FM 1 worked Solutions

①

$I = 1.5\{vi - (4i + 6j)\}$

$= 1.5\{(v-4)i + 6j\}$

$|I| = 15$

$15^2 = 1.5^2[(v-4)^2 + 6^2]$

$\frac{15^2}{1.5^2} = 100$ $\quad$ $100 = (v-4)^2 + 36$

Now $64 = (v-4)^2$ So $v-4 = \pm 8$

So $v = +8 + 4 = \underline{12}$ or $v = -8 + 4 = \underline{-4}$

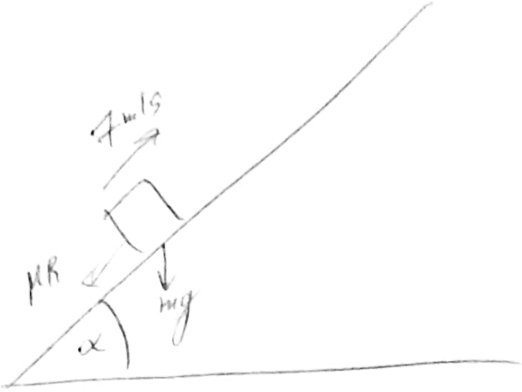
②

$\tan(\alpha) = \frac{3}{4}$

$\cos(\alpha) = \frac{4}{5}$

$\sin(\alpha) = \frac{3}{5}$

a)



Max Friction $= \frac{1}{4} mg\cos(\alpha) = \frac{1}{4} \times \frac{4}{5} mg$ ~~cos(α)~~

$= \frac{1}{5} mg$

weight $mg\sin(\alpha) = \frac{3mg}{5} > \frac{mg}{5}$ ∴ The weight is larger than the friction, so the box will slide back down.

b) $\frac{1}{2} \cancel{m} (7)^2 = 2 \times \left[\frac{\cancel{m}g}{5} \times \frac{25}{8}\right] + \frac{1}{2} \cancel{m} V^2$

$\frac{49}{2} = \frac{10g}{8} + \frac{V^2}{2} \Rightarrow \frac{V^2}{2} = \frac{49}{4} \Rightarrow V^2 = \frac{49}{2}$

so $V = 4.9$ (4.95) m/s

c) Include air resistance & have a more accurate value of g

3) $P = 15 \times F_1$ and $P = 10 \times F_2$

$\therefore F_1 - R = 600 \times 0.2$

So $\frac{P}{15} - R = 600 \times 0.2$

Up the slope, we get:

$F_2 - R - 600g \sin(\theta) = 0$

So $\frac{P}{10} - R - 30g = 0$

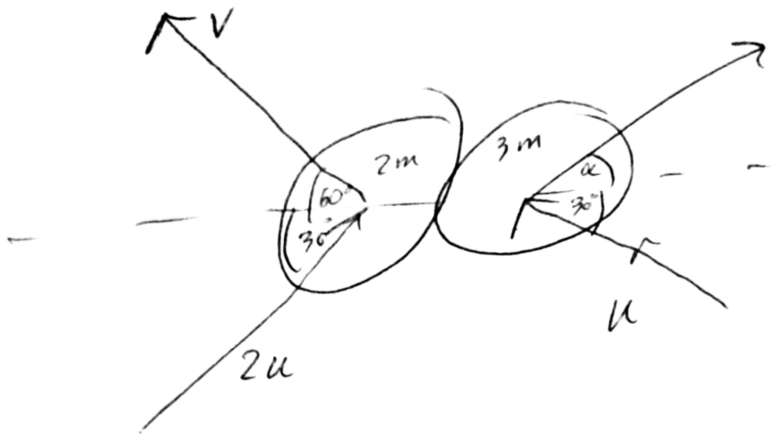
$\frac{P}{15} - R = 120$

$-\left(\frac{P}{10} - R - 30g = 0\right)$

$\frac{P}{15} - \frac{P}{10} = 120 - 30g$

$-\frac{P}{30} = -174$ $\therefore P = 5220$

4)



a) $2u\sin(30) = V\sin(60) \quad \therefore V = \frac{2u}{\sqrt{3}}$ or $\frac{2\sqrt{3}u}{3}$

b) CLM

$2\cancel{m}(2u\cos(30)) - (3\cancel{m} \times u\cos(30)) = 3\cancel{m} \times W\cos(\alpha) - 2\cancel{m} \times V\cos(\alpha)$

$u\cos(30) + V = 3w\cos(\alpha)$

$w\cos(\alpha) = \frac{1}{3}\left(\cancel{\frac{2u}{\sqrt{3}}}\frac{\sqrt{3}}{2}u + \frac{2\sqrt{3}}{3}u\right) = \frac{7\sqrt{3}}{18}u$

