Year 12 Full AS Pure Mock Dec 2022

This exam has 14 questions, for a total of 100 marks.

- Print in pdf "booklets" will allow all questions to be on the left hand side.
- If instead printing in "2 in 1" settings, then first print from page 2 up to the last page, then print page 1 separately.

Question	Marks	Score
1	5	
2	7	
3	6	
4	7	
5	7	
6	7	
7	10	
8	6	
9	5	
10	11	
11	8	
12	8	
13	9	
14	4	
Total:	100	

Andrew Chan
13th February 2023

In this question you should show all stages of your working.
Solutions relying on calculator technology are not acceptable.

1. Find

$$\int \left(\frac{4\sqrt{x} - 3}{2x^2} \right) \mathrm{d}x \qquad x > 0$$

Give your answer in simplest form.

(5)

Question 1 continued

(Total for Question 1 is 5 marks)

2.

$$f(x) = (x^2 - 2)(2x - 3) - 21$$

(a) Use the factor theorem to show that $(x - 3)$ is a factor of $f(x)$

(2)

(b) Hence,

(i) factorise $f(x)$

(ii) show that the equation $f(x) = 0$ has only one real root.

(5)

Question 2 continued

(Total for Question 2 is 7 marks)

3. The triangle PQR is such that $\overrightarrow{PQ} = 3\mathbf{i} + 5\mathbf{j}$ and $\overrightarrow{PR} = 13\mathbf{i} - 15\mathbf{j}$

(a) Find $\overrightarrow{QR}$

(2)

(b) Hence find $|\overrightarrow{QR}|$, giving your answer as a simplified surd.

(2)

The point S lies on the line segment QR so that $QS : SR = 3 : 2$

(c) Find $\overrightarrow{PS}$

(2)

Question 3 continued

(Total for Question 3 is 6 marks)

4.

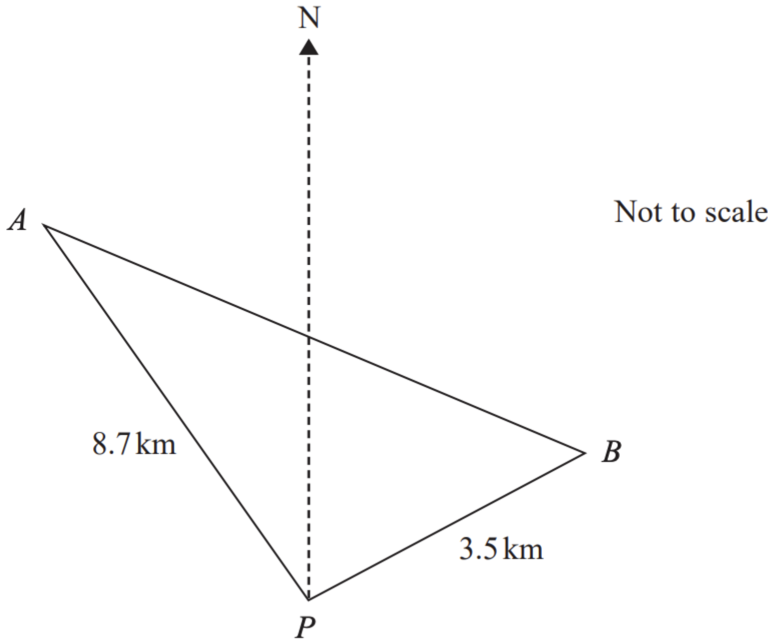

Figure 1

Figure 1 shows the position of two stationary boats, A and B, and a port P which are assumed to be in the same horizontal plane.

- Boat A is 8.7 km on a bearing of 314° from port P.
- Boat B is 3.5 km on a bearing of 052° from port P.

(a) Show that $\angle APB = 98°$

(1)

(b) Find the distance of boat B from boat A, giving your answer to one decimal place.

(2)

(c) Find the bearing of boat B from boat A, giving your answer to one decimal place.

(4)

Question 4 continued

(Total for Question 4 is 7 marks)

5. The percentage, P, of the population of a small country who have access to the internet, is modelled by the equation

$$P = ab^t$$

where a and b are constants and t is the number of years after the start of 2005.

Using the data for the years between the start of 2005 and the start of 2010, a graph is plotted of $\log_{10} P$ against t.

