13Fm Further Pure (FP1) Mock Paper 2021.05.26

- Advised to print in "A3-booklets", this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

This exam paper has 9 questions, for a total of 75 marks.

Question	Marks	Score
1	6	
2	6	
3	7	
4	7	
5	8	
6	13	
7	11	
8	4	
9	13	
Total:	75	

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Last updated: 24th February 2023

1. The rectangular hyperbola H has equation $xy = 9$

The line l_1 with equation $y = mx + c$ is a tangent to H at the point P.

(a) Show that

$$c^2 + km = 0$$

where k is a constant to be determined.

(2)

The parabola C has equation $y^2 = 32x$.

The directrix of C crosses the x-axis at the point Q.

The line l_1 passes through Q and the line l_2 is the normal to H at the point P.

(b) Determine the exact gradient of l_2

(4)

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Question 1 continued

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Question 1 continued

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Question 1 continued

(Total for Question 1 is 6 marks)

2. The height, y kilometres, of a rocket above horizontal ground is modelled by the differential equation

$$2\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + y\frac{\mathrm{d}y}{\mathrm{d}t} - 5 = 0$$

where t is the time in minutes after the rocket is launched.

Given that three minutes after the launch, the rocket is at a height of 5 km and is ascending vertically at a rate of 0.72 kilometres per minute,

(a) find the vertical acceleration of the rocket three minutes after the launch.

(2)

(b) Use each of the approximations

$$\left(\frac{\mathrm{d}^2y}{\mathrm{d}t^2}\right)_n \approx \frac{y_{n+1} - 2y_n + y_{n-1}}{h^2} \text{ and } \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)_n \approx \frac{y_{n+1} - y_{n-1}}{2h}$$

once to estimate the height of the rocket above the ground 3 minutes and 15 seconds after the launch.

(4)

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Question 2 continued

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 6 marks)

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3. Use algebra to obtain the range of values of x for which

$$|x^2 + x - 2| < \frac{1}{2}(x + 5)$$

leave your answers in set notation.

(7)

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Question 3 continued

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Question 3 continued

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Question 3 continued

(Total for Question 3 is 7 marks)

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4.

$$\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} = 2\cos x$$

(a) Express $\frac{d^3y}{dx^3}$ in terms of x, $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$

(3)

Given that at $x = 0$, $y = 2$ and $\frac{dy}{dx} = 5$

(b) Determine the value of $\frac{d^3y}{dx^3}$ at $x = 0$

(1)

(c) Express y as a series in ascending powers of x, up to and including the term in x^3

(3)

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Question 4 continued

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 7 marks)

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5.

(a) Show that the substitution $y^2 = \frac{1}{z}$ transforms the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = 3xy^3 \qquad y \neq 0 \qquad \text{(I)}$$

into the differential equation

$$\frac{\mathrm{d}z}{\mathrm{d}x} - 4z = -6x \qquad \text{(II)}$$

(3)

(b) Obtain the general solution of differential equation (II)

(5)

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Question 5 continued

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Question 5 continued

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Question 5 continued

(Total for Question 5 is 8 marks)

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6. The coordinates of the points A, B and C relative to a fixed origin O are $(1, 2, 3)$, $(-1, 3, 4)$ and $(2, 1, 6)$ respectively. The plane Π contains the points A, B and C.

(a) Find a Cartesian equation of the plane Π.

(5)

The point D has coordinates $(k, 4, 14)$ where k is a positive constant.

Given that the volume of the tetrahedron $ABCD$ is 6 cubic units,

(b) find the value of k.

(4)

$$\mathbf{M} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 2 & 4 \end{pmatrix}$$

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix $\mathbf{M}$.

The equation of l_2 is

$$\Big(\mathbf{r} - (12\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})\Big) \times (-6\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}) = 0$$

(c) Find a vector equation for the line l_1

(4)

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 13 marks)

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7. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

The line l_1 is a tangent to E at the point $P(a\cos\theta, b\sin\theta)$.

(a) Using calculus, show that the equation for l_1 is

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$

(4)

The circle C has equation

$$x^2 + y^2 = a^2$$

The line l_2 is a tangent to C at the point $Q(a\cos\theta, a\sin\theta)$.

(b) Find an equation for the line l_2

(2)

Given that l_1 and l_2 meet at the point R,

(c) find, in terms of a, b and θ, the coordinates of R.

(3)

(d) Find the locus of R, as θ varies.

(2)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 11 marks)

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8. Given that k is a real non-zero constant and that

$$y = x^3 \sin kx$$

use Leibnitz's theorem to show that

$$\frac{\mathrm{d}^5 x}{\mathrm{d}x^5} = \left(k^2x^2 + A\right)k^3 x \cos kx + B\left(k^2x^2 + C\right)k^2 \sin kx$$

where A, B and C are integers to be determined.

(4)

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Question 8 continued

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 4 marks)

9. (a) Show that an indeterminate form results when the evaluation of

$$\lim_{x \to 5} \left(\frac{-4 + \sqrt{1 + 3x}}{x^4 - 24x^2 - 25} \right)$$

is attempted by direct substitution of $x = 5$

(1)

(b) Find the Taylor series expansion of $\sqrt{1 + 3x}$ in ascending powers of $(x - 5)$ up to and including the term in $(x - 5)^2$, giving all coefficients in their simplest form.

(5)

(c) Hence determine the exact value of

$$\lim_{x \to 5} \left(\frac{-4 + \sqrt{1 + 3x}}{x^4 - 24x^2 - 25} \right)$$

(4)

(d) Showing your method clearly, use L'Hospital's rule to verify your answer to part (c).

(3)

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Question 9 continued

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Question 9 continued

(Total for Question 9 is 13 marks)

Total for paper is 75 marks

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