Question 1

A particle P of mass 0.4 kg is attached to one end of a light elastic string of natural length 0.5 m and modulus of elasticity 6 N. The other end of the string is attached to a fixed point O. The particle P is released from rest at the point $(0.5 + x)$ m vertically below O. The particle P comes to instantaneous rest at O. **Take $g=10\text{ms}^{-2}$.**

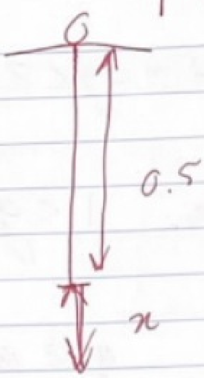
(i) Find x. [3]

(ii) Find the greatest speed of P. [5]

Question	Answer	Marks	Guidance
5(i)	$0.4g(0.5+x)=\frac{6x^2}{(2\times 0.5)}$	**M1**	Set up an energy equation
	$6x^2-4x-2=0$ or $3x^2-2x-1=0$	**M1**	Attempt to solve a 3 term quadratic equation
	$x=1$ (ignore $-\frac{1}{3}$ if seen)	**A1**	
		3	
5(ii)	$0.4g=\frac{6e}{0.5}$	**M1**	Use $T=\frac{\lambda x}{l}$ to find the extension at the equilibrium position
	$e=\frac{1}{3}$	**A1**	
	PE change $=0.4g\left(0.5+\frac{1}{3}\right)$	**B1ft**	Ft for candidate's e
	$\frac{0.4V^2}{2}=0.4g\left(0.5+\frac{1}{3}\right)-\frac{6\left(\frac{1}{3}\right)^2}{(2\times 0.5)}$	**M1**	Set up a three term energy equation
	$V=3.65\ \text{ms}^{-1}$	**A1**	
		5	

FM1 - Model Answers

① i



$g = 10$

$$0.4g(0.5+x) = \frac{6x^2}{2\times 0.5} = 6x^2$$

$$\Leftrightarrow \quad 6x^2 - 0.4g(x) - 0.2g = 0$$

$$6x^2 - 4x - 2 = 0$$

$x = 1$ or ~~$-\frac{1}{3}$~~ → ignore

$x = 1$ only

ii) $0.4g = \frac{6x}{0.5}$ $\therefore x = \frac{1}{3}$

P.E is $0.4g(0.5+\frac{1}{3})$

$\text{EPE} + \frac{0.4v^2}{2} = \text{PE}$ $\therefore$ $\frac{6(\frac{1}{3})^2}{2\times 0.5} + \frac{0.4v^2}{2} = 0.4g(0.5+\frac{1}{3})$

$v = 3.65\,\text{m/s}$

(a) A firework is instantaneously at rest in the air when it explodes into two parts. One part is the body B of mass 0.06 kg and the other a cap C of mass 0.004 kg. The total kinetic energy given to B and C is 0.8 J. B moves off horizontally in the **i** direction.

By considering both kinetic energy and linear momentum, calculate the velocities of B and C immediately after the explosion. **[8]**

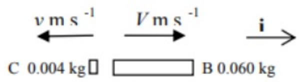
(b) A car of mass 800 kg is travelling up some hills.

In one situation the car climbs a vertical height of 20 m while its speed decreases from $30\,\mathrm{m\,s^{-1}}$ to $12\,\mathrm{m\,s^{-1}}$. The car is subject to a resistance to its motion but there is no driving force and the brakes are not being applied.

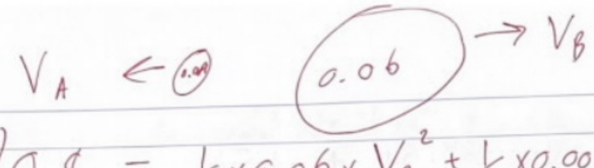
(i) Using an energy method, calculate the work done by the car against the resistance to its motion. **[4]**

In another situation the car is travelling at a constant speed of $18\,\mathrm{m\,s^{-1}}$ and climbs a vertical height of 20 m in 25 s up a uniform slope. The resistance to its motion is now 750 N.

