Q1.

Question Number	Scheme	Marks
	$\dfrac{4\sqrt{x}-3}{2x^2}=\dfrac{4\sqrt{x}}{2x^2}-\dfrac{3}{2x^2}=2x^{-\frac{3}{2}}-\dfrac{3}{2}x^{-2}$	M1 A1
	$\displaystyle\int\frac{4\sqrt{x}-3}{2x^2}\,dx=-4x^{-\frac{1}{2}}+\frac{3}{2}x^{-1}+c$	dM1 A1 A1
		(5 marks)

M1 Attempts to write as a sum of two terms. Award if any index is correct and processed

Look for Ax^m+Bx^n where $m=-\dfrac{3}{2}$ or $n=-2$.

A1 $2x^{-\frac{3}{2}}-\dfrac{3}{2}x^{-2}$ oe such as $\dfrac{1}{2}\left(4x^{-\frac{3}{2}}-3x^{-2}\right)$ on one line with the indices processed

dM1 Raises the power by one. One index must have been correct and processed

Look for one of the following $\rightarrow \ldots x^{-\frac{1}{2}}+\ldots x^{-1}$

A1 For one correct term either $-4x^{-\frac{1}{2}}$ or $+\dfrac{3}{2}x^{-1}$ which must be simplified.

If candidate attempts to integrate $\dfrac{1}{2}\left(4x^{-\frac{3}{2}}-3x^{-2}\right)$ you may award for one of $\dfrac{1}{2}\left(-8x^{-\frac{1}{2}}+3x^{-1}\right)$

A1 Fully correct including an arbitrary constant Eg. $-4x^{-\frac{1}{2}}+\dfrac{3}{2}x^{-1}+c$.

Accept exact equivalent answers such as $-\dfrac{4}{\sqrt{x}}+\dfrac{3}{2x}+c$ or $-\dfrac{4}{\sqrt{x}}+\dfrac{1.5}{x}+c$

Also accept the factorised form $\dfrac{-8x^{-\frac{1}{2}}+3x^{-1}}{2}$ or equivalent

Q2.

Question Number	Scheme	Marks
(a)	States -21	B1
		(1)
(b)	Attempts $f(3)=\left(3^2-2\right)(2\times3-3)-21$	M1
	Achieves $f(3)=0\Rightarrow\ (x-3)$ is a factor of $f(x)$ *	A1*
		(2)
(c) (i)	$f(x)=2x^3-3x^2-4x-15=(x-3)\left(2x^2....\pm5\right)$	B1 M1
	$=(x-3)\left(2x^2+3x+5\right)$	A1
(ii)	Attempts b^2-4ac for their $2x^2+3x+5$	M1
	Achieves $b^2-4ac<0$ and states that only root is $x=3$ o.e. *	A1*
		(5)
		(8 marks)

(a)

B1: -21

(b)

M1: Attempts to substitute $x=3$ into $f(x)=\left(x^2-2\right)(2x-3)-21$ or its expanded form.

Condone slips but don't accept just $f(3)=0$. Attempts via long division score M0.

A1*: Achieves $f(3)=0$ and states that $(x-3)$ is a factor of $f(x)$. If this latter is given in a preamble accept a minimal conclusion such as // or QED

(c)(i)

B1: Expands $f(x)=\left(x^2-2\right)(2x-3)-21$ to reach correct 4 term cubic. Allow if the cubic is seen anywhere.

M1: Correct attempt to find quadratic factor using either division by $(x-3)$ or inspection. To score the M mark look for e.g. correct first and last terms by inspection, or first two terms by division. (Two correct terms will imply the mark.) Allow for work seen in (b) a long as it is referred to in (c).

A1: $(f(x)=)(x-3)\left(2x^2+3x+5\right)$ following a correct cubic. Must be seen together on one line.

(c) (ii)

M1: Attempts to show their quadratic factor has no real roots. Factorisation attempts are M0. Accept via

- an attempt at b^2-4ac for their quadratic factor
- an attempt to solve their $2x^2+3x+5$ using the quadratic formula or calculator
- an attempt to complete the square

A1*: Requires correct factorisation, correct calculation, reason and conclusion

For example after (c)(i) $f(x)=(x-3)\left(2x^2+3x+5\right)$ accept e.g.

- $2x^2+3x+5$ has no roots as $3^2-4\times2\times5<0$ so $f(x)=0$ only has root at $x=3$
- $2x^2+3x+5=0\Rightarrow x=-\frac{3}{4}\pm i\frac{\sqrt{31}}{4}$, $(x-3)=0\Rightarrow x=3$. So only one real root.
- $2x^2+3x+5=0\Rightarrow 2\left(x+\frac{3}{4}\right)^2-\frac{9}{8}+5\ldots\frac{31}{8}>0$, so no roots hence 3 is the only root

There must be some reference to the root either by stating it or indicating the linear term has a root (e.g. writing "root" next to it), but do not accept incorrect statements such as only real root is $(x-3)$

Do not allow statements such as "Math error" without interpretation of what this means.

Q3.

Question	Scheme	Marks	AOs
(a)	$\overrightarrow{QR}=\overrightarrow{PR}-\overrightarrow{PQ}=13\mathbf{i}-15\mathbf{j}-(3\mathbf{i}+5\mathbf{j})$	M1	1.1a
	$=10\mathbf{i}-20\mathbf{j}$	A1	1.1b
		(2)	
(b)	$\lvert\overrightarrow{QR}\rvert=\sqrt{"10"^2+"(-20)"^2}$	M1	2.5
	$=10\sqrt{5}$	A1ft	1.1b
		(2)	
(c)	$\overrightarrow{PS}=\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{QR}=3\mathbf{i}+5\mathbf{j}+\frac{3}{5}("10\mathbf{i}-20\mathbf{j}")=\ldots$ or $\overrightarrow{PS}=\overrightarrow{PR}+\frac{2}{5}\overrightarrow{RQ}=13\mathbf{i}-15\mathbf{j}+\frac{2}{5}("-10\mathbf{i}+20\mathbf{j}")=\ldots$	M1	3.1a
	$=9\mathbf{i}-7\mathbf{j}$	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

M1: Attempts subtraction either way round. This cannot be awarded for adding the two vectors. If no method shown it may be implied by one correct component. eg $10\mathbf{i}-10\mathbf{j}$ on its own can score M1.

