1 It is known that 20% of plants of a certain type suffer from a fungal disease, when grown under normal conditions. Some plants of this type are grown using a new method. A random sample of 250 of these plants is chosen, and it is found that 36 suffer from the disease. Test, at the 2% significance level, whether there is evidence that the new method reduces the proportion of plants which suffer from the disease. **[7]**

2 It is known that under the standard treatment for a certain disease, 9.7% of patients with the disease experience side effects within one year. In a trial of a new treatment, 450 patients with this disease were selected and the number, X, that experienced side effects within one year was noted.

It was found that 51 of the 450 patients experienced side effects within one year.

(a) Test, at the 10% significance level, whether the proportion of patients experiencing side effects within one year is greater under the new treatment than under the standard treatment. **[7]**

(b) It was later discovered that all 450 patients selected for the trial were treated in the same hospital. Comment on the validity of the model used in part **(a)**. **[1]**

3 **(a)** André throws a fair six-sided dice 30 times. The number of throws on which the score is six is denoted by X.

(i) State a suitable model for X, including the values of any parameters. **[1]**

(ii) Find $P(X = 9)$. **[1]**

(iii) Find $P(X \geq 9)$. **[2]**

(b) André has another six-sided dice. He suspects that this dice is biased so that it is more likely to show a six than if it were fair. He throws the dice 30 times and it shows a six on 9 throws. Test at the 5% significance level whether André's suspicion is justified. **[6]**

4 It is known that, under the standard treatment for a certain disease, 9.7% of patients with the disease experience side effects within one year.

In a trial of a new treatment, a random sample of 450 patients with this disease was selected and the number X who experienced side effects within one year was noted.

It was found that 51 of the 450 patients experienced side effects within one year.

Test, at the 10% significance level, whether the proportion of patients experiencing side effects within one year is greater under the new treatment than under the standard treatment. **[7]**

5(a) Some packets of a certain kind of biscuit contain a free gift. The manufacturer claims that the proportion of packets containing a free gift is 1 in 4. Marisa suspects that this claim is not true, and that the true proportion is less than 1 in 4. She chooses 20 packets at random and finds that exactly 1 contains the free gift.

Use a binomial model to test the manufacturer's claim, at the 2.5% significance level. **[7]**

(b) The packets are packed in boxes, with each box containing 40 packets. Marisa chooses three boxes at random and finds that one box contains 19 packets with the free gift and the other two boxes contain no packets with the free gift.

Give a reason why this suggests that the binomial model used in part **(a)** may not be appropriate. **[1]**

END OF QUESTION PAPER

Question	Answer/Indicative content	Marks	Guidance	
1	H_0: p = 0.2 where p = P(A plant gets disease)	B1 (AO1.1)	Allow "possibility" or "proportion". Not p = %age having disease	**If 2-tail test:** H_o: p = 0.2 (defined p) B1
	H_1: $p < 0.2$ (not $p \leq 0.2$)	B1 (AO2.5)	Undefined p: B1B0	H_1: $p \neq 0.2$ B0 M1 A1
	$X \sim$ B(250, 0.2) and X = 36 (allow 35)	M1 (AO3.3)	Stated or implied eg by 0.0139 (or 0.00884)	
	P(X ≤ 36) = 0.0139 or 0.014	A1 (AO3.4)	cao **BC**	
	0.0139 < 0.02	A1f (AO1.1)	NB dep attempt P(X ≤ 36) ft their P(X ≤ 36) (< 0.02) Must see this statement	0.0139 > 0.01 A1 No more marks
	Reject H_0 (Allow Accept H_1)	M1 (AO2.2b)	NB dep attempt P(X ≤ 36) or P(X < 36) and dep comp 0.02, ft their P($X \leq$ 36), possibly not reject H_0	
	There is evidence that new method reduces prop of diseased plants	A1f (AO3.5a)	In context, not definite ft only their P($X \leq$ 36) or P(X < 36) possibly "no evidence.." Ignore all else P(X < 36): max B1B1M1A0A0M1A1	
		[7]	<u>Examiner's Comments</u> Many candidates had clearly been well prepared for a hypothesis test question. However, even these often made errors. Examples of such errors were as follows. • Failure to define "p" in the hypotheses • H1: p ≠ 0.02	