(ii) Calculate the power of the driving force required. **[5]**

Q 2		m a r k	notes
(a)	 Energy: $\frac{1}{2}\times 0.004\times v^2+\frac{1}{2}\times 0.060\times V^2=0.8$ $v^2+15V^2=400$ PCLM in **i** direction: $0.06V-0.004v=0$ $v=15V$ Solving $(15V)^2+15V^2=400$ so $V^2=\frac{400}{240}=\frac{5}{3}$ and $\mathbf{V}=\sqrt{\frac{5}{3}}\mathbf{i}$ $\mathbf{v}=-15\sqrt{\frac{5}{3}}\mathbf{i}$ $(=-\sqrt{375}\mathbf{i})$	 M1 A1 M1 A1 M1 A1 F1 A1 8	 Use of KE in two terms in an equation. Any form PCLM. Accept sign errors. Any form Valid method for elimination of *v* or *V* from a linear and a quadratic Accept 1.29099…**i** Accept no direction Accept – 19.3649…**i** Accept no direction Second answer follows from first (Relative) directions indicated - accept diagram. Both speeds correct.
(b) (i)	W is work done by resistances on car $\frac{1}{2}\times 800\times(12^2-30^2)=-800\times 9.8\times 20+\text{W}$ W = – 145 600 so 145 600 J done by car against resistances	 M1 B1 A1 A1 4	 Use of WE. Must have KE, W and GPE. Allow -*W* Both KE terms. Accept sign error All correct with *W* or -*W* cao

Q 2		mark	notes
(ii)	**either**		
	The slope is 18 × 25 = 450 m long	B1	
	$\frac{800 \times 9.8 \times 20 + 750 \times 450}{25}$	M1	Use of P = (Work done) / (elapsed time) used for at least one work done term
		M1	WD is force × distance used for at least one force
		A1	Allow only sign errors both terms
	= 19 772 W	A1	cao.
	or		
	The angle of the slope is arcsin (1/22.5)	B1	
	$\left(800 \times 9.8 \times \frac{1}{22.5} + 750\right) \times 18$	M1	Use of $P = Fv$ used for at least one term
		M1	Attempt at weight component
		A1	Allow only sign errors both terms
	= 19 772 W	A1	cao.
		5	
		17	



(6) a) $0.8 = \frac{1}{2} \times 0.06 \times V_B^2 + \frac{1}{2} \times 0.004 \times V_A^2$

$V_A^2 + 15V_B^2 = 400$ (1)

PCLM:

$0.06 V_B - 0.004 V_A = 0V_A + 0V_B$

$\therefore 15V_B = V_A$ (2)

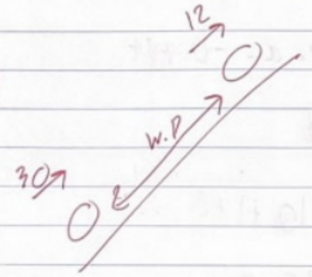
(2) → (1)

$(15V_B)^2 + 15V^2 = 400 \quad \therefore V_B^2 = \frac{400}{240} = \frac{5}{3}$

$\therefore V_B = \sqrt{\frac{5}{3}}$ to the left

$V_A = -15\sqrt{\frac{5}{3}}$ to the right

bi)



$\frac{1}{2} \times 800 \times (30^2 - 12^2) = 800g \times 20 + W.D$

$\therefore WD = 145\,600$ J

$= 146{,}000$ J

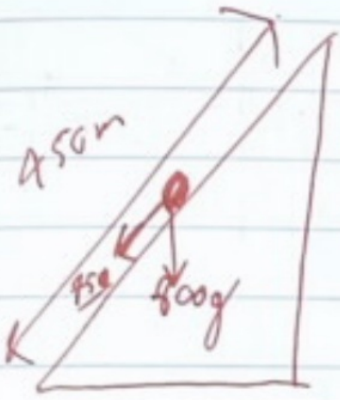
c) Power = $\frac{\text{work done}}{\text{time}}$ 18 m/s × 25s = 450 m

weight, Resistance

$= \frac{800 \times 9.8 \times 20 + 750 \times 450}{25\,s} = 19772\,W$

$= 19{,}700\,W$

time

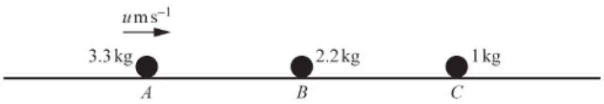


Question 7

OCR AS 2020 (FM1)

4(a) Three particles A, B and C are free to move in the same straight line on a large smooth horizontal surface. Their masses are 3.3 kg, 2.2 kg and 1 kg respectively. The coefficient of restitution in collisions between any two of them is e.

