Mr. Chan's 13Fm Further Mechanics 1
Oblique Collisions
Questions by Topic Pack

- Questions cover Year 13 Further Maths: Further Mechanics, Oblique Collisions
- Question by topic pack from Old specification M4, 2007-2019
- There are generally 2-3 questions per paper, with at least one easier one and one harder one per year
- Feel to to take a look at