Year 13 Mathematics Mock Set#03c
Mechanics Paper
Year 13 final revision session 2023

- Advised to print in "A3-booklets", this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

This exam paper has 5 questions, for a total of 50 marks.

Question	Marks	Score
1	7	
2	10	
3	11	
4	13	
5	9	
Total:	50	

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1. A particle P is moving with constant acceleration $(-4\mathbf{i} + \mathbf{j})$ ms^{-2}.
At time $t = 0$, P has velocity $(14\mathbf{i} - 5\mathbf{j})$ ms^{-1}.

(a) Find the size of the angle between the direction of $\mathbf{i}$ and the direction of motion of P at time $t = 2$ seconds.

(3)

At time $t = T$ seconds, P is moving in the direction of vector $(2\mathbf{i} - 3\mathbf{j})$

(b) Find the value of T.

(4)

either suvat or diff/Int:

$a = \begin{pmatrix} -4 \\ 1 \end{pmatrix} \quad \int a\,dt = \begin{matrix} -4t + C_1 \\ t + C_2 \end{matrix} = v$

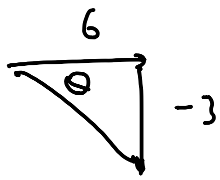
$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = \begin{pmatrix} 14 \\ -5 \end{pmatrix}$

$v = \begin{pmatrix} 14-4t \\ -5+t \end{pmatrix}$

or $v = u + at$

$= \begin{pmatrix} 14 \\ -5 \end{pmatrix} + \begin{pmatrix} -4 \\ 1 \end{pmatrix} t = \begin{pmatrix} 14-4t \\ -5+t \end{pmatrix}$

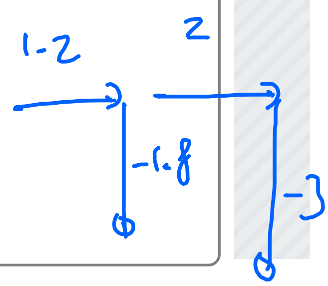
at time $t=2$, $v = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$



$\theta = 26.6°$

b) $\begin{pmatrix} 14-4t \\ -5+t \end{pmatrix} /\!/ \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ $\quad \nearrow \begin{pmatrix} 1.2 \\ -1.8 \end{pmatrix}$

$\dfrac{14-4t}{2} = \dfrac{-5+t}{-3} \qquad t = \dfrac{16}{5} = 3.2$ //



2. [In this question, the perpendicular unit vectors **i** and **j** are in a horizontal plane.]

A particle Q of mass 1.5 kg is moving on a smooth horizontal plane under the action of a single force **F** newtons.

At time t seconds ($t \geq 0$), the position vector of Q, relative to a fixed point O, is **r** metres and the velocity of Q is **v** ms^{-1}.

Given that

$$\mathbf{v} = (3t^2 + 2t)\mathbf{i} + (t^3 + kt)\mathbf{j}$$

where k is a constant.

Given also that when $t = 2$, particle Q is moving in the direction of the vector $\mathbf{i} + \mathbf{j}$.

(a) Show that $k = 4$.

(2)

(b) Find the magnitude of **F** when $t = 2$.

(4)

Given that $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j}$ when $t = 0$,

(c) find **r** when $t = 2$.

(4)

a)

$$v = \begin{pmatrix} 3t^2+2t \\ t^3+kt \end{pmatrix}$$

$t=2 \quad v \parallel \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$$v = \begin{pmatrix} 16 \\ 8+2k \end{pmatrix} \parallel \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\frac{16}{1} = \frac{8+2k}{1} \quad \Rightarrow \quad k=4$$

b) $F = ma \quad \Rightarrow \quad a = \begin{pmatrix} 6t+2 \\ 3t^2+k \end{pmatrix}$

$$t=2 \quad a = \begin{pmatrix} 12+2 \\ 12+4 \end{pmatrix} = \begin{pmatrix} 14 \\ 16 \end{pmatrix}$$

$$F = 1.5\begin{pmatrix} 14 \\ 16 \end{pmatrix}$$

$$F = \begin{pmatrix} 21 \\ 24 \end{pmatrix} \qquad |F| = 3\sqrt{113} \simeq 31.9 \text{ N}$$

