

# 12Fm October Exam 2022

Total 90 marks, 1 hour 45 minutes

## Topic List

- Pure:
  1. Indices
  2. Quadratics
  3. Equations and Inequalities
  4. Graphs and Transformations
  5. Lines and Circles (Coordinate Geometry)
  6. Factor Theorem
  7. First half of Pure Chapter 11, Vectors.
- Stats:
  1. Probability
  2. Sampling (no LDS)
- Mechanics:
  1. Kinematics (with equations)
  2. Kinematics (with graphs)

## Section A: Pure

1.

$$f(x) = 2x^3 - 7x^2 - 10x + 24$$

(a) Use the factor theorem to show that  $(x + 2)$  is a factor of  $f(x)$

**(2)**

(b) Hence factorise  $f(x)$  completely

**(3)**

In this question you should show all stages of your working.  
Solutions relying on calculator technology are not acceptable.

**2.**

(a) Solve

$$3^x = \frac{1}{27}$$

**(2)**

(b)

$$\frac{2^{x+1}}{4^{y-2}} = 64$$

Express  $y$  in terms of  $x$ , writing your answer in simplest form

**(3)**

**3.**

(a) On separate axes sketch the graphs of

(i)  $y = -3x + c$  where  $c$  is a positive constant

(ii)  $y = \frac{1}{x} + 5$

On each sketch show the coordinates of any point at which the graph crosses the  $y$ -axis and the equation of any horizontal asymptote.

(4)

Given that the two curves in part (a) meet at two distinct points,

(b) Show that  $(5 - c)^2 > 12$

(3)

(c) **Hence** find the set of possible values for  $c$ .

(4)

4.

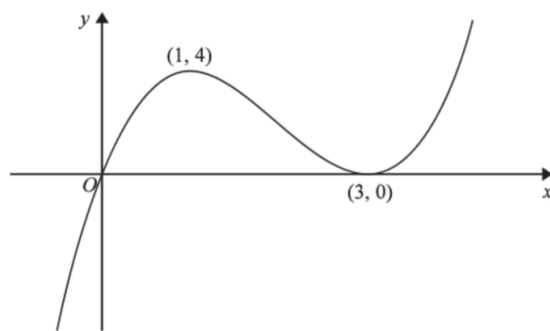


Figure 1

Figure 1 shows a sketch of the curve with equation  $y = f(x)$  where

$$f(x) = x(3 - x)^2 \quad x \in \mathbb{R}$$

The curve passes through the origin and touches the  $x$ -axis at the point  $(3, 0)$ . There is a maximum point at  $(1, 4)$  and a minimum point at  $(3, 0)$ .

(a) On separate diagrams, sketch the curve with equation

(i)  $y = f\left(\frac{1}{2}x\right)$

(ii)  $y = f(x + 2)$

On each sketch indicate clearly the coordinates of

- any points where the curve crosses or touches the  $x$ -axis,
- any points where the curve crosses or touches the  $y$ -axis,
- any maximum or minimum points.

(6)

Given that the curve with equation  $y = f(x) + k$ , where  $k$  is a non-zero constant, has a maximum point at  $(a, 0)$ .

(b) State the values of  $a$  and  $k$

(2)

5.

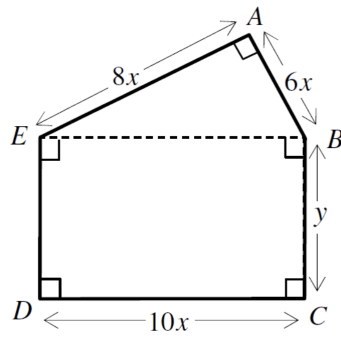


Figure 2

Figure 2 shows a pentagon  $ABCDE$  whose measurements, in cm, are in terms of  $x$  and  $y$ .

(a) If the perimeter of the pentagon is 120 cm, shows that its area  $A$  cm<sup>2</sup> is given by

$$A = 600x - 96x^2 \tag{4}$$

(b) Determine the maximum value for the area of the pentagon and the corresponding value of  $x$  which produces this maximum area.

(5)

**6.**

Given that the point  $A$  has position vector  $3\mathbf{i} - 7\mathbf{j}$  and the point  $B$  has position vector  $8\mathbf{i} + 3\mathbf{j}$ .

(a) Find  $\overrightarrow{AB}$  (2)

(b) Find  $|\overrightarrow{AB}|$  (2)

(c) Find a unit vector in the direction of  $\overrightarrow{AB}$  (2)

(d) Find, as a bearing, the direction of  $\overrightarrow{AB}$  (2)

7.