A1: Correct answer. Allow $10\mathbf{i}-20\mathbf{j}$ and $\begin{pmatrix}10\\-20\end{pmatrix}$ but not $\begin{pmatrix}10\mathbf{i}\\-20\mathbf{j}\end{pmatrix}$

(b)

M1: Correct use of Pythagoras. Attempts to "square and add" before square rooting. The embedded values are sufficient. Follow through on their $\overrightarrow{QR}$

A1ft: $10\sqrt{5}$ following (a) of the form $\pm10\mathbf{i}\pm20\mathbf{j}$

(c)

M1: Full attempt at finding a $\overrightarrow{PS}$. They must be attempting $\overrightarrow{PQ}\pm\frac{3}{5}\overrightarrow{QR}$ or

$\overrightarrow{PS}=\overrightarrow{PR}\pm\frac{2}{5}\overrightarrow{RQ}$ but condone arithmetical slips after that.

Cannot be scored for just stating eg $\overrightarrow{PQ}\pm\frac{3}{5}\overrightarrow{QR}$

Follow through on their $\overrightarrow{QR}$. Terms do not need to be collected for this mark. If no method shown it may be implied by one correct component following through on their $\overrightarrow{QR}$

A1: Correct vector as shown. Allow $9\mathbf{i}-7\mathbf{j}$ and $\begin{pmatrix} 9 \\ -7 \end{pmatrix}$.

Only withhold the mark for $\begin{pmatrix} 9\mathbf{i} \\ -7\mathbf{j} \end{pmatrix}$ if the mark has not already been withheld in (a) for $\begin{pmatrix} 10\mathbf{i} \\ -20\mathbf{j} \end{pmatrix}$

Alt (c) (Expressing $\overrightarrow{PS}$ in terms of the given vectors) They must be attempting $\frac{2}{5}\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{PR}$

M1: $(\overrightarrow{PS}=\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{QR}=\overrightarrow{PQ}+\frac{3}{5}(\overrightarrow{PR}-\overrightarrow{PQ}))$

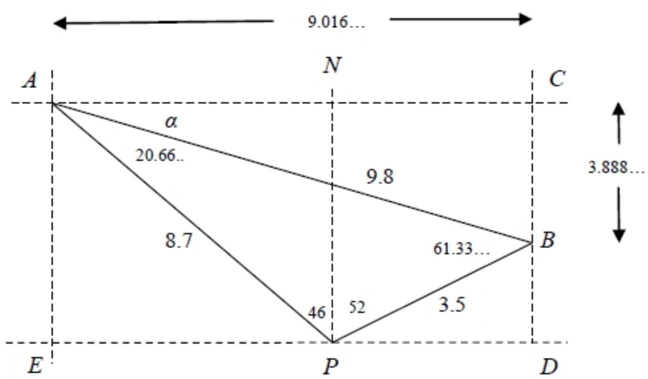
$\Rightarrow \frac{2}{5}\overrightarrow{PQ}+\frac{3}{5}\overrightarrow{PR}=\frac{2}{5}(3\mathbf{i}+5\mathbf{j})+\frac{3}{5}(13\mathbf{i}-15\mathbf{j})=\ldots$

A1: Correct vector as shown. Allow $9\mathbf{i}-7\mathbf{j}$ and $\begin{pmatrix} 9 \\ -7 \end{pmatrix}$.

Only withhold the mark for $\begin{pmatrix} 9\mathbf{i} \\ -7\mathbf{j} \end{pmatrix}$ if the mark has not already been withheld in (a) for $\begin{pmatrix} 10\mathbf{i} \\ -20\mathbf{j} \end{pmatrix}$

Q4.

Question Number	Scheme	Notes	Marks
(a)	($APN =$) $360° - 314° = 46°$ ($APB =$) $46° + 52° = 98°$ or (Reflex APB) $= 314° - 52° = 262°$ ($APB =$) $360° - 262° = 98°$ or Shows on a sketch the 314 and 46 And states $46° + 52° = 98°$	Correct explanation that **explains why** APN is 46° (e.g. 360° – 314°) and adds that to 52° **or** shows/states that reflex $APB = 262°$ and so $APB = 360° - 262° = 98°$. Do not be overly concerned how they use the letters to reference angles as long as the correct calculations are seen. **Do not allow the use of $AB = 9.8$ from (b).**	B1
			(1)
(b)	$(AB^2 =) 8.7^2 + 3.5^2 - 2\times 8.7\times 3.5\cos 98°$	Correct use of cosine rule. You can ignore the lhs for this mark so just look for $8.7^2 + 3.5^2 - 2\times 8.7\times 3.5\cos 98°$	M1
	$AB = 9.8$ (km)	Awrt 9.8 km (you can ignore their intermediate value for AB^2 provided awrt 9.8 is obtained for AB)	A1
			(2)
(c) **Way 1**	$\frac{"9.8"}{\sin 98°} = \frac{3.5}{\sin PAB}$ or $3.5^2 = 8.7^2 + "9.8"^2 - 2\times 8.7\times "9.8"\cos PAB$ $\Rightarrow PAB = \ldots$	Correct sine or cosine rule method to obtain angle PAB. May be implied by awrt 21°	M1
	$PAB = 20.66\ldots°$	Allow awrt 21°. May be implied by a correct bearing.	A1
	Bearing is $180° -$ "$20.66°$" $- 46°$	Fully correct method	M1
	$= 113°$ or $114°$	Awrt 113° or awrt 114°	A1
(c) **Way 2**	$\frac{"9.8"}{\sin 98°} = \frac{8.7}{\sin PBA}$ or $8.7^2 = 3.5^2 + "9.8"^2 - 2\times 3.5\times "9.8"\cos PBA$ $\Rightarrow PBA = \ldots$	Correct sine or cosine rule method to obtain angle PBA. May be implied by awrt 61° or 62°	M1
	$PBA = 61.33\ldots°$	Allow awrt 61° or awrt 62°. May be implied by a correct bearing.	A1
	Bearing is $52° +$ "$61.33\ldots°$"	Fully correct method	M1
	$= 113°$ or $114°$	Awrt 113° or awrt 114°	A1
			(4)
(c) **Way 3**	Let $\alpha =$ Bearing $- 90°$		
	$\tan\alpha = \frac{BC}{AC} = \frac{8.7\cos 46° - 3.5\cos 52°}{8.7\sin 46° + 3.5\sin 52°}$	Correct method for α	M1
	$\alpha = 23.33°$	Allow awrt 23°. May be implied by a correct bearing.	A1
	Bearing is $90° +$ "$23.33°$"	Fully correct method	M1
	$= 113°$ or $114°$	Awrt 113° or awrt 114°	A1
			(4)
			Total 7