Initially, B and C are at rest and A is moving towards B with speed u ms^{-1} (see diagram). A collides directly with B and B then goes on to collide directly with C.



The velocities of A and B immediately after the first collision are denoted by v_A ms^{-1} and v_B ms^{-1} respectively.

- Show that $v_A = \frac{u(3-2e)}{5}$.
- Find an expression for v_B in terms of u and e. **[4]**

(b) Find an expression in terms of u and e for the velocity of B immediately after its collision with C. **[4]**

(c) After the collision between B and C there is a further collision between A and B.

Determine the range of possible values of e. **[4]**

4	a		1st Collision: $3.3u = 3.3v_A + 2.2v_B$ 1st Collision: $\pm e = \frac{v_B - v_A}{u}$ $3u = 3v_A + 2v_B$ and $2eu = 2v_B - 2v_A$ $\Rightarrow 5v_A = 3u - 2eu \Rightarrow v_A = \frac{u(3-2e)}{5}$ $3u = 3v_A + 2v_B$ and $3eu = 3v_B - 3v_A$ $\Rightarrow 5v_B = 3u + 3eu \Rightarrow v_B = \frac{3u(1+e)}{5}$	M1 (AO3.1b) M1 (AO3.1b) A1 (AO1.1) A1 (AO1.1) [4]	Conservation of momentum NEL AG find v_B by elimination or substitution	Must be seen Must be seen AEF - award if seen in (b)
	b		2nd Collision: $2.2 \times \frac{3u(1+e)}{5} = 2.2V_B + V_C$ 2nd Collision: $\pm e = \frac{V_C - V_B}{\left(\frac{3u(1+e)}{5}\right)}$ $\frac{33}{25}u(1+e) = \frac{11}{5}V_B + V_C$ & $\frac{3eu(1+e)}{5} = V_C - V_B$ $\Rightarrow \frac{16}{5}V_B = \frac{33}{25}u(1+e) - \frac{3eu(1+e)}{5}$ $\Rightarrow V_B = \frac{3u(1+e)(11-5e)}{80}$	M1ft (AO3.3) M1ft (AO3.3) M1ft (AO3.1b) A1 (AO1.1) [4]	Conservation of momentum (ft their value of V_B) NEL (ft their value of V_B) Attempt to eliminate V_C oe	May be in terms of V_A $V_C = \frac{33u(1+e)^2}{80}$ Must be in terms of e and u only.
	c		$\frac{u(3-2e)}{5} > \frac{3u(1+e)(11-5e)}{80}$ $3e^2 - 10e + 3 > 0$ A and B collide again => $e \neq 0$ $(3e - 1)(e - 3) > 0$ and $0 \le e \le 1$ and $e \neq 0$ $\Rightarrow 0 < e < \frac{1}{3}$	M1ft (AO3.4) M1 (AO1.1) B1 (AO2.2a) A1 (AO1.1) [4]	Correct condition for further collision (ft their V_B from (b) Rearranging to 3 term quadratic inequality in e $e < \frac{1}{3}$ is not sufficient for A1	If B1 not awarded then award A1 for $0 \le e < \frac{1}{3}$
			Total	12		

3. A light elastic string of natural length 1·5 m and modulus of elasticity 490 N has one end attached to a fixed point *A* and the other end attached to a particle *P* of mass 30 kg. Initially, *P* is held at rest vertically below *A* such that the distance *AP* is 0·6 m. It is then allowed to fall vertically.

(a) Calculate the distance *AP* when *P* is instantaneously at rest for the first time, giving your answer correct to 2 decimal places. [8]

(b) Estimate the distance *AP* when *P* is instantaneously at rest for the second time and clearly state one assumption that you have made in making your estimate. [2]

Q	**Solution**	**Mark**	**Notes**
3(a)	Let x be the extension in the string when P is instantaneously at rest for the 1st time.		
	Loss in PE = mgh	M1	attempted, h a distance.
	$= 30\times9.8(0.9+x)$	A1	
	Gain in EE = $\frac{1}{2}\times\lambda\frac{x^2}{l}$	M1	attempted
	$= \frac{1}{2}\times490\frac{x^2}{1.5}$	A1	
	Conservation of energy	M1	
	$\frac{1}{2}\times490\frac{x^2}{1.5} = 30\times9.8(0.9+x)$	A1	
	$x^2 - 1.8x - 1.62 = 0$	m1	attempt to solve quadratic.
	$x = \frac{1\cdot8\pm\sqrt{1\cdot8^2+4\times1\cdot62}}{2}$		
	$x = 2.4588$		
	$AP = 3.96$ (m)	A1	cao
3(b)	When P is instantaneously at rest for the 2nd time $AP = 0.6$ (m)	B1	
	External resistance to motion have been assumed to be negligible.	B1	

11 Two uniform small smooth spheres A and B have equal radii and equal masses. The spheres are on a smooth horizontal surface. Sphere A is moving at an acute angle α to the line of centres, when it collides with B, which is stationary.