Question 2 continued

Given that $\mathbf{r} = 3\mathbf{i} + 4\mathbf{j}$ when $t = 0$,

(c) find $\mathbf{r}$ when $t = 2$.

$$v = \begin{pmatrix} 3t^2 + 2t \\ t^3 + kt \end{pmatrix}$$

$$r_0 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$$

$$\int v \, dt = r = \begin{pmatrix} t^3 + t^2 + C_1 \\ \frac{t^4}{4} + \frac{4t^2}{2} + C_2 \end{pmatrix}$$

$$r = \begin{pmatrix} t^3 + t^2 + 3 \\ \frac{t^4}{4} + 2t^2 + 4 \end{pmatrix}$$

$$r = \begin{pmatrix} 15 \\ 16 \end{pmatrix} // = \underline{\underline{15i + 16j}}$$

3.

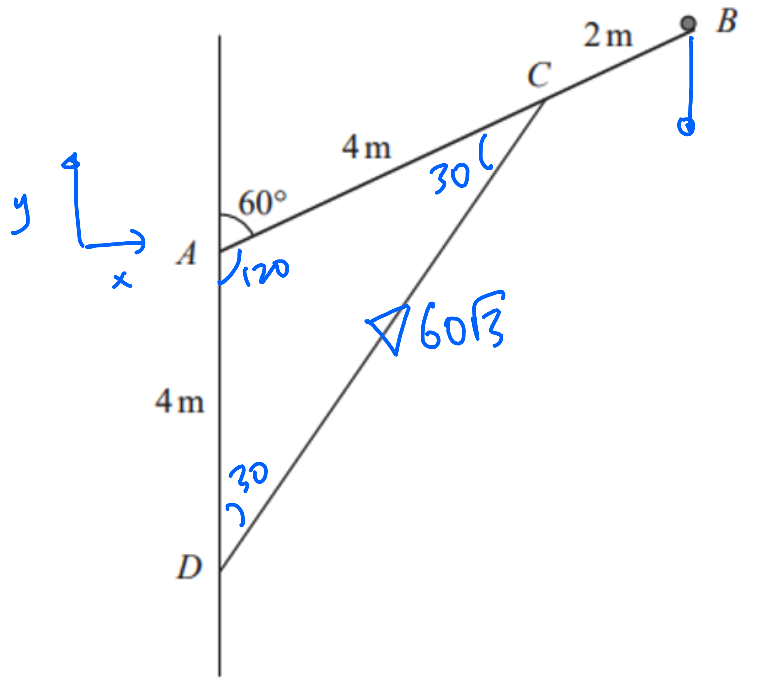


Figure 1

A uniform pole AB, of weight 50 N and length 6 m.
A particle of weight W newtons is attached at one end B, the other end A is freely hinged to a vertical wall.
A light rod holds the particle and pole in equilibrium with the pole at 60° to the wall.
One end of the light rod is attached to the pole at C, where $AC = 4$ m, the other end of the rod is attached to the wall at the point D.
The point D is vertically A with $AD = 4$ m, as shown in Figure 1.
The pole and the light rod lie in a vertical plane which is perpendicular to the wall.
The pole is modelled as a rod.
Given that the thrust in the light rod is $60\sqrt{3}$ N.

(a) Show that $W = 15$

(4)

(b) (i) the horizontal component of the force exerted by the hinge on the rod at A

(ii) the vertical component of the force exerted by the hinge on the rod at A

(5)