Q5.

Question Number	Scheme	Marks
(a)	States $\log a = 0.68$ or $\log b = 0.09$	M1
	$a = 4.79$ **or** $b = 1.23$	A1
	States $\log a = 0.68$ and $\log b = 0.09$	M1
	$a = 4.79$ **and** $b = 1.23$ **CSO**	A1
		(4)
(b)	The percentage of the population with access to the internet at the start of 2005	B1
		(1)
(c)	$P = 4.79 \times 1.23^{10} =$ awrt 38	M1, A1
		(2)
		Total 7

(a)

M1: Either states any of $\log a = 0.68$, $a = 10^{0.68}$, $a =$ awrt 4.8

or any of $\log b = 0.09$, $b = 10^{0.09}$, $b =$ awrt 1.2

A1: Achieves either $a =$ awrt 4.79 **or** $b =$ awrt 1.23

M1: States a correct equation for both a and b. See first M mark

A1: Achieves $a = 4.79$ **and** $b = 1.23$ **with no incorrect work**.

Implied by $P = 4.79 \times 1.23^t$ **with no incorrect work**

These are NOT awrt values

Examples of incorrect work are

- $P = ab^t \Rightarrow \log P = \log a \times t \log b$
- $\log P = 0.68 + 0.09t \Rightarrow P = 10^{0.68} + 10^{0.09t} \Rightarrow P = 4.79 \times 1.23^t$

(b)

B1: A correct interpretation. The emboldened words must be present or stated in a similar way

"The **percentage** of the population with access to the **internet** at the start of **2005**"

A minimal answer is "the percentage with access to the internet in 2005"

Also allow "the initial percentage with internet access".

It is acceptable to state 4.79% of the population had access to the in 2005

(c)

M1: For attempting 4.79×1.23^{10} following through on their 4.79 and 1.23, (Ignore subsequent work)

Alternatively attempting $\log P = 0.68 + 10 \times 0.09 \Rightarrow P = ...$

Condone an attempt at 4.79×1.23^{11}

A1: AWRT 38. ISW after sight of awrt 38 and condone misinterpretations such as stating 38 people.

Q6.

Question Number	Scheme	Notes	Marks
3(a)	$\left(2-\frac{kx}{4}\right)^8 = 2^8 + \binom{8}{1}2^7\left(-\frac{kx}{4}\right)+\binom{8}{2}2^6\left(-\frac{kx}{4}\right)^2+\binom{8}{3}2^5\left(-\frac{kx}{4}\right)^3+\ldots$ Or $\left(1-\frac{kx}{8}\right)^8 = 1+\binom{8}{1}\left(-\frac{kx}{8}\right)+\binom{8}{2}\left(-\frac{kx}{8}\right)^2+\binom{8}{3}\left(-\frac{kx}{8}\right)^3+\ldots$		M1
	$= 256-256kx+112k^2x^2-28k^3x^3+\ldots$	$256-256kx$	B1
		$112k^2x^2$ **or** $-28k^3x^3$ (unsimplified)	A1
		$112k^2x^2$ **and** $-28k^3x^3$ (simplified)	A1
			(4)
(b)	$f(x)=(5-3x)\left(2-\frac{kx}{4}\right)^8=(5-3x)\left(256-256kx+112k^2x^2-28k^3x^3+\ldots\right)$ Coefficient of x is $5\times -256k - 3\times 256$		M1
	$5\times 256 = 3(-1280k-768)\Rightarrow k=\ldots$	Sets $5\times$their constant term from (a) = $3\times$their coefficient of x from f(x) and solves for k	M1
	$k=-\frac{14}{15}$	Correct value.	A1
			(3)
			Total 7

Notes

(a)

M1: Attempts the binomial expansion on $\left(2\pm\frac{kx}{4}\right)^8$ or $\left(1\pm\frac{kx}{\beta}\right)^8$ up to at least the third (x^2) term with an acceptable structure. Look for the correct binomial coefficient (accept alternative notation ${}^nC_r$) combined with the correct power of x but allow if powers of 2 are incorrect and if brackets are missing. M0 for descending powers.

B1: for $256-256kx$, may be listed, must be simplified. Allow for $256(1-kx+\ldots)$ if the 2^8 is taken out first.

A1: Correct third or fourth term, may be listed. Need not be simplified but the binomial coefficients must be numerical. Allow for one term from $256\left(\ldots+\frac{28}{64}(kx)^2-\frac{56}{512}(kx)^3\ldots\right)$ if the 2^8 is taken out first. May have powers as $(kx)^n$ for this mark. Allow for the correct x^2 term if the sign was incorrect in their bracket.

A1: Correct simplified third and fourth terms as shown in scheme, may be listed. Must have k^nx^n terms.

