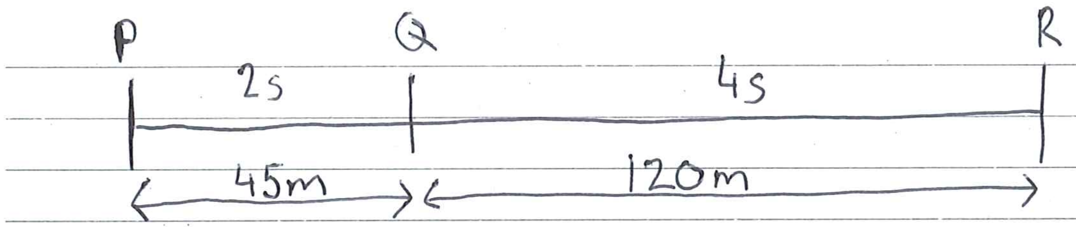
1. Three posts P, Q and R, are fixed in that order at the side of a straight horizontal road. The distance from P to Q is 45 m and the distance from Q to R is 120 m. A car is moving along the road with constant acceleration a m s^{-2}. The speed of the car, as it passes P, is u m s^{-1}. The car passes Q two seconds after passing P, and the car passes R four seconds after passing Q. Find

(i) the value of u,

(ii) the value of a.

(7)



PQ: $s = 45$ $\quad s = ut + \frac{1}{2}at^2$

u $\quad 45 = 2u + 2a$

v

a

$t = 2$

PR: $s = 165$ $\quad s = ut + \frac{1}{2}at^2$

u $\quad 165 = 6u + 18a$

v

a

$t = 6$

$2u + 2a = 45$ $\quad \times 3 \quad \rightarrow \quad 6u + 6a = 135$

$6u + 18a = 165$ $\quad\quad - \; 6u + 18a = 165$

$12a = 30$

$\boxed{a = 2.5\,\text{ms}^{-2}}$

$2u + 2(2.5) = 45$

$2u = 40$

$\boxed{u = 20\,\text{ms}^{-1}}$

2. A particle P travels along a straight line through a point O so that at time t s after passing through O its displacement from O is x m, where $x = t^3 - 15t^2 + 62t$

Find

(a) the initial velocity of P, **(3)**

(b) the value of t for which P has zero acceleration. **(2)**

a) $v = 3t^2 - 30t + 62$

When $t = 0$

$v = 3(0)^2 - 30(0) + 62$

$v = 62ms^{-1}$

b) $a = 6t - 30$

$6t - 30 = 0$

$6t = 30$

$t = 5s$

3.

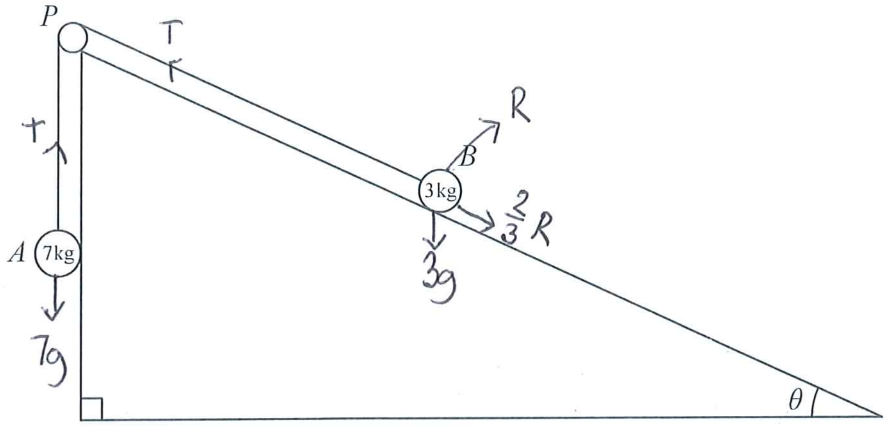


Figure 4

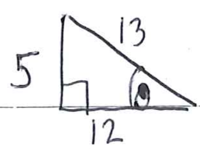
Two particles A and B, of mass 7 kg and 3 kg respectively, are attached to the ends of a light inextensible string. Initially B is held at rest on a rough fixed plane inclined at angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$. The part of the string from B to P is parallel to a line of greatest slope of the plane. The string passes over a small smooth pulley, P, fixed at the top of the plane. The particle A hangs freely below P, as shown in Figure 4. The coefficient of friction between B and the plane is $\frac{2}{3}$. The particles are released from rest with the string taut and B moves up the plane.