Question			Answer/Indicative content	Marks	Guidance
					• Finding P(X < 36) instead of P(X ≤ 36) • Finding P(X = 36) instead of P(X ≤ 36) • Writing P(X ≤ 0.2) instead of P(X ≤ 36) • Comparing the probability with 0.2 instead of 0.02 • Omitting to state "Reject H0" in the conclusion • Giving a definite conclusion such as "Reject Ho. The new method reduces the proportion . . . ." • Stating that because 0.0139 < 0.02, we do not reject H_0 There was a significant minority that could not access this question on Hypothesis Testing.
			Total	7	

Question			Answer/Indicative content	Marks	Guidance	
2		a	$H_0 : p = 0.097$ $H_1 : p > 0.097$ where p is the proportion of patients experiencing side effects within a year	B1 (AO1.1) B1 (AO2.5)	Must be stated in terms of parameters Undefined p B1B0	
			$X \sim B(450, 0.097)$ and $X = 50$	M1 (AO3.3)	Stated or implied	Only 0.138 seen without parameters/ distribution
			$P(X \geq 51) = 1 - 0.862 = 0.138$(3 s.f.)	A1 (AO3.4)	BC	M1AO
			Comparison with 0.1 Do not reject H_0	A1 (AO1.1) M1 (AO1.1)		
			No evidence (at 10% level) that proportion under new treatment greater than under standard treatment	A1 (AO2.2b) [7]	In context, not definite, e.g. Proportion not greater A0	FT their 0.138, but not comparison with 0.1
		b	E.g. The patients could be treated together so they are not independent, so the binomial model is not valid. E.g. The 450 patients are not a random sample from the population, so the binomial model is not valid. E.g. It is not known whether the proportion of patients experiencing side effects under the standard treatment is 9.7%, so the binomial model used may not be valid.	B1 (AO3.5a) [1]	In context, referring to independence or random sampling. Must include a comment on appropriateness.	
			Total	8		

Question			Answer/Indicative content	Marks	Guidance	
3		a	i) $B(30, \frac{1}{6})$	B1(AO3.3) [1]		
		a	ii) 0.0309	B1(AO1.1) [1]	BC	
		a	iii) $1 - P(X \leq 8)$ = 0.0506	M1(AO3.4) A1(AO1.1) [2]		
		b	$H_0: p = \frac{1}{6}$ where $p = P(\text{score is } 6)$ $H_1: p > \frac{1}{6}$ $P(X \geq 9)$ = '0.0506' or their (a)(ii) comp 0.05 Not reject H_0 No evidence that dice biased towards 6	B1(AO1.1) B1(AO2.5) B1(AO3.4) A1(AO1.1) M1(AO2.2b) A1f(AO3.5a) [6]	Undefined *p*: B0B1 BC ft their (a)(ii) ft their (a)(ii) In context, not definite	dice is unbiased B0 dice is biased towards 6 B1
			Total	**10**		

Question			Answer/Indicative content	Marks	Guidance	
4			H_0: $p = 0.097$ where p is the proportion of patients experiencing side-effects (within a year) or 9.7% of patients experience side-effects	B1(AO1.1)	Allow "possibility" or, "probability" Not p = percentage having disease	If 2-tail test: H_o: $p = 0.097$ B1 (defined p)
			H_1: $p > 0.097$ or > 9.7% exp side-effects	B1(AO2.5)	One error, eg undefined p, p = 9.7% B1B0	H_1: $p \neq 0.097$ B0
			B(450, 0.097) & $X = 50$ (allow 51)	M1(AO3.3)	stated or implied eg by 0.138 or 0.107 or 0.108 or 0.862 or 0.893 or 0.0308 or 0.0366 NB 0.138 seen (or 0.107 or 0.108 or 0.862 or 0.893) implies M1 even if part of incorrect statement, eg $P(X \leq 51) = 0.107$ or $P(X \geq 51) = 0.138$	M1
			$P(X \geq 51) = 1 - 0.862 = 0.138$ (3 sf)	A1(AO3.4)	cao **BC**	A1
			Compare 0.1	A1f(AO1.1)	Dep 0.138 or 0.107 or 0.108 only	Comp 0.05 A1 No more marks
			Insufficient evidence to reject H_0 Allow "Not reject H_0"	M1(AO1.1)	Must see this statement oe Dep 0.138 or 0.107 or 0.108 or $P(X \geq 51$ or $50)$ stated	Condone "Reject H_1" Might be implied by conclusion NB possible opposite conclusion on ft
			Allow Accept H_0 No evidence (at 10% level) that proportion experiencing side-effects in one year under new treatment is greater than under standard treatment	A1f(AO2.2b) [7]	Any equivalent statement, in context, not definite, eg allow "likelihood", "percentage" Ignore all else ft only their $P(X \geq 51)$ and NB possible	NOT eg "change" NOT "proportion experiencing side effects is not significantly greater" Context can be implied by, eg, "patients", "proportion"