Note: isw after correct terms are seen if they try to divide through.

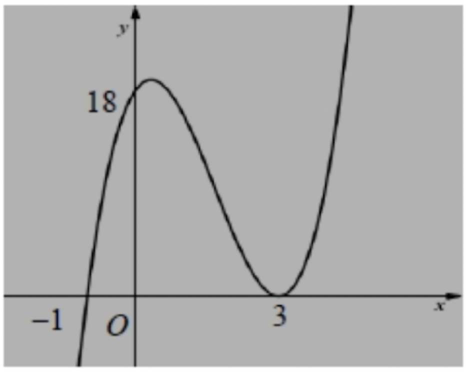
(b)

M1: Correct strategy for the coefficient of x or the x term. E.g. $5\times$their $-256k$ – $3\times$their 256 or may be part of a full expansion – look for ($5\times$their $-256k$ – $3\times$their 256)x but terms must have been combined.

M1: Sets $5\times$their constant term from (a) = $3\times$their coefficient of x from f(x) and solves for k. Should be an equation in k only, but allow recovery if they initially include the x but later cross it out to give a constant for the answer. The attempt at the x coefficient must have been an attempt at a sum of two terms from their expansion of f(x)

A1: Correct value, must be exact. Allow $-0.9\dot{3}$ but not a terminating decimal.

Q7. (need a marking scheme for part c)

Question Number	Scheme	Marks
(a)	 Positive cubic shape anywhere with 1 maximum and 1 minimum	M1
	Positive cubic shape that at least reaches the x-axis at $x=-1$ and with a minimum on the x-axis at $x=3$	A1
	y intercept at 18. Must correspond with their sketch	B1
	For the intercepts allow as numbers as above or allow as coordinates e.g. (18, 0), (0, −1), (0, 3) as long as they are marked in the correct place.	
		(3)
(b)	E.g. $(2x+2)(x^2-6x+9)=\dots$	M1
	$=2x^3-10x^2+6x+18$	A1 A1
		(3)
(c)	$(\mathrm{f}'(x)=)6x^2-20x+6$	M1
	$y=\frac{512}{27}$	A1
	$(x=3)\quad y=0$	B1
	$\{k: 0<k<\frac{512}{27}\}$ OR $\{k: k>0\}\cap\{k: k<\frac{512}{27}\}$	A1ft
		(4)
		(10 marks)

(a)

M1 Correct shape for a $y=+x^3$ graph. Do not be too concerned if the "ends" become vertical or even go beyond the vertical slightly. Condone with no axes and condone cusp like appearance for the turning points.

A1 $y=+x^3$ shape, intersects (or at least reaches the x-axis) at -1, minimum at $x=3$ but must not stop or cross at $x=3$

B1 y intercept at 18

You can ignore the position of the maximum i.e. it may be to the left of or right of or on the y-axis.

(b) Mark (b) and (c) together.

M1 Attempts to multiply out.

E.g. Look for an attempt to square $(x-3)$ to obtain $x^2\pm6x\pm9$ and then an attempt to multiply by $(x+1)$ or $(2x+2)$ or an attempt to multiply $(x+1)$ or $(2x+2)$ by $(x-3)$ and then multiply the result by $(x-3)$

Condone slips e.g. attempting $(2x+1)(x-3)^2$ but expect to see an expression of the required form

A1	Two correct terms of $2x^3 - 10x^2 + 6x + 18$
A1	Fully correct $2x^3 - 10x^2 + 6x + 18$. (Ignore any spurious "= 0")
	Special case: if they obtain $2x^3 - 10x^2 + 6x + 18$ but then attempt to "simplify" as e.g. $f(x) = x^3 - 5x^2 + 3x + 9$ then score A1A0 **but note that all marks are available in (c) in such cases.**
(c)	
M1	Correctly differentiates their $2x^3 - 10x^2 + 6x + 18$. Allow follow through but only from a 4 term cubic.
	Allow use of product rule e.g. $f(x) = 2(x+1)(x-3)^2 \to f'(x) = 2(x-3)^2 + 4(x+1)(x-3)$ You can condone poor notation so just look for the correct or correct ft expression.

(c) ***Be aware the value of *y* can be solved directly using a calculator which is not acceptable***

M1: Uses a correct strategy for the y value of either the maximum or minimum. E.g. differentiates to achieve a quadratic, solves $\frac{dy}{dx} = 0$ and uses their x to find y

A1: $y = \frac{512}{27}$

Cannot be scored for an answer without any working seen.

B1 $y = 0$ OR $x = 3$

A1ft: Correct answer in any acceptable set notation following through their $\frac{512}{27}$ and/or 0

Condone $\left\{\text{" } 0 \text{ "} < k < \text{"}\frac{512}{27}\text{"}\right\}$ or $\left\{\text{" } 0 \text{ "} < k\right\} \cap \left\{k < \text{"}\frac{512}{27}\text{"}\right\}$ but not $\left\{\text{" } 0 \text{ "} < k\right\} \cup \left\{k < \text{"}\frac{512}{27}\text{"}\right\}$

Note: If there is a contradiction of their solution on different lines of working do not penalise intermediate working and mark what appears to be their final answer.
Must be in terms of k

Q8. (b and c swapped, please award according to marking scheme, So Diff part 2 marks, Finding time part 3 marks)

Question	Scheme	Marks	AOs
(a)	$(k=)\ 0.8$	B1	1.1b
		(1)	
(b)	$1=0.8+1.4\mathrm{e}^{-0.5t}\Rightarrow 1.4\mathrm{e}^{-0.5t}=0.2$	M1	3.1b
	$-0.5t=\ln\left(\dfrac{0.2}{1.4}\right)\Rightarrow t=...$	M1	1.1b
	awrt 3.9 minutes	A1	1.1b
		(3)	
(c)	$\left(\dfrac{\mathrm{d}P}{\mathrm{d}t}=\right)\ -0.7\mathrm{e}^{-0.5t}$ $\left(\dfrac{\mathrm{d}P}{\mathrm{d}t}\right)_{t=2}=-0.7\mathrm{e}^{-0.5\times 2}$	M1	3.1b
	= awrt 0.258 (kg/cm^2 per minute)	A1	1.1b
		(2)	
			(6 marks)

Notes

(a)

B1: Completes the equation for the model by obtaining $(k=)\ 0.8$ or equivalent.