$u\sin(30) = w\sin(\alpha) = \frac{u}{2}$

$\tan(\alpha) = \frac{u}{2} \div \frac{7\sqrt{3}}{18}u = \frac{u}{2} \times \frac{18}{7\sqrt{3}u} = \frac{9}{7\sqrt{3}}$

~~angle~~ deflection is $150° - \alpha = 113.4°$.

c) NLR: $W\cos(\alpha) + V\cos(60) = e(3u\cos(30))$

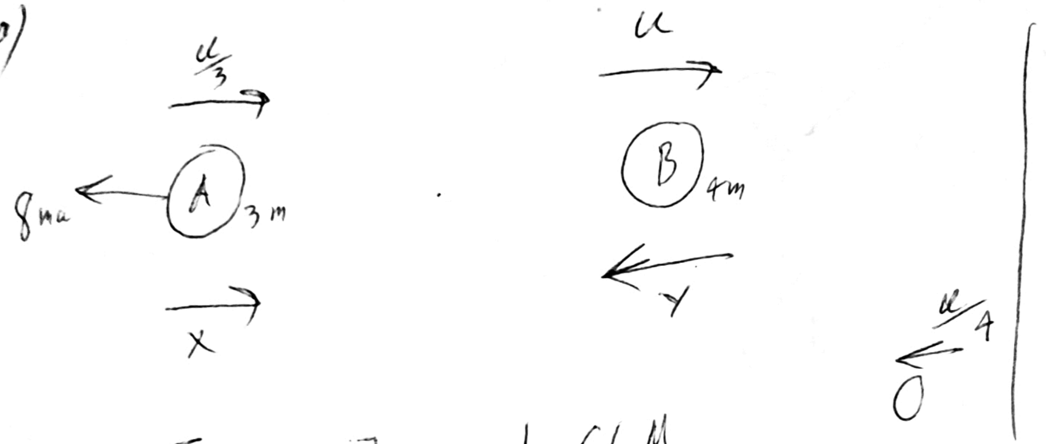
$\frac{7\sqrt{3}}{18}\cancel{u} + \frac{2\sqrt{3}}{3}\cancel{u} \times \frac{1}{2} = e \times \frac{3\sqrt{3}}{2}\cancel{u}$

$\frac{13\sqrt{3}}{..} = e\frac{3\sqrt{3}}{..} \quad \therefore e = \frac{13}{27}$

d) Impulse between spheres acts parralled to the plane

$\therefore$ momentum is conserved horizontally

(9) a)



$$-8mu = 3m\left[\frac{u}{3} - X\right]$$

$$\therefore \; -8mu = mu - 3mX$$

$$\therefore \; 3mX = 9mu$$

$$\underline{X = 3u}$$

CLM

$$3m(X) - 4mY = 3m\left(\frac{u}{3}\right) + 4mu$$

$$9u - 4Y = u + 4u$$

$$4u = 4Y \text{ so } \underline{u = Y}$$

$$e = \frac{u - \frac{u}{3}}{3u + u} = \frac{\frac{2u}{3}}{4u} = \frac{2u}{12u} = \frac{1}{6}$$

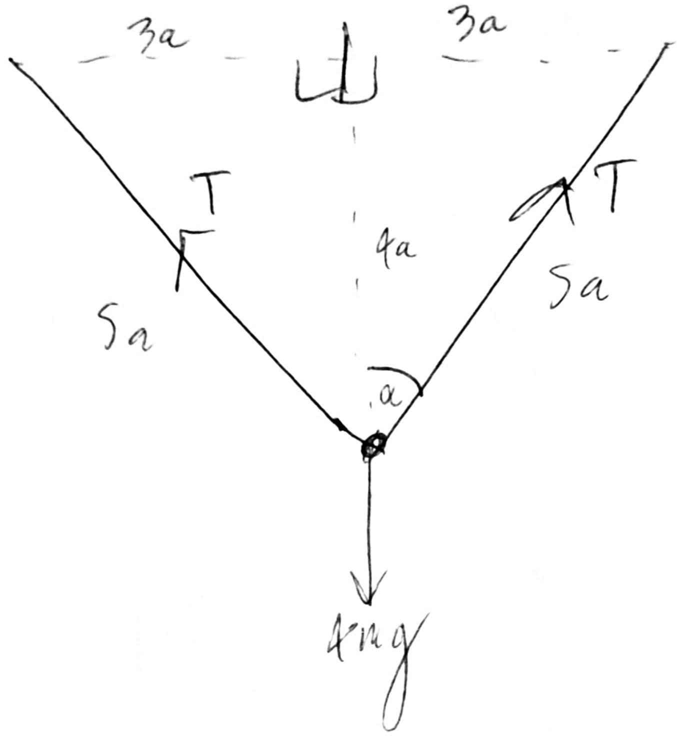
b) $e_{wall} = \frac{1}{4}$

When B hits wall, gap is $\frac{2d}{3}$ as relative speed is $\underline{u - \frac{u}{3} = \frac{2u}{3}}$

