Question 1:

[**5 marks**] Find, using calculus and showing each step of your working,

$$\int_1^4 \left(6x - 3 - \frac{2}{\sqrt{x}}\right) \mathrm{d}x$$

Question 2:

(a) [**4 marks**] Find

$$\int \frac{2 + 4x^3}{x^2} \mathrm{d}x$$

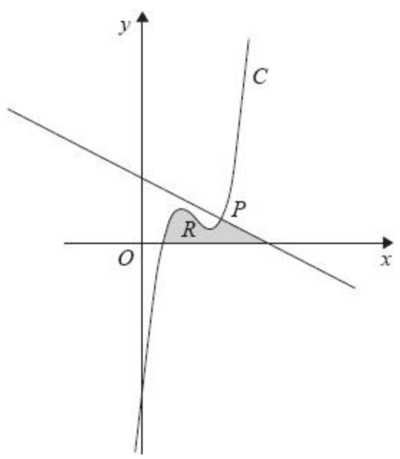
giving each term in its simplest form.

(b) [**5 marks**] Given that k is a constant and

$$\int_2^4 \left(\frac{4}{\sqrt{x}} + k\right) \mathrm{d}x = 30$$

find the exact valule of k.

Question 3:



The diagram shows part of the curve C with equation

$$y = x^3 - 9x^2 + 26x - 18$$

The point $P(4, 6)$ lies on C.

(a) [**5 marks**] Use calculus to show that the normal to C at the point P has equation $2y + x = 16$

(b) [**1 marks**] Show that C cuts the x-axis at $(1, 0)$

The shaded region is bounded by the curve C, the x-axis and the normal to C at P.

(c) [**6 marks**] Showing all your working, use calculus to find the exact area of R.

Question 4:

The curve C has equation

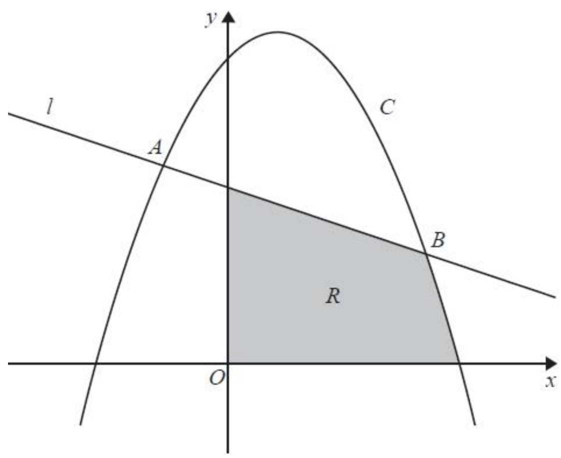
$$y = 12x^{\frac{5}{4}} - \frac{5}{18}x^2 - 1000 \qquad x > 0$$

(a) [**2 marks**] Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(b) [**5 marks**] Hence find the coordinates of the stationary point on C.

(c) [**3 marks**] Use $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ to determine the nature of this stationary point.

Question 5:



The line l has equation $y = 8 - x$ and the curve C has equation $y = 14 + 3x - 2x^2$

The line l and the curve C intersect at points A and B as shown.

(a) [**5 marks**] Use algebra to find the coordinates of A and the coordinates of B.

The shaded region is bounded by the coordinate axes, the line l, and the curve C.

(a) [**8 marks**] Use algebraic integration to calculate the exact area of R.

Question 6:

The curve C has equation

$$y = \frac{(x-3)(3x-25)}{x} \qquad x > 0$$

(a) [**3 marks**] Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in a fully simplified form.

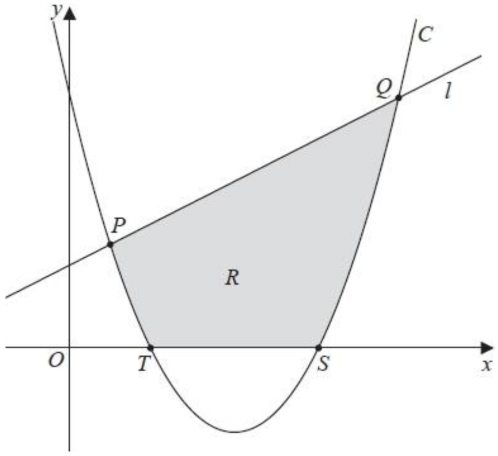
(b) [**4 marks**] Hence find the coordinates of the turning point on the curve C.

(c) [**2 marks**] Determine the nature of this turning point, justifying your answer.

The point P, with x coordinate $\frac{5}{2}$, lies on the curve C.

(d) [**5 marks**] Find the equation of the normal at P, in the form $ax + by + c = 0$ where a, b, c are integers.

Question 7:



The straight line l with equation $y = \frac{1}{2}x + 1$ cuts the curve C with equation $y = x^2 - 4x + 3$ at points P and Q.

(a) [**5 marks**] Use algebra to find the coordinates of the points P and Q.

The curve C crosses the x-axis at the points T and S.

(b) [**2 marks**] Write down the coordinates of the points T and S.

The finite region R is bounded by the line segment PQ, the line segment TS, and the arcs PT and SQ of the curve.

(c) [**8 marks**] Use integration to find the exact area of the shaded region R.

Very Difficult Challenge:

Remember that

$$\frac{\mathrm{d}}{\mathrm{d}x} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Prove the following from first principles. (**Hint:** You might want to start from the RHS)

(a) $\frac{\mathrm{d}}{\mathrm{d}x} f(x)g(x) = f(x)\frac{\mathrm{d}}{\mathrm{d}x} g(x) + g(x)\frac{\mathrm{d}}{\mathrm{d}x} f(x)$

(**Hint:** Use the fact that $f(x) = \lim_{h \to 0} f(x+h)$)

(b) $\frac{\mathrm{d}}{\mathrm{d}x} \frac{f(x)}{g(x)} = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

(**Hint:** Split up the fraction on the RHS, and use the fact that $\frac{1}{g(x)} = \lim_{h \to 0} \frac{1}{g(x+h)}$)

Numerical Answers:

(1) 32

(2) (a) $-2x^{-1} + 2x^2 + C$

(b) $k = 7 + 4\sqrt{2}$

(3) (a) $2y + x = 16$

(b) $y = 0$ when $x = 1$

(c) $\frac{207}{4}$

(4) (a) $15x^{1/4} - \frac{5}{9}x$

(b) $\left(81, \frac{187}{2}\right)$

(c) Maximum

(5) (a) $A = (-1, 9)$ and $B = (3, 5)$

(b) $\frac{499}{24}$

(6) (a) $3 - 75x^{-2}$

(b) $(5, -4)$

(c) Minimum

(d) $x - 9y + 29 = 0$

(7) (a) $P = \left(\frac{1}{2}, \frac{5}{4}\right)$ and $Q = (4, 3)$

(b) $T = (1, 0)$ and $S = (3, 0)$

(c) $\frac{93}{16}$