(a) Find the magnitude of the acceleration of B immediately after release. **(10)**

(b) Find the speed of B when it has moved 1 m up the plane. **(2)**

When B has moved 1 m up the plane the string breaks. Given that in the subsequent motion B does not reach P,

(c) find the time between the instants when the string breaks and when B comes to instantaneous rest. **(4)**



$\sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$

a) $R(\downarrow)_A: 7g - T = 7a$

$R(\nwarrow)_B: T - \frac{2}{3}R - 3g\sin\theta = 3a$

$R(\nearrow)_B: R = 3g\cos\theta$

$R = \frac{36g}{13}$

$7g - T = 7a$

$+ \quad T - \frac{24g}{13} - \frac{15g}{13} = 3a$

$7g - \frac{24g}{13} - \frac{15g}{13} = 10a$

$4g = 10a \rightarrow \boxed{a = \frac{2}{5}g}$

Question 3 continued

b) $s = 1$ $\quad v^2 = u^2 + 2as$

$u = 0$ $\quad v^2 = \frac{4}{5}g$

v $\quad v = 2.8 \text{ ms}^{-1}$

$a = \frac{2}{5}g$

t

c) When the string breaks:

$R(\nwarrow)_B : -\frac{2}{3}R - 3g\sin\theta = 3a$

$$\frac{-24g}{13} - \frac{15g}{13} = 3a$$

$$-3g = 3a$$

$$a = -g$$

s $\quad v = u + at$

$u = 2.8$ $\quad 0 = 2.8 - gt$

$v = 0$ $\quad gt = 2.8$

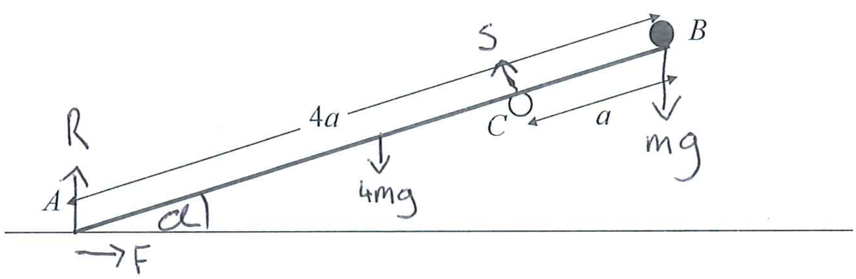
$a = -g$ $\quad t = \frac{2.8}{g}$

t

$$t = 0.29$$

4.

Figure 2

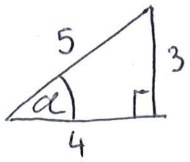


A wooden plank AB has mass $4m$ and length $4a$. The end A of the plank lies on rough horizontal ground. A small stone of mass m is attached to the plank at B. The plank is resting on a small smooth horizontal peg C, where $BC = a$, as shown in Figure 2. The plank is in equilibrium making an angle α with the horizontal, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between the plank and the ground is μ. The plank is modelled as a uniform rod lying in a vertical plane perpendicular to the peg, and the stone as a particle. Show that

(a) the reaction of the peg on the plank has magnitude $\frac{16}{5}\,mg$, **(3)**