After the impact A is moving at an acute angle β to the line of centres. The coefficient of restitution between A and B is $\frac{1}{3}$.

(a) Show that $\tan\beta = 3\tan\alpha$. **[5]**

(b) Explain why the assumption that the contact between the spheres is smooth is needed in answering part **(a)**. **[1]**

It is given that A is deflected through an angle γ.

(c) Determine, in terms of α, an expression for $\tan\gamma$. **[2]**

(d) Determine the maximum value of γ. You do not need to justify that this value is a maximum. **[5]**

11	(a)		$mu\cos\alpha = mv_1 + mv_2$	M1*	3.3	Use of conservation of linear momentum – correct number of terms	*m* is the mass of A and B, v_1 is the component of the velocity of A parallel to the line of centres after impact and v_2 is the equivalent component for B
			$v_1 - v_2 = -\frac{1}{3}u\cos\alpha$	M1*	3.3	Use of Newton's experimental law – correct number of terms and consistent with conservation of linear momentum	
			$v_1 = \frac{1}{3}u\cos\alpha$	A1	1.1		
			$\tan\beta = \frac{u\sin\alpha}{v_1}$	M1dep*	3.4	Use of tan ratio for β with their v_1	
			$\tan\beta = \frac{u\sin\alpha}{\frac{1}{3}u\cos\alpha} \Rightarrow \tan\beta = 3\tan\alpha$	A1	2.2a	**AG** – sufficient working must be shown as answer given	
				[5]			
11	(b)		The component of the velocity of A perpendicular to the line of centres does not change	B1	3.5b		
				[1]			

11	(c)		$\tan\gamma = \tan(\beta-\alpha) = \dfrac{3\tan\alpha - \tan\alpha}{1+(3\tan\alpha)(\tan\alpha)}$	M1	3.1b	Use of a correct compound-angle formula for $\tan(\beta \pm \alpha)$ and substitute given result from **(a)**	
			$\tan\gamma = \dfrac{2\tan\alpha}{1+3\tan^2\alpha}$	A1	1.1		
				[2]			
11	(d)			M1*	3.1b	Attempt to differentiate using quotient rule	
			$\left(\sec^2\gamma\dfrac{d\gamma}{d\alpha}=\right)$ $\dfrac{(1+3\tan^2\alpha)(2\sec^2\alpha)-(2\tan\alpha)(6\tan\alpha\sec^2\alpha)}{(1+3\tan^2\alpha)}=0$	A1	1.1	Correct derivative equated to zero	
			$1+3\tan^2\alpha-6\tan^2\alpha=0 \Rightarrow \tan\alpha=\dfrac{1}{\sqrt{3}}$	M1dep*	1.1	Find value of $\tan\alpha$ or $\tan^2\alpha$	$\alpha=\dfrac{\pi}{6}$
			$\tan\gamma=\dfrac{2\left(\frac{\sqrt{3}}{3}\right)}{1+3\left(\frac{\sqrt{3}}{3}\right)^2} \Rightarrow \gamma=...$	M1	1.1	Substitute their value for α or $\tan\alpha$ into their expression for $\tan\gamma$ - dependent on both previous M marks	$\tan\gamma=\dfrac{\sqrt{3}}{3}$
			$\gamma=\dfrac{\pi}{6}$	A1	1.1		
				[5]			

THE ASSOCIATED EXAMINING BOARD
General Certificate of Education

Advanced Level Examination

June 1993

MATHEMATICS—APPLIED **A/MATH/4**

Mathematics Paper 4
(0602/2)

Monday 21 June 9.30 am to 12.30 pm

Time allowed: 3 hours

4. Figure 1 shows a horizontal rectangular billiard table $ABCD$ with pockets at A, B, C and D. A small uniform smooth billiard ball P is stationary at a point on the table whose distances from AD, BC and AB are $9a$, $16a$ and $12a$ respectively, where a is a constant. A second identical billiard ball Q is travelling with speed u on the table in a direction parallel to DA when it strikes ball P obliquely.