(b) ***Be aware this could be solved entirely using a calculator which is not acceptable***

M1: For using the model with $P=1$ and their value for k from (a) and proceeding to $A\mathrm{e}^{\pm 0.5t}=B$. Condone if A or B are negative for this mark.

M1: Uses correct log work to solve an equation of the form $A\mathrm{e}^{\pm 0.5t}=B$ leading to a value for t. They cannot proceed directly to awrt 3.9 without some intermediate working seen.

Eg $t=2\ln 7$ or $-2\ln\left(\dfrac{1}{7}\right)$ is acceptable.

Also allow $1.4\mathrm{e}^{-0.5t}=0.2\Rightarrow -0.5t=-1.9459...\Rightarrow t=...$

This cannot be scored from an unsolvable equation (eg when their $k \ldots 1$ so that $\mathrm{e}^{\pm 0.5t}$ „ 0).

A1: Accept awrt 3.9 minutes or t = awrt 3.9 with correct working seen.
eg $1.4\mathrm{e}^{-0.5t}=0.2\Rightarrow t=3.9$ would be M1M0A0

(c) ***Be aware this can be solved entirely using a calculator which is not acceptable***

M1: Links rate of change to gradient and differentiates to obtain an expression of the form $A\mathrm{e}^{-0.5t}$ and substitutes $t=2$. Do not accept $At\mathrm{e}^{-0.5t}$ as the derivative.
Beware that substituting $t=2$ and proceeding from e^{-1} to e^{-2} is M0A0

A1: Obtains awrt 0.258 with differentiation seen. (Units not required) Condone awrt -0.258
Awrt ± 0.258 with no working is M0A0. Isw after a correct answer is seen.

(Ignore in (c) any spurious notation on the LHS when differentiating such as $P=\ldots$ or $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\ldots$)

Q9.

Question Number	Scheme	Notes	Marks
	$2\log_3(1-x)=\log_3(1-x)^2$ or $3=\log_3 3^3$	Correct power law used or implied	B1
	$\log_3(32-12x)-\log_3(1-x)^2=\log_3\dfrac{32-12x}{(1-x)^2}$ Combines 2 log terms correctly		M1
	$\dfrac{32-12x}{(1-x)^2}=27$	Obtains this equation in any form	A1
	$\Rightarrow 27x^2-42x-5=0\Rightarrow x=...$	Solves 3TQ	M1
	$x=-\dfrac{1}{9}$	This value only i.e. the $\dfrac{5}{3}$ must clearly be discarded if seen.	A1
			(5)
			Total 5

Q10.

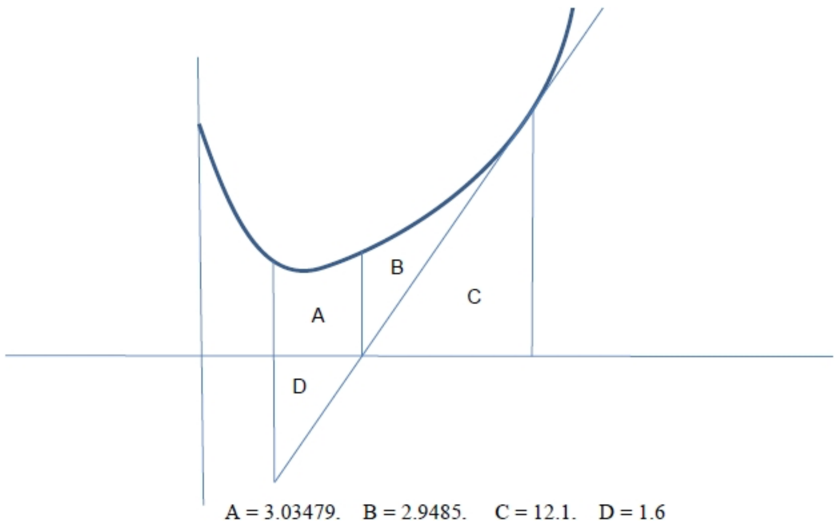
Question Number	Scheme		Marks
(a)	$y=\dfrac{3}{4}x^2-4\sqrt{x}+7\Rightarrow\dfrac{dy}{dx}=\dfrac{3}{2}x-2x^{-0.5}$	M1: Differentiates to obtain at least one correct power for one of the terms in x. (may be un-simplified) e.g. $x^2\to x^{2-1}$ or $\sqrt{x}\to x^{\frac{1}{2}-1}$ A1: Correct derivative. Allow un-simplified e.g. $2\times\dfrac{3}{4}x^{2-1}$ or $-4\times\dfrac{1}{2}x^{\frac{1}{2}-1}$	M1A1
	At $x=4$ $\dfrac{dy}{dx}=\dfrac{3}{2}(4)-2(4)^{-0.5}=...$	Substitutes $x=4$ into a changed function in an attempt to find the gradient.	M1
	$y-11="5"(x-4)$ or $y=mx+c\Rightarrow 11="5"\times4+c\Rightarrow c=...$	Correct straight line method using (4, 11) correctly placed and their dy/dx at $x=4$ for the tangent **not the normal**. If using $y=mx+c$, must reach as far as finding a value for c. **Dependent on the previous M.**	dM1
	$y=5x-9$	Correct printed equation with no errors seen. **Beware of the "5" appearing from wrong working.**	A1*
	<u>Important Note:</u> **Following a correct derivative, if candidate states $x=4$ so dy/dx = 5, this is fine if they then complete correctly – allow full marks. However, following a correct derivative, if the candidate <u>just</u> states dy/dx = 5 and then proceeds to obtain the correct straight line equation, the final mark can be withheld. Some evidence is needed that the candidate is considering the gradient at $x=4$.**		
			(5)