Time to close the gap is $\dfrac{\frac{2d}{3}}{X + \text{rebound speed}} = \dfrac{\frac{2d}{3}}{\frac{u}{3} + \frac{u}{4}} = \dfrac{\frac{2d}{3}}{\frac{7u}{12}} = \underline{\frac{8d}{7u}}$

Distance of B from wall is $\frac{8d}{7u} \times \frac{u}{4} = \underline{\frac{2d}{7}}$

6 a)



$\cos(\alpha) = \frac{4}{5}$

$$2T \times \frac{4}{5} = 4mg$$

$$T = \frac{\lambda\left(5a - \frac{x_c}{2}\right)}{\frac{x_c}{2}} = \frac{5mg\left(5a - \frac{x_c}{2}\right)}{3\frac{x_c}{2}}$$

$$\text{So } 2T \times \frac{4}{5} = \frac{20mg\left(5a - \frac{x_c}{2}\right)}{3x_c} \times \frac{4}{5} = \frac{16mg\left(5a - \frac{x_c}{2}\right)}{3x_c}$$

$$3x_c \times 4\cancel{mg} = 16\cancel{mg}\left(5a - \frac{x_c}{2}\right)$$

$$12x_c = 80a - 8x_c \quad \therefore 20x_c = 80a$$

$$\text{so } x_c = \underline{4a}$$

b) Max speed when $a = 0$, i.e. at equilibrium

So Elastic Energy before + GPE before = Find K.E + E.P.E after

$$\frac{5mg}{3\times 2\times 4a}(2a)^2 + 4mg(4a) = \frac{1}{2}(4m)V^2 + \frac{5mg}{3\times 2\times 4a}(6a)^2$$

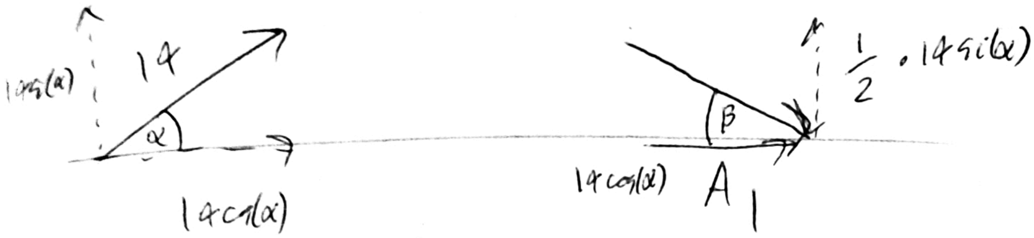
$$\Rightarrow \frac{20mg\,a^2}{24a} + 16mga = 2mV^2 + \frac{180mga^2}{24a}$$

$$\frac{5mga}{6} + 16mga = 2mV^2 + \frac{15mga}{2}$$

$$\therefore \frac{28mga}{3} = 2mV^2 \Rightarrow \frac{14ga}{3} = V^2$$

$$\text{So } V = \sqrt{\frac{14ga}{3}}$$

7)



$$\tan(\beta) = \frac{7\sin(\alpha)}{14\cos(\alpha)} = \frac{1}{2}\tan(\alpha)$$

$$\tan(\alpha) = \frac{3}{4} \quad \therefore \quad \frac{3}{8} = \tan(\beta) \quad \text{so} \quad \beta = 20.6^\circ$$

b) Freely under gravity $\Rightarrow$ No resistance to vertical motion from air.

So vertical component down is the same the one up at start.

c)

$$-14\sin(\alpha) = 14\sin(\alpha) - gt_1 \quad \therefore t_1 = \frac{28\sin(\alpha)}{g}$$

$$-7\sin(\alpha) = 7\sin(\alpha) - gt_2 \quad \therefore t_2 = \frac{14\sin(\alpha)}{g}$$

$$T = t_1 + t_2 = \frac{42\sin(\alpha)}{g} = \frac{42}{g} \times \frac{3}{5} = \frac{18}{7} \approx 2.57s$$

d) Horizontal component is always $14\cos(\alpha)$

Vertical components form a geometric sequence.

When at A_n, Vertical component is $\left(\frac{1}{2}\right)^n 14\sin(\alpha)$

$$\tan(\phi) = \frac{\left(\frac{1}{2}\right)^n 14\sin(\alpha)}{14\cos(\alpha)} = \left(\frac{1}{2}\right)^n \tan(\alpha) = \frac{3}{4\cdot 2^n} = \frac{3}{2^{n+2}}$$

e) Infinitely many bounces, with the max height decreasing each time.

f) All ~~energy~~ momentum vertically will be removed, so only the horizontal component remains. The ball will move with speed $14\cos(\alpha) = 14\times\frac{4}{5} = \underline{11.2\ m/s}$.