Year 13 Further Mathematics Mock Set#02b
Core Pure Paper 1

- Advised to print in "A3-booklets", this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

This exam paper has 10 questions, for a total of 75 marks.

Question	Marks	Score
1	5	
2	5	
3	11	
4	13	
5	6	
6	8	
7	3	
8	7	
9	7	
10	10	
Total:	75	

Andrew Chan
Last updated: 24th February 2023

1. The equation

$$x^3 \quad 8x^2 + cx + d = 0$$

where c and d are real numbers, has roots α, β, γ.

When plotted on an Argand diagram, the triangle with vertices at α, β, γ has an area of 8.

Given that $\alpha = 2$, find the values of c and d.

Fully justify your solution.

(5)

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Question 1 continued

Question 1 continued

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Question 1 continued

(Total for Question 1 is 5 marks)

2. The cubic equation

$$9x^3 \quad 5x^2 + 4x + 7 = 0$$

has roots α, β, γ.

Without solving the equation, find the cubic equation whose roots are $(3\alpha \quad 2)$, $(3\beta \quad 2)$ and $(3\gamma \quad 2)$.
Give your answer in the form $aw^3 + bw^2 + cw + d = 0$, where a, b, c and d are integers to be determined.

(5)

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Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 5 marks)

3. Given that $z = e^{i\theta}$

(a) show that

$$z^n + \frac{1}{z^n} = 2\cos(n\theta)$$

where n is a positive integer.

(2)

(b) Show that

$$\cos^6\theta = \frac{1}{32}\left(\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta + 10\right)$$

(5)

(c) Hence solve the equation

$$\cos 6\theta + 6\cos 4\theta + 15\cos 2\theta = 0 \qquad f0 \quad \theta \quad \pi g$$

Give your answers to 3 significant figures.

(4)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 11 marks)

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4.

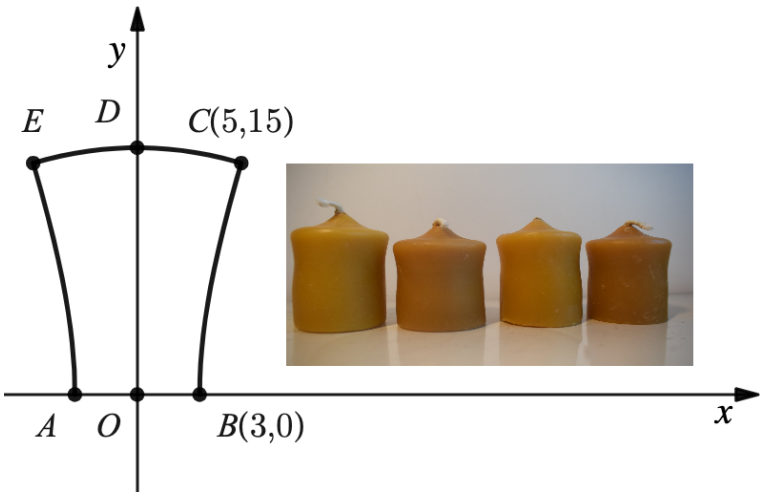

Figure 1: https://www.desmos.com/calculator/7ygzajifqg

Beeswax candles are slightly concave.

Figure 1 shows the vertical cross-section, $AOBCDE$, through the centre of a wax candle.

The point B has coordinates $(3, 0)$ and the point C has coordinates $(5, 15)$

The units are in centimetres.

The curve BC is represented by the equation

$$y = \frac{p\overline{225x^2 \quad 2025}}{a} \qquad f3 \quad x \quad 5g$$

where a is a constant.

The curve CD is represented by the equation

$$y = 16 \quad 0.04x^2 \qquad f0 \quad x \quad 5g$$

In a model, Mrs Chan rotate the region bounded by the y-axis, the line OB, the curve BC, and the curve CD through 360° about the y-axis to form a candle.

(a) Determine the value of a according to Mrs Chan's model.