The points are found to lie approximately on a straight line with gradient 0.09 and intercept 0.68 on the $\log_{10} P$ axis.

(a) Find, according to the model, the value of a and the value of b.
Give your answer to 2 decimal places.

(4)

(b) In the context of the model, give a practical interpretation of the constant a.

(1)

(c) Use the model to estimate the percentage of the population who had access to the internet at the start of 2015

(2)

Question 5 continued

(Total for Question 5 is 7 marks)

Turn over

6. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{kx}{4}\right)^8$$

where k is a non-zero constant. Give your answer in simplest form.

(4)

$$f(x) = (5 - 3x)\left(2 - \frac{kx}{4}\right)^8$$

In the expansion of $f(x)$, the constant term is 3 times the coefficient of x.

(b) Find the value of k.

(3)

Question 6 continued

(Total for Question 6 is 7 marks)

7. **In this question you should show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation $y = f(x)$ where

$$f(x) = 2(x+1)(x-3)^2$$

(a) Sketch a graph of C
Show on your graph the coordinates of the points where C cuts or meets the coordinate axes.

(3)

(b) Write $f(x)$ in the form $ax^3 + bx^2 + cx + d$, where a, b, c and d are constants to be found.

(3)

The line l has equation $y = k$, where k is a constant.
Given that C and l intersect at 3 distinct points,

(c) find the range of values for k, writing your answer in set notation.

(4)

Question 7 continued

(Total for Question 7 is 10 marks)

8. **In this question you should show all stages of your working. Solutions relying on calculator technology are not acceptable.**

The air pressure, P kg/cm^2, inside a car tyre, t minutes from the instant when the tyre developed a puncture is given by the equation

$$P = k + 1.4e^{-0.5t} \qquad t \geq 0$$

where k is a constant.

Given that the initial air pressure inside the tyre was 2.2 kg/cm^2,

(a) state the value of k.

(1)

(b) Hence find the rate at which the air pressure in the tyre is decreasing exactly 2 minutes from the instant when the tyre developed the puncture.

Give your answer in kg/cm^2 per minute to 3 significant figures.

(2)

From the instant when the tyre developed the puncture,

(c) find the time taken for the air pressure to fall to 1 kg/cm^2.

Give your answer in minutes to one decimal place.

(3)

Question 8 continued

(Total for Question 8 is 6 marks)

9. Solve

$$\log_3(32 - 12x) = 2\log_3(1 - x) + 3$$

(5)

Question 9 continued

(Total for Question 9 is 5 marks)

10. **In this question you should show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Figure 2 shows a sketch of part of the curve C with equation

$$y = \frac{3}{4}x^2 - 4\sqrt{x} + 7 \qquad x > 0$$

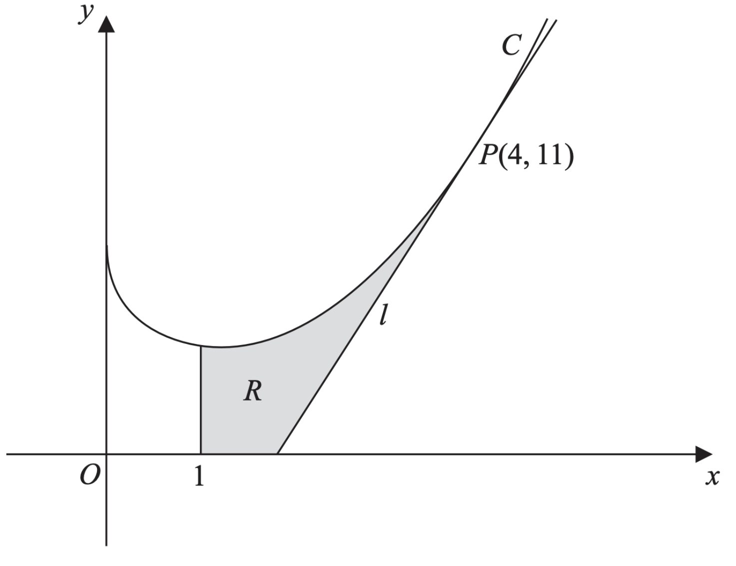

Figure 2

- Point $P(4,11)$ lies on C
- Line l is a tangent to C at the point P

(a) Use calculus to show that l has equation $5x - y - 9 = 0$

(5)

The finite region R, shown in Figure 2, is bounded by the curve C, the line $x = 1$, the x-axis and the line l.