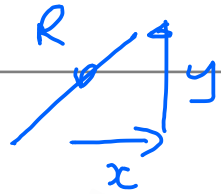
(c) Hence,

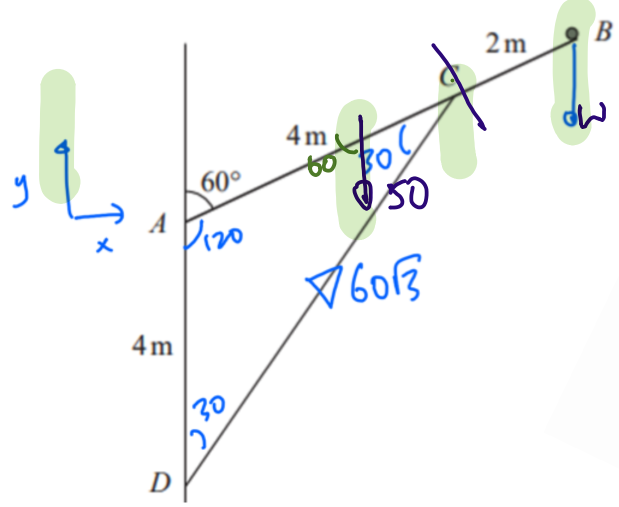
(i) find the magnitude of the resultant force acting on the pole at A.

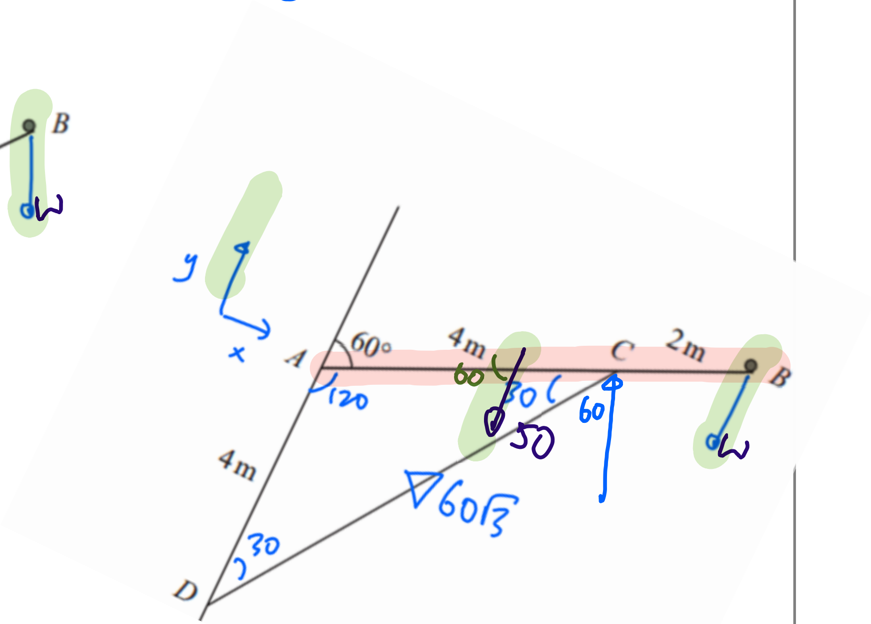
(ii) find the direction of the resultant force acting on the pole at A.

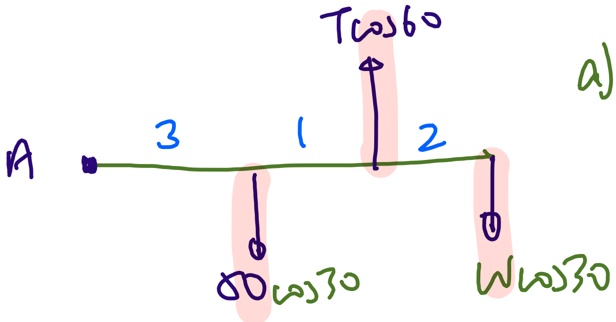
(2)

Question 3 continued









a)

$M(A) = 3(50\cos 30) + 6(W\cos 30) = 4(60\sqrt{3})\cos 60$

$W = 15$ //

$\uparrow y \quad \downarrow 50 \quad \downarrow W \quad \uparrow 60\sqrt{3}\cos 30$

$\vec{x} \quad \overrightarrow{60\sqrt{3}\sin 30}$

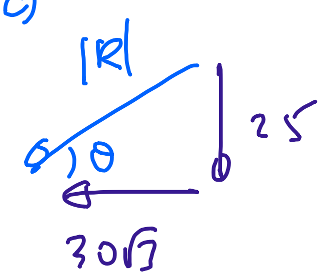
b)

$y + 60\sqrt{3}\cos 30 = 50 + W$

$y = -25$

$X = -30\sqrt{3}$

c)



$\theta = 25.7°$ //

with horizontal, downward

$|R| = 5\sqrt{133}$ //

4.

