Given that $(x - 3)$ is a factor of $f(x)$,

(a) **[2 marks]** show that $a = -9$

(b) **[4 marks]** factorise $f(x)$ completely.

Given that

$$g(y) = 2(3^{3y}) - 5(3^{2y}) - 9(3^y) + 18$$

(c) **[3 marks]** find the values of y that satisfy $g(y) = 0$, giving your answer to 2dp where appropriate.

Question 8:

The value of a car is modelled by the formula

$$V = 16000e^{-kt} + A \qquad t \geq 0$$

where V is the value of the car in pounds, t is the age of the car in years, and k and A are **positive** constants.

Given that the value of the car is £ 17500 when new and £ 13500 two years later,

(a) **[1 mark]** find the value of A

(b) **[4 marks]** show that $k = \ln\left(\frac{2}{\sqrt{3}}\right)$

(c) **[4 marks]** find the age of the car, in years, when the value of the car is £ 6000, giving your answer to 2dp.

Question 9:

The mass, m grams, of a leaf t days after it has been picked from a tree is given by

$$m = pe^{-kt}$$

where k and p are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) **[1 mark]** Write down the value of p.

(b) **[4 marks]** Show that $k = \frac{1}{4}\ln(3)$

(c) **[6 marks]** Find the value of t when $\frac{\mathrm{d}m}{\mathrm{d}t} = -0.6\ln(3)$

Question 10:

A study is being carried out on two colonies of ants.

The number of ants N_A in colony A, t years after the start of the study, is modelled by the equation

$$N_A = 3000 + 600e^{0.12t} \qquad t \geq 0$$

(a) **[5 marks]** Find the time taken, from the start of the study, for the number of ants in colony A to double. Give your answer, in years, to 2dp.

(b) **[3 marks]** Show that $\dfrac{\mathrm{d}N_A}{\mathrm{d}t} = pN_A + q$ where p and q are constants to be determined.

The number of ants N_B in colony B, t years after the start of the study, is modelled by the equation

$$N_B = 2900 + Ce^{kt} \qquad t \geq 0$$

where C and k are positive constants.

According to this model, there will be 3100 ants in colony B one year after the start of the study and 3400 ants in colony B two years after the start of the study.

(c) **[4 marks]** Show that $k = \ln\left(\dfrac{5}{2}\right)$ and find the value of C.

Numerical Answers:

(1) (a) $x = 1.43$

(b) $x = \dfrac{7}{3}$

(2) $x = 3$ or $x = 6$

(3) (a) $\log_3(y) = 1 + 2\log_3(x)$

(b) $x = \dfrac{1}{3}$ or $x = 9$

(4) $x = \dfrac{4}{5}$

(5) (a) $\dfrac{2x-1}{x-3}$

(b) $x = \dfrac{3e-1}{e-2}$

(6) (a) $x = 8$

(b) $x = \dfrac{3}{2}$

(7) (a) $a = -9$

(b) $(x-3)(2x-3)(x+2)$

(c) $y = 0.37$ or $y = 1$

(8) (a) $A = 1500$

(b) $k = \ln\left(\dfrac{2}{\sqrt{3}}\right)$

(c) $T = 8.82$

(9) (a) $p = 7.5$

(b) $k = \dfrac{1}{4}\ln(3)$

(c) $t = 4.15$

(10) (a) $t = 16.22$ years

(b) $p = \dfrac{3}{25}$ and $q = -360$

(c) $k = \ln\left(\dfrac{5}{2}\right)$ and $C = 80$