(2)

(b) Use algebraic integration, determine, according to the model, the exact volume of wax that would be required to make the candle.

(9)

(c) State a limitation of the model.

(1)

When the candle was manufactured, 700 cm^3 of wax were required.

(d) Use this information and your answer to part(b) to evaluate the model, explaining your reasoning.

(1)

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Question 4 continued

Question 4 continued

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Question 4 continued

(Total for Question 4 is 13 marks)

5.

$$\mathbf{A} = \begin{pmatrix} 2 & k & 2 \\ 2 & 2 & k \\ 1 & 2 & 2 \end{pmatrix}$$

where k is a constant.

(a) Determine the values of k for which $\mathbf{A}$ is singular.

(2)

Given that $\mathbf{A}$ is non-singular,

(b) find $\mathbf{A}^{-1}$, giving your answer in terms of k.

(4)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 6 marks)

6. (a) Express $\frac{2}{r(r^2 \quad 1)}$ in partial fractions.

(3)

(b) Hence find, in terms of n,

$$\sum_{r=2}^{n} \frac{1}{r(r^2 \quad 1)}$$

Give your answer in the form

$$\frac{n^2 + An + B}{Cn(n+1)}$$

where A, B and C are constants to be found.

(5)

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Question 6 continued

Question 6 continued

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Question 6 continued

(Total for Question 6 is 8 marks)

7. Evaluate the improper integral

$$\int_{2}^{1} 3e^{(4\ \ 2x)}\,dx$$

(3)

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Question 7 continued

(Total for Question 7 is 3 marks)

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8. (a) Write $\dfrac{4}{4x+1} \quad \dfrac{3}{3x+2}$ in the form $\dfrac{C}{(4x+1)(3x+2)}$, where C is a constant.

(1)

(b) Evaluate the improper integral

$$\int_1^{\infty} \frac{10}{(4x+1)(3x+2)}\,\mathrm{d}x$$

showing the limiting process used and giving your answer in the form $\ln k$, where k is a constant.

(6)

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Question 8 continued

Question 8 continued

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Question 8 continued

(Total for Question 8 is 7 marks)

9. Given that $y = \text{artanh}\,(\cos x)$

(a) show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \quad \text{cosec}\, x$$

(2)

(b) Hence find the exact value of

$$\int_0^{\frac{\pi}{6}} \cos x \quad \text{artanh}\,(\cos x)\,\mathrm{d}x$$

giving your answer in the form $a \ln\left(b + c\sqrt{3}\right) + d\pi$, where a, b, c and d are rational numbers to be found.

(5)

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Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 7 marks)

10.

Figure 2

Distilled Water and antifreeze are being mixed together in a tank, as shown in Figure 2.

The mixture of distilled water and antifreeze are assumed to be instantly dispersed evenly throughout the tank.

Initially the tank holds a mixture of 8 litres of distilled water and 2 litres of antifreeze, so that the concentration of antifreeze in the mixture is said to be 20%.

The concentration of antifreeze in the mixture is now increased by

- adding distilled water to the tank at a rate of 0.1 litres per second
- adding antifreeze to the tank at a rate of 0.3 litres per second
- pumping the mixture from the tank at a rate of 0.4 litres per second

Let x litres be the amount of antifreeze in the tank at time t seconds after the mixture starts to be altered.

(a) Show that the change in the amount of antifreeze in the tank can be modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.3 \quad \frac{x}{k}$$

where k is a positive constant to be determined.

(2)

(b) By solving the differential equation in part (a), determine how long it will take for the concentration of antifreeze in the mixture to reach 40%, according to the model.

Give your answer to the nearest tenth of a second.

(6)

As t becomes large, the concentration of antifreeze in the mixture approaches c%, where c is a constant.

(c) Find the value of c.

(2)

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Question 10 continued

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Question 10 continued

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Question 10 continued

Question 10 continued

(Total for Question 10 is 10 marks)

Total for paper is 75 marks

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