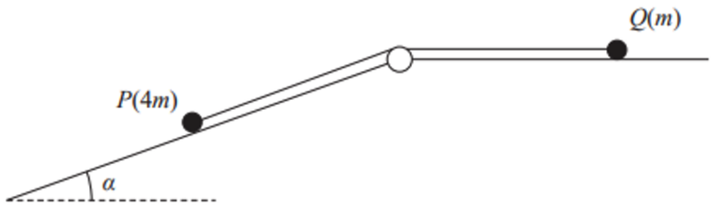


Figure 2

A particle of mass $4m$ lies on the surface of a fixed rough inclined plane.

The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$.

The particle P is attached to one end of a light inextensible string.

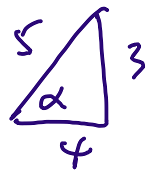
The string passes over a small smooth pulley that is fixed at the top of the plane.
The other end of the string is attached to a particle Q of mass m which lies on a smooth horizontal plane.

The string lies along the horizontal plane and in the vertical plane that contains the pulley and a line of greatest slope of the inclined plane.

The system is released from rest with the string taut, as shown in Figure 2, and P moves down the plane.

The coefficient of friction between P and the plane is $\frac{1}{4}$.

For the motion before Q reaches the pulley,



(a) write down an equation of motion for Q,

(1)

(b) find, in terms of m and g, the tension in the string,

(7)

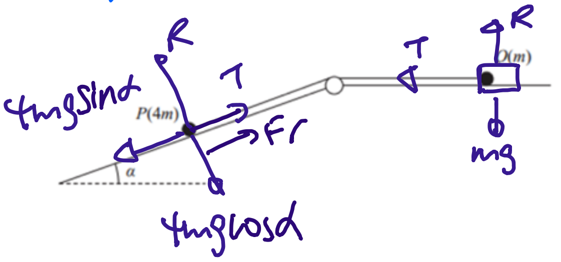
(c) find the magnitude of the force exerted on the pulley by the string.

(4)

(d) State where in your working you have used the information that the string is light.

(1)

a) $\mu = \frac{1}{4}$ $\tan\alpha = \frac{3}{4}$



a)

$Q: T = ma$

b)

$4mg\sin\alpha - T - F_r = 4ma$

$R = 4mg\cos\alpha$

$F_r = \frac{1}{4}(4mg\cos\alpha)$

$= mg\left(\frac{4}{5}\right)$

Question 4 continued

$$\therefore 4mg\left(\tfrac{3}{5}\right) - \tfrac{4}{5}mg = T + 4ma \quad -(1)$$

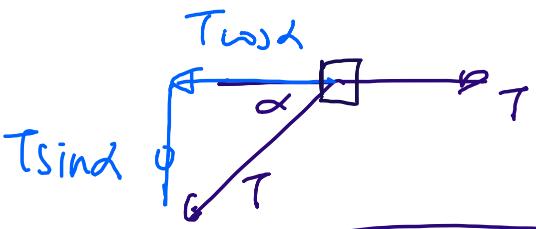
$$0 = T - ma \quad -(2)$$

$$5ma = 0.32mg$$

$$a = \frac{8}{25}g$$

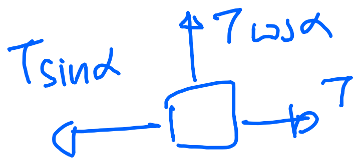
$$T = \frac{8}{25}mg$$

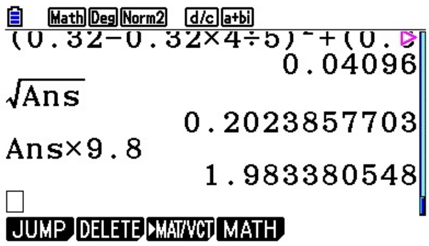
c)



$$\sqrt{(T - T\cos\alpha)^2 + (T\sin\alpha)^2}$$

$$= 1.98\ m$$





d) tension is equal in magnitude

5. A particle P is projected from a fixed point O on a horizontal ground.
The particle is projected with speed u at an angle α above the horizontal.
At the instant when the horizontal distance of P from O is x, the vertical distance of P above the ground is y.
The motion of P is modelled as that of a particle freely under gravity.