For part (b), in all cases, look to apply the appropriate scheme that gives the candidate the best mark

	Finds area under curve between 1 and 4 and subtracts triangle C (see diagram at end)		
(b) Way 1	$\int \frac{3}{4}x^2 - 4\sqrt{x} + 7\,\mathrm{d}x = \frac{1}{4}x^3 - \frac{8}{3}x^{1.5} + 7x(+c)$	M1: $x^n \to x^{n+1}$ on any term. May be un-simplified e.g. $x^2 \to x^{2+1}$, $x^{0.5} \to x^{0.5+1}$, $7 \to 7x^1$ A1: Correct integration. May be un-simplified e.g. terms such as $\frac{1}{3}\times\frac{3}{4}x^{2+1}$, $-\frac{2}{3}\times 4x^{0.5+1}$, $7x^1$ and $+\,c$ is not required.	M1A1
	Tangent meets x axis at $x = 1.8$	This may be embedded within a triangle area below or may be seen on a diagram.	B1
	Area of triangle $= \frac{1}{2}\times(4 - '1.8')\times 11 = (12.1)$ Correct method for the area of a triangle - look for $\frac{1}{2}\times(4-'1.8')\times 11$ This may be implied by the evaluation of $\int_{'1.8'}^{4} 5x - 9\,\mathrm{d}x = \left[5\frac{x^2}{2} - 9x\right]_{'1.8'}^{4}$		M1
	Correct method for area = Area A + Area B + Area C − Area C $\left(\frac{1}{4}4^3 - \frac{8}{3}\times 4^{1.5} + 7\times 4\right) - \left(\frac{1}{4}1^3 - \frac{8}{3}\times 1^{1.5} + 7\times 1\right) - '12.1'$ Correct combination of areas. **Dependent on both previous method marks.**		ddM1
	= awrt 5.98	Area of R = awrt 5.98 or allow the exact answer of $\frac{359}{60}$ or equivalent.	A1
			(6)
			(11 marks)

	Finds area under curve between 1 and "1.8" and adds "line – curve" or "curve – line" between "1.8" and 4		
(b) Way 2	$\int \frac{3}{4}x^2 - 4\sqrt{x} + 7\,\mathrm{d}x = \frac{1}{4}x^3 - \frac{8}{3}x^{1.5} + 7x(+c)$	M1: $x^n \to x^{n+1}$ on any term. May be un-simplified e.g. $x^2 \to x^{2+1}$, $x^{0.5} \to x^{0.5+1}$, $7 \to 7x^1$ A1: Correct integration. May be un-simplified e.g. terms such as $\frac{1}{3}\times\frac{3}{4}x^{2+1}$, $-\frac{2}{3}\times 4x^{0.5+1}$, $7x^1$ and $+\,c$ is not required.	M1A1
	Tangent meets x axis at $x = 1.8$	This may be seen on a diagram.	B1
	Area between "1.8" and 4 = $\pm\int_{'1.8'}^{4}\left(\frac{3}{4}x^2 - 4\sqrt{x} + 7\right) - (5x - 9)\,\mathrm{d}x = \pm\left[\frac{1}{4}x^3 - \frac{8}{3}x^{1.5} - \frac{5x^2}{2} + 16x\right]_{'1.8'}^{4}$ $= \frac{56}{3} - 15.7182...(= 2.9485...)$ Attempts to integrate "curve – line" or "line – curve", substitute the limits "1.8" and 4 and subtracts.		M1
	Correct method for area = Area A + Area B $\left(\left(\frac{1}{4}"1.8"^3 - \frac{8}{3}"1.8"^{1.5} + 7\times"1.8"\right) - \left(\frac{1}{4}1^3 - \frac{8}{3}1^{1.5} + 7\times 1\right) + '2.9485...'\right)$ Correct combination of areas. **Dependent on both previous method marks.**		ddM1
	= awrt 5.98	Area of R = awrt 5.98 or allow the exact answer of $\frac{359}{60}$ or equivalent.	A1
			(6)

	Uses "line – curve" or "curve – line" between 1 and 4 and subtracts triangle below x axis		
(b) Way 3	$\pm\left(\frac{3}{4}x^2-4\sqrt{x}+7-5x+9\right)=\pm\left(\frac{3}{4}x^2-4\sqrt{x}-5x+16\right)$ $\pm\int \frac{3}{4}x^2-4\sqrt{x}-5x+16\,dx=\pm\left(\frac{1}{4}x^3-\frac{8}{3}x^{1.5}-\frac{5x^2}{2}\right)+kx(+c)$ M1: $x^n\to x^{n+1}$ on any term. May be un-simplified e.g. $x^2\to x^{2+1}$, $x^{0.5}\to x^{0.5+1}$, $x\to x^{1+1}$, $16\to 16x^1$. If terms are not collected when subtracting then the same condition applies. A1: Correct integration as shown. May be un-simplified for coefficients and powers and + c is not required.		M1A1
	Tangent meets x axis at $x=1.8$	This may be embedded within a triangle area below or may be seen on a diagram.	B1
	Area of triangle $=\frac{1}{2}\times('1.8'-1)\times\lvert 5\times1-9\rvert=(1.6)$ Correct method for the area of a triangle - look for $\frac{1}{2}\times('1.8'-1)\times\lvert 5\times1-9\rvert$		M1
	Correct method for area = Area A + Area B + Area D – Area D $\left(\left(\frac{1}{4}4^3-\frac{8}{3}4^{1.5}-\frac{5\times4^2}{2}+16\times4\right)-\left(\frac{1}{4}1^3-\frac{8}{3}1^{1.5}-\frac{5\times1^2}{2}+16\times1\right)-'1.6'\right)$ Correct combination of areas. **Dependent on both previous method marks.**		ddM1
	= awrt 5.98	Area of R = awrt 5.98 or allow the exact answer of $\frac{359}{60}$ or equivalent.	A1
			(6)



A = 3.03479. B = 2.9485. C = 12.1. D = 1.6

Q11.