(b) Use calculus to find the area of R, giving your answer to 2 decimal places.

(6)

Question 10 continued

(Total for Question 10 is 11 marks)

11.

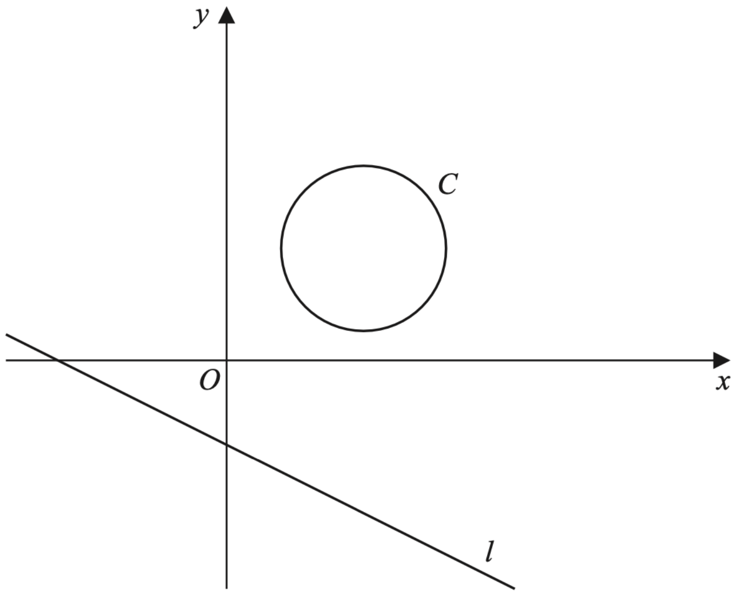

Figure 3

Figure 3 shows the circle C with equation

$$x^2 + y^2 - 10x - 8y + 32 = 0$$

and the line l with equation

$$2y + x + 6 = 0$$

(a) Find

(i) the coordinates of the centre of C

(ii) the radius of C

(3)

(b) Find the shortest distance between C and l.

(5)

Question 11 continued

(Total for Question 11 is 8 marks)

12. A company makes a particular type of watch.

The annual profit made by the company from sales of these watches is modelled by the equation

$$P = 12x - x^{\frac{3}{2}} - 120$$

where P is the annual profit measured in thousands of pounds and £x is the selling price of the watch.

According to this model,

(a) find, using calculus, the maximum possible annual profit.

(6)

(b) Justify, also using calculus, that the profit you have found is a maximum.

(2)

Question 12 continued

(Total for Question 12 is 8 marks)

13. **In this question you should show all stages of your working. Solutions relying on calculator technology are not acceptable.**

(i) A student's attempt to solve a question is set out below:

"Solve for $-90° < x < 90°$, the equation $3\tan x - 5\sin x = 0$"

$$3\tan x - 5\sin x = 0 \qquad (1)$$
$$3\left(\frac{\sin x}{\cos x}\right) - 5\sin x = 0 \qquad (2)$$
$$3\sin x - 5\sin x\cos x = 0 \qquad (3)$$
$$3 - 5\cos x = 0 \qquad (4)$$
$$\cos x = \frac{3}{5} \qquad (5)$$
$$x = 53.1° \qquad (6)$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

(2)

(ii) The first four positive solutions, in order of size, of the equation

$$\cos(5x + 40°) = \frac{3}{5}$$

are x_1, x_2, x_3 and x_4

Find, to the nearest degree, the value of x_4

(2)

(iii) Solve, for $0 \leq x \leq 450°$, the equation

$$5\cos^2 x = 6\sin x$$

giving your answers to one decimal place.

(5)

Question 13 continued

(Total for Question 13 is 9 marks)

14. Given $n \in \mathbb{N}$, prove, by exhaustion, that $n^2 + 2$ is not divisible by 4.

(4)

Question 14 continued

Question 14 continued

Question 14 continued

Question 14 continued

(Total for Question 14 is 4 marks)

–Total for paper is 100 marks–

–End of Paper–