(a) Show that

$$y = x\tan\alpha - \frac{gx^2}{2u^2}(1+\tan^2\alpha)$$

(6)

A small ball is projected from a fixed point O on horizontal ground.
The ball is projected with speed 20 ms^{-1} at angle $\theta°$ above the horizontal.
A vertical pole AB, of height 2 m, stands on the ground with $OA = 10$ m, as shown in Figure 3.

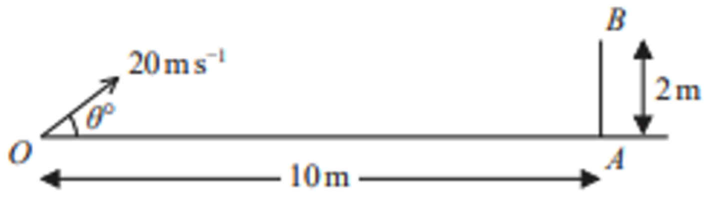


Figure 3

- The path of the ball lies in the vertical plane containing O, A and B.
- The ball is modelled as a particle moving freely under gravity
- The pole is modelled as a rod.

Using the model,

(b) find the range of values of θ for which the ball will pass over the pole.

(3)

5a) $\rightarrow$ $s = x$, $u = u\cos\alpha$, $v = u\cos\alpha$, $a = 0$, $t = t$

$\uparrow$ $s = y$, $u = u\sin\alpha$, $v = ?$, $a = -g$, $t = t$

$s = ut + \frac{1}{2}at^2$

$\Rightarrow$ $x = u\cos\alpha\, t$ ① $\quad y = u\sin\alpha\, t - \frac{g}{2}t^2$ ②

Sub ① into ②

$y = u\sin\alpha\left(\frac{x}{u\cos\alpha}\right) - \frac{g}{2}\left(\frac{x}{u\cos\alpha}\right)^2$

Question 5 continued

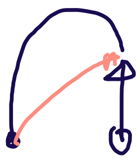
$$y = x\tan\alpha - \frac{gx^2}{2u^2}\left(\frac{1}{\cos^2\alpha}\right)$$

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\left(\sec^2\alpha\right)$$

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\left(\tan^2\alpha + 1\right)$$

b) $y = 2,\ u = 20,\ x = 10$

$\boxed{\alpha = \theta}$



anything in between

$$2 = 10\tan\theta - \frac{g(10^2)}{2(20^2)}\left(1 + \tan^2\theta\right)$$

$$2 = 10\tan\theta - \frac{49}{40}\left(1 + \tan^2\theta\right)$$

$$\frac{49}{40}\tan^2\theta - 10\tan\theta + 2 + \frac{49}{40} = 0$$

```
Math Rad Norm2  d/c a+bi
aX²+bX+c=0
X1 [7.8269]
X2 [0.3363]
            7.826905948
REPEAT
```

```
Math Rad Norm2  d/c a+bi
aX²+bX+c=0
X1 [7.8269]
X2 [0.3363]
            7.826905948
REPEAT
```

```
Math Deg Norm2  d/c a+bi
Ply Result
        [7.826905948 ]
        [0.3363593582]
Mat Ans[1,1]→A
            7.826905948
tan⁻¹ A
            82.71908495
JUMP DELETE MAT/VCT MATH
```

```
Math Deg Norm2  d/c a+bi
tan⁻¹ A
            82.71908495
Mat Ans[2,1]→B
           0.3363593582
tan⁻¹ B
            18.59084752
JUMP DELETE MAT/VCT MATH
```

$\tan\theta = 7.8269\ldots$

$\theta = 82.7190\ldots$

or

$\tan\theta = 0.3363\ldots$

$\theta = 18.5908\ldots$

$$18.6^\circ < \theta < 82.7^\circ \quad (3sf)$$ //