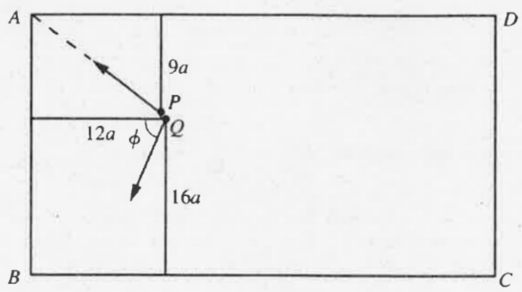


Figure 1

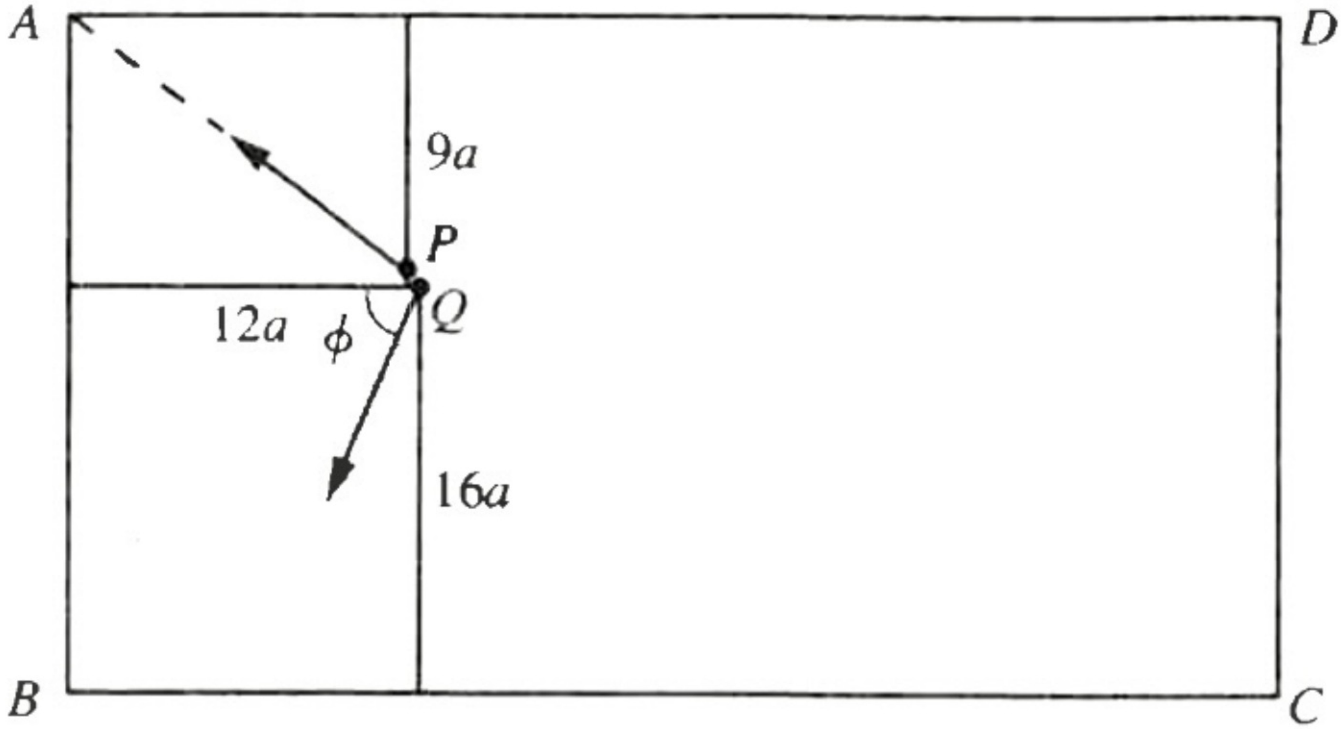
As a result of the collision ball P falls into the pocket at A and the direction of motion of ball Q is deflected through an angle ϕ. Given that P and Q are of equal mass and that the coefficient of restitution between the balls is e, show that

$$\tan\phi = \frac{6(e+1)}{17-8e}.$$ *(11 marks)*

Given that ball Q falls into the pocket at B, find

(a) the coefficient of restitution between the balls, *(2 marks)*

(b) the angle between the directions of motion of P and Q immediately after the impact. *(2 marks)*


A
D
9a
P
12a
ϕ
Q
16a
B
C

4. Figure 1 shows a horizontal rectangular billiard table $ABCD$ with pockets at A, B, C and D. A small uniform smooth billiard ball P is stationary at a point on the table whose distances from AD, BC and AB are $9a$, $16a$ and $12a$ respectively, where a is a constant. A second identical billiard ball Q is travelling with speed u on the table in a direction parallel to DA when it strikes ball P obliquely.