Question	Answer/Indicative content	Marks	Guidance
			opposite conclusion on ft 0.108 seen: max B1B1M1A0A1M1A1 Bin(450, 0.1) used: max B1B1M0A0A0M1A1

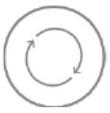
Examiner's Comments

Many candidates had clearly been well prepared for hypothesis test questions. However, a few candidates appeared to have no familiarity at all with this topic.

Many candidates lost a mark because they correctly stated the hypotheses in terms of *p*, but did not define *p*. A few gave incorrect forms of the hypotheses such as H_0: P(X = 0.975) and P(H_0 = 0.975). A few used 0.1 in their hypotheses instead of 0.097.

Most candidates used the correct binomial distribution and some stated that they were finding P($X \geq 51$). However, the calculations that many carried out were incorrect. Many actually calculated 1 – P($X \leq 51$) or P($X \leq 51$) or P($X \leq 50$) or P(X = 51) or P(X = 50).

Most candidates gave a conclusion referring to H_0 and a conclusion in context, and many of these were correct. Some candidates gave their conclusion as "There is evidence to accept H_0" or "There is evidence to reject H_1" rather than "There is insufficient evidence to reject H_0". These forms of the conclusion are not quite correct. Some gave the opposite to the correct conclusion. For example: "0.138 > 0.1 so reject H_0". A few compared their calculated probability with 0.097 instead of 0.1. A few gave their contextual conclusion in a definite form, for example "The proportion experiencing side effects under the new treatment is not greater than under

Question			Answer/Indicative content	Marks	Guidance
					the standard treatment." These lost the final mark.  AfL A common issue in hypothesis tests is candidates not securing the final marks for their conclusion, which need to be non-assertive and in context. Spec reference 2.05a gives examples of how the conclusion should be worded.
			Total	7	

Question			Answer/Indicative content	Marks	Guidance
5	a		Allow 2 sf throughout H_0: $p = 0.25$ where p = P(a packet contains gift)	B1(AO1.1)	or p = proportion of packets containing gift
			H_1: $p < 0.25$	B1(AO2.5)	One error, eg undefined p B1B0
			B(20, 0.25) & $X = 1$	M1(AO3.3)	soi
			$P(X \leq 1) = 0.0243$	A1(AO3.4)	Condone P($X = 1$) = 0.0243 but not P($X = 1$) = 0.0211 or other incorrect
			comp 0.025 Reject H_0	A1(AO1.1) M1(AO1.1)	dep 0.0243 and 0.025 Allow eg "H_0 is incorrect" Dep 0.0243 or P($X \leq 1$) stated or 0.0211 Can be implied by correct conclusion as for A1 below
			Sufficient evidence that proportion containing gift is less than 0.25	A1(AO2.2b) [7]	In context, not definite, eg not "Proportion is less"

Examiner's Comments

There were some very good responses to this question. However, it was clear that the candidates fell into two classes - those that had been trained to answer hypothesis test questions and those that had not been so well trained. A common error was to omit to define p in the hypotheses. Another, more serious, error was to find P($X = 1$) rather than P($X \leq 1$). A third common error was to give a definite conclusion such as "The proportion of packets containing a gift is less than 25%", rather than a correct, more limited, statement such as "There is significant evidence to suggest that the proportion of packets containing a gift is less than 25%".

Question			Answer/Indicative content	Marks	Guidance
	b		EITHER whether a packet contains a free gift is not independent of whether other nearby packets contain the free gift OR eg The probability that a packet contains a gift is not the same for each packet or The proportion of packets with gifts in each box is not constant OR Free gifts not distributed randomly	B1(AO3.5b) [1]	Allow The probability of packet containing a gift is not independent Explanation, **in context** of why either the independence condition or the constant probability condition is not met. NOT The number of gifts in each box is not constant
			Total	8	