(b) $\mu \geqslant \frac{48}{61}$. **(6)**

(c) State how you have used the information that the peg is smooth. **(1)**



$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

a) $M(A)$: $4mg \times 2a\cos\alpha + mg \times 4a\cos\alpha = S \times 3a$

$12amg\cos\alpha = 3Sa$

$4mg\cos\alpha = S$

$4mg\left(\frac{4}{5}\right) = S$

$\frac{16mg}{5} = S$

b) $R(\uparrow)$: $R + S\cos\alpha = 5mg \rightarrow R + \frac{64mg}{25} = 5mg$

$R(\rightarrow)$: $F = S\sin\alpha \rightarrow F = \frac{48mg}{25}$

$R = 5mg - \frac{64mg}{25}$ $\quad R = \frac{125-64}{25}mg$ $\quad R = \frac{61}{25}mg$

Question 4 continued

~~F ≤ μR~~

$F \leq \mu R$

$\frac{48}{25} mg \leq \mu \times \frac{61}{25} mg$

$48 \leq 61\mu$

$\mu \geq \frac{48}{61}$

c) No friction at the peg

5.

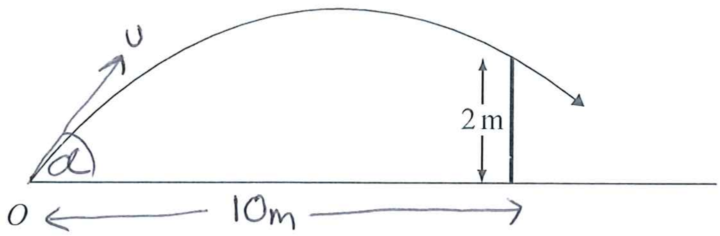


Figure 3

A child playing cricket on horizontal ground hits the ball towards a fence 10 m away. The ball moves in a vertical plane which is perpendicular to the fence. The ball just passes over the top of the fence, which is 2 m above the ground, as shown in Figure 3.

The ball is modelled as a particle projected with initial speed u m s^{-1} from point O on the ground at an angle α to the ground.

(a) By writing down expressions for the horizontal and vertical distances, from O of the ball t seconds after it was hit, show that

$$2 = 10\tan\alpha - \frac{50g}{u^2\cos^2\alpha}.$$

(6)

Given that $\alpha = 45°$,

(b) find the speed of the ball as it passes over the fence.

(6)

a) $(\rightarrow)$: $x = (u\cos\alpha)t$

$(\uparrow)$: $y = (u\sin\alpha)t - \frac{1}{2}gt^2$

$10 = (u\cos\alpha)t$

$t = \dfrac{10}{u\cos\alpha}$

y: $2 = (u\sin\alpha)\left(\dfrac{10}{u\cos\alpha}\right) - \dfrac{1}{2}g\left(\dfrac{10}{u\cos\alpha}\right)^2$

$2 = 10\tan\alpha - \dfrac{50g}{u^2\cos^2\alpha}$

Question 5 continued

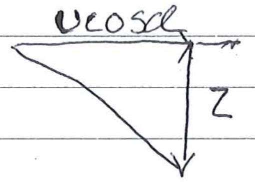
b) $2 = 10(1) - \frac{50g}{u^2(\frac{1}{\sqrt{2}})^2}$

$2 = 10 - \frac{100g}{u^2}$

$\frac{100g}{u^2} = 8$

$100g = 8u^2$

$u = \sqrt{\frac{100g}{8}}$ $\quad u = 11.1 \text{ ms}^{-1}$



$s = 2$ $\quad v^2 = u^2 + 2as$

$u = \frac{\sqrt{2}}{2}u$ $\quad z^2 = (\frac{\sqrt{2}}{2}u)^2 - 2(g)(2)$

$v = z$

$a = g$ $\quad z^2 = \frac{1}{2}u^2 - 4g$

t $\quad$ ~~$z^2 = \frac{1}{2}(11.7)^2 - 4g$~~

~~$z =$~~

$z = \sqrt{\frac{1}{2}\left(\frac{100g}{8}\right) - 4g}$

$z = \frac{21\sqrt{5}}{10}$

$\text{Speed} = \sqrt{\left(\frac{21\sqrt{5}}{10}\right)^2 + \left(\sqrt{\frac{100g}{8}} \times \frac{1}{\sqrt{2}}\right)^2}$

$= 9.13 \text{ ms}^{-1}$