Question	Scheme	Marks	AOs
(a)	$(x\pm5)^2+(y\pm4)^2$	M1	1.1b
	(i) Centre is (5, 4)	A1	1.1b
	(ii) Radius is 3	A1	1.1b
		(3)	
(b)	$2y+x+6=0 \Rightarrow y=-\frac{1}{2}x+\ldots \Rightarrow -\frac{1}{2} \to 2$	B1	2.2a
	$m_N=2 \Rightarrow y-4=2(x-5)$ $y-4=2(x-5),\ 2y+x+6=0 \Rightarrow x=\ldots,\ y=\ldots$	M1	3.1a
	Intersection is at $\left(\frac{6}{5},\ -\frac{18}{5}\right)$ oe	A1	1.1b
	Distance from centre to intersection is $\sqrt{\left(5-\frac{6}{5}\right)^2+\left(4+\frac{18}{5}\right)^2}$ So distance required is $\sqrt{\left(\text{"}5\text{"}-\text{"}\frac{6}{5}\text{"}\right)^2+\left(\text{"}4\text{"}+\text{"}\frac{18}{5}\text{"}\right)^2}-\text{"}3\text{"}$	dM1	3.1a
	$=\frac{19\sqrt{5}}{5}-3$ (or awrt 5.50)	A1	1.1b
		(5)	
			(8 marks)

Notes
(a)
M1: Attempts to complete the square for both x and y terms $(x\pm5)^2 \ldots (y\pm4)^2$ which may be implied by a centre of (±5, ±4)
A1: Centre (5, 4)
A1: Radius 3
(b)
B1: Deduces the gradient of the perpendicular to l is 2. May be seen in the equation for the perpendicular line to l
M1: A fully correct strategy for finding the intersection. This requires use of their gradient of the perpendicular which cannot be the gradient of l Look for $y-\text{"}4\text{"}=\text{"}2\text{"}(x-\text{"}5\text{"})$ where $(5,4)$ is their centre being solved simultaneously with the equation of l Do not be concerned with the mechanics of their rearrangement when solving simultaneously. Many are finding the y-intercept of l $(0,-3)$ which is M0

A1: $\left(\frac{6}{5},\ -\frac{18}{5}\right)$ or equivalent eg $(1.2,\ -3.6)$

They do not have to be written as coordinates and may be seen within their working rather than explicitly stated. They may also be written on the diagram.

dM1: Fully correct strategy for finding the required distance e.g. correct use of Pythagoras to find the distance between their centre and their intersection and then completes the problem by subtracting their radius. Condone a sign slip subtracting their $-\frac{18}{5}$.

It is dependent on the previous method mark.

Alternatively, they solve simultaneously their $y = 2x - 6$ with the equation of the circle and then find the distance between this intersection point and the point of intersection between l and the normal. They must choose the smaller positive root of the solution to their quadratic.

Eg

$(x-5)^2 + (2x-6-4)^2 = 9 \Rightarrow 5x^2 - 50x + 125 = 9$

$x = \frac{25-3\sqrt{5}}{5},\ y = \frac{20-6\sqrt{5}}{5}$

Distance between two points:

$$\sqrt{\left("\frac{25-3\sqrt{5}}{5}" - "\frac{6}{5}"\right)^2 + \left("\frac{20-6\sqrt{5}}{5}" + "\frac{18}{5}"\right)^2}$$

A1: Correct value e.g. $\sqrt{\frac{361}{5}} - 3$ or $\frac{19\sqrt{5}-15}{5}$). Also allow awrt 5.50

Isw after a correct answer is seen.

Alt (b) Be aware they may use vector methods:

B1M1: Attempts to find the perpendicular distance between their $(5,4)$ and $x + 2y + 6 = 0$ by substituting the values into the formula to find the distance between a point (x, y) and a line $ax + by + c = 0$

$$\Rightarrow \frac{|ax+by+c|}{\sqrt{a^2+b^2}} = \frac{|"5"\times"1"+"4"\times"2"+"6"|}{\sqrt{"1"^2+"2"^2}}$$

A1: $\frac{|5\times1+4\times2+6|}{\sqrt{1^2+2^2}}\ \left(=\frac{19}{\sqrt{5}}\right)$

dM1: Distance $= "\frac{19\sqrt{5}}{5}" - 3$

A1: $\frac{19\sqrt{5}-15}{5}$

Q12.

Question Number	Scheme	Marks
(a)	$\frac{dP}{dx}=12-\frac{3}{2}x^{\frac{1}{2}}$	M1A1
	Sets $\frac{dP}{dx}=0\to 12-\frac{3}{2}x^{\frac{1}{2}}=0\to x^n=...$	dM1
	$x=64$	A1
	When $x=64\Rightarrow P=12\times 64-64^{\frac{3}{2}}-120=...$	M1
	Profit $=$ (£) 136 000	A1
		(6)
(b)	$\left(\frac{d^2P}{dx^2}\right)=-\frac{3}{4}x^{-\frac{1}{2}}$ and substitutes in their $x=64$ to find its value or state its sign	M1
	At $x=64$ $\frac{d^2P}{dx^2}=-0.09375<0\Rightarrow$ maximum	A1
		(2)
		(8 marks)

Notes

You should mark parts a and b together. You may see work in (a) from (b)

(a)

M1 Attempts to differentiate $x^n\to x^{n-1}$ seen at least once. It must be an x term and **not** the $120\to 0$

A1 $\frac{dP}{dx}=12-\frac{3}{2}x^{\frac{1}{2}}$ with no need to see the lhs. Condone $\frac{dy}{dx}$ all of the way through part (a).

dM1 Sets their $\frac{dP}{dx}=0$ and proceeds to $x^n=k, k>0$. Dependent upon the previous M. Don't be too concerned with the mechanics of process. Condone an attempted solution of $\frac{dP}{dx}...0$ where ... could be an inequality

A1 $x=64$. Condone $x=\pm 64$ here

M1 Substitutes their solution for $\frac{dP}{dx}=0$ into P and attempts to find the value of P.
The value of x must be positive. If two values of x are found, allow this mark for any attempt using a positive value.