The circle  $C$  has centre  $X(3, 5)$  and radius  $r$ .

The line  $l$  has equation  $y = 2x + k$ , where  $k$  is a constant.

(a) Show that  $l$  and  $C$  intersect when

$$5x^2 + (4k - 26)x + k^2 - 10k + 34 - r^2 = 0$$

(3)

Given that  $l$  is a tangent to  $C$ ,

(b) Show that  $5r^2 = (k + p)^2$ , where  $p$  is a constant to be found.

(3)

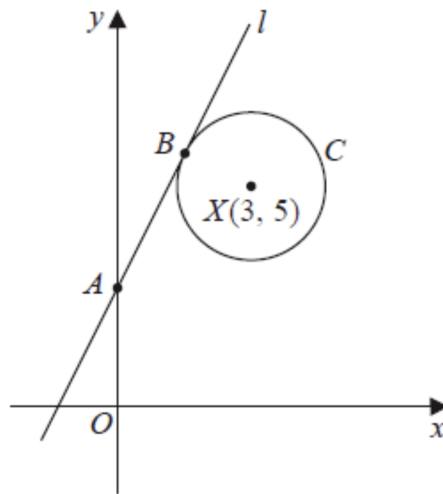


Figure 3

Figure 3 shows the line  $l$  which

- cuts the  $y$ -axis at the point  $A$
- and touches the circle  $C$  at the point  $B$

Given that  $AB = 2r$

(c) find the value of  $k$ .

(6)



## Section B: Mechanics

8.

Two small children, Ajaz and Beth, are running a 100m race along a straight horizontal track.

- They both start from rest, leaving the start line at the same time.
- Ajaz accelerates at  $0.8 \text{ ms}^{-2}$  up to a speed of  $4 \text{ ms}^{-1}$  and then maintains this speed until he crosses the finish line.
- Beth accelerates at  $1 \text{ ms}^{-2}$  for  $T$  seconds and then maintains a constant speed until she crosses the finish line.
- Ajaz and Beth cross the finish line at the same time.

- (a) Sketch on the same axes, a speed-time graph for each child, from the instant when they leave the start line to the instant when they cross the finish line. (3)
- (b) Find the time taken by Ajaz to complete the race. (4)
- (c) Find the value of  $T$  (4)
- (d) Find the difference in the speeds of the two children as they cross the finish line. (2)

**9.**

At time  $t = 0$ , a particle is projected vertically upwards with speed  $u$  from a point  $A$ . The particle moves freely under gravity. At time  $T$  the particle is at its maximum height  $H$  above  $A$ .

(a) Find  $T$  in terms of  $u$  and  $g$  **(2)**

(b) Show that  $H = \frac{u^2}{2g}$  **(2)**

The point  $A$  is at a height  $3H$  above the ground.

(c) Find, in terms of  $T$ , the total time from the instant of projection to the instant when the particle hits the ground. **(4)**

## Section C: Statistics

10.

The names of the 720 members of a swimming club are listed alphabetically in the club's membership book. The chairman of the swimming club wishes to select a systematic sample of 40 names. The names are numbered from 001 to 720 and a number between 001 and  $w$  is selected at random. The corresponding name and every  $x^{\text{th}}$  name thereafter are included in the sample.

(a) Find the value of  $w$ . (1)

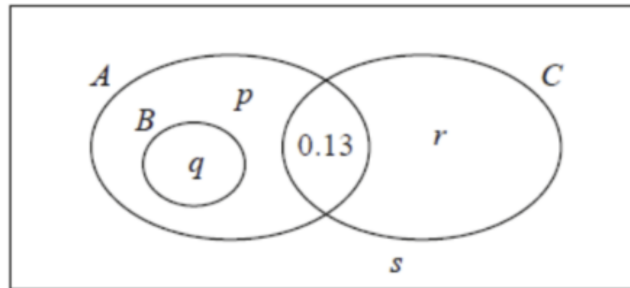
(b) Find the value of  $x$ . (1)

(c) Write down the probability that the sample includes both the first name and the third name in the club's membership book. (1)

(d) State one advantage and one disadvantage of systematic sampling in this case. (2)

11.

In the Venn diagram below,  $A$ ,  $B$ ,  $C$  are events and  $p$ ,  $q$ ,  $r$  and  $s$  are probabilities. The events  $A$  and  $C$  are independent and  $P(A) = 0.65$



(a) State which two of the events  $A$ ,  $B$ ,  $C$  are mutually exclusive.

(1)

(b) Find the value of  $r$  and the value of  $s$

(5)