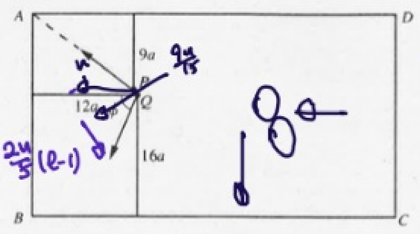


Figure 1

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$$\tan\phi = \frac{6(e+1)}{17-8e}. \qquad (11 \text{ marks})$$

Given that ball Q falls into the pocket at B, find

(a) the coefficient of restitution between the balls, (2 marks)

(b) the angle between the directions of motion of P and Q immediately after the impact. (2 marks)

+ve ↑ , → +ve

$9a$ ↑ , α , $12a$, u

$u_Q = \begin{pmatrix} u \\ 0 \end{pmatrix}$ $u_P = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$V_Q = \begin{pmatrix} ? \end{pmatrix}$ $V_P = \begin{pmatrix} 12k \\ 9k \end{pmatrix}$

impulse $= mv - mu$

$= k\begin{pmatrix} 12 \\ 9 \end{pmatrix}$ $m = \frac{9}{12}$

common tangent $\Rightarrow m = \frac{-12}{9}$

$\frac{\begin{pmatrix} u \\ 0 \end{pmatrix}\cdot\begin{pmatrix} 12 \\ 9 \end{pmatrix}}{15} = \frac{12u}{15} = u_Q$ in // direction

$\frac{12u}{15}$ → [Q] [P] rest

← x → y

$\frac{12u}{15} = -x + y$ (CLM) → ①

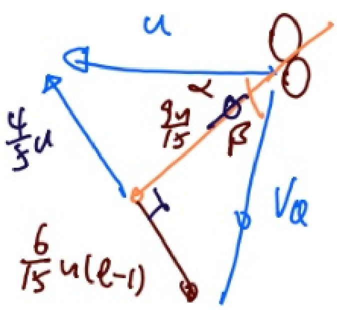
$e = \frac{x+y}{\frac{12u}{15}}$ (NLE) → ②

$x + y = \frac{12ue}{15}$ — ②

$\frac{\begin{pmatrix} u \\ 0 \end{pmatrix}\begin{pmatrix} -9 \\ 12 \end{pmatrix}}{15} = \frac{-9u}{15}$ in ⊥ direction

① + ② $2y = \frac{12u}{15}(e+1)$

$y = \frac{6u}{15}(e+1)$



② − ① $2x = \frac{12u}{15}(e-1)$

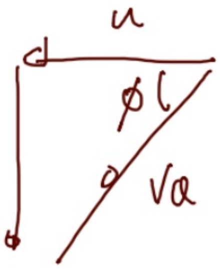
$x = \frac{6u}{15}(e-1)$

$\tan\alpha = \frac{4}{3}$

$\tan\beta = \frac{2}{3}(e-1)$

$\frac{\frac{4}{3} + \frac{2}{3}(e-1)}{1 - \frac{8}{9}(e-1)} = \tan(\alpha+\beta)$

$= \frac{12 + 6e - 6}{9 - 8e + 8} = \frac{6(e+1)}{17-8e}$ //



$$\tan\phi = \frac{16}{12} = \frac{6(e+1)}{17-8e}$$

$$\frac{4}{3} = \frac{6(e+1)}{17-8e}$$

$$68 - 32e = 18e + 18$$

$$50e = 50$$

$$e = 1$$

$$y = \frac{12}{15}u$$

$$x = 0$$

Note that,
Since one particle was at rest, it'll move in the direction of the impulse which is arctan(9/12)
So the other particle must move perpendicular to that, since it goes in the other pocket
So x must be zero {arctan(x/y) + arctan(y/x) = 90}