A1 CSO. Profit = (£) 136 000 or 136 thousand but not 136 or $P=136$.
This cannot follow two values for x, eg $x=\pm 64$ Condone a lack of units or incorrect units such as \$

(b)

M1 Achieves $\frac{d^2P}{dx^2}=kx^{-\frac{1}{2}}$ and attempts to find its value at $x="64"$
Alternatively achieves $\frac{d^2P}{dx^2}=kx^{-\frac{1}{2}}$ and attempts to state its sign. Eg $\frac{d^2P}{dx^2}=-\frac{3}{4}x^{-\frac{1}{2}}<0$
Allow $\frac{d^2P}{dx^2}$ appearing as $\frac{d^2y}{dx^2}$ for the both marks.

A1 Achieves $x=64$, $\frac{d^2P}{dx^2}=-\frac{3}{4}x^{-\frac{1}{2}}$ and states $\frac{d^2P}{dx^2}=-\frac{3}{32}<0$ (at $x=64$) then the profit is maximised.

This requires the correct value of x, the correct value of the second derivative (allowing for awrt -0.09) a reason + conclusion.

Alt: Achieves $x=64$, $\frac{d^2P}{dx^2}=-\frac{3}{4}x^{-\frac{1}{2}}$ and states as $x>0$ or $\sqrt{x}>0$ means that $\frac{d^2P}{dx^2}<0$ then the profit is maximised.

Part (b) merely requires the use of calculus so allow

M1 Attempting to find the value of $\frac{dP}{dx}$ at two values either side, but close to their 64. Eg. For 64 , allow the lower value to be $63.5\leqslant x<64$ and the upper value to be $64<x\leqslant 64.5$

A1 Requires correct values, correct calculations with reason and conclusion

Q13.
Part 2 first, then part 3, then part 1

Question	Scheme	Marks	AOs
(i)	Uses $\cos^2\theta = 1-\sin^2\theta$ $5\cos^2\theta = 6\sin\theta \Rightarrow 5\sin^2\theta + 6\sin\theta - 5 = 0$	M1 A1	1.2 1.1b
	$\Rightarrow \sin\theta = \dfrac{-3+\sqrt{34}}{5} \Rightarrow \theta = ...$	dM1	3.1a
	$\Rightarrow \theta = 34.5°, 145.5°, 394.5°$	A1 A1	1.1b 1.1b
		(5)	
(ii) (a)	One of • They cancel by $\sin x$ (and hence they miss the solution $\sin x = 0 \Rightarrow x = 0$) • They do not find all the solutions of $\cos x = \frac{3}{5}$ (in the given range) or they missed the solution $x = -53.1°$	B1	2.3
	Both of the above	B1	2.3
		(2)	
(ii) (b)	Sets $5\alpha + 40° = 720° - 53.1°$	M1	3.1a
	$\alpha = 125°$	A1	1.1b
		(2)	
			(9 marks)

Notes

(i)

M1: Uses $\cos^2\theta = 1-\sin^2\theta$ to form a 3TQ in $\sin\theta$

A1: Correct 3TQ=0 $5\sin^2\theta + 6\sin\theta - 5 = 0$

dM1: Solves their 3TQ in $\sin\theta$ to produce one value for θ. It is dependent upon having used $\cos^2\theta = \pm 1 \pm \sin^2\theta$

A1: Two of awrt $\theta = 34.5°, 145.5°, 394.5°$ (or if in radians two of awrt 0.60, 2.54, 6.89)

A1: All three of awrt $\theta = 34.5°, 145.5°, 394.5°$ and no other values

(i) (a)

See scheme

(ii)(b)

M1: Sets $5\alpha + 40° = 666.9°$ o.e.

A1: awrt $\alpha = 125°$

Q14.

Question	Scheme	Marks
	A solution based around a table of results	
	(table below)	
	When n is odd, n^2 is odd (odd $\times$ odd = odd) so n^2+2 is also odd	M1
	So for all odd numbers n, n^2+2 is also odd and so cannot be divisible by 4 (as all numbers in the 4 times table are even)	A1
	When n is even, n^2 is even **and a multiple** of 4, so n^2+2 cannot be a multiple of 4	M1
	Fully correct and exhaustive proof. Award for both of the cases above plus a final statement "So for all n, n^2+2 cannot be divisible by 4"	A1*
		(4)

n	n^2	n^2+2	
1	**1**	**3**	**Odd**
2	**4**	**6**	**Even**
3	**9**	**11**	**Odd**
4	**16**	**18**	**Even**
5	**25**	**27**	**Odd**
6	**36**	**38**	**Even**

Question	Scheme	Marks
	Alternative - (algebraic) proof	
	If n is even, $n=2k$, so $\frac{n^2+2}{4}=\frac{(2k)^2+2}{4}=\frac{4k^2+2}{4}=k^2+\frac{1}{2}$	M1
	If n is odd, $n=2k+1$, so $\frac{n^2+2}{4}=\frac{(2k+1)^2+2}{4}=\frac{4k^2+4k+3}{4}=k^2+k+\frac{3}{4}$	M1
	For a partial explanation stating that • either of $k^2+\frac{1}{2}$ or $k^2+k+\frac{3}{4}$ are not a whole numbers. • with some valid reason stating why this means that n^2+2 is not a multiple of 4.	A1
	Full proof with no errors or omissions. This must include • The conjecture • Correct notation and algebra for both even and odd numbers • A full explanation stating why, for all n, n^2+2 is not divisible by 4	A1*
		(4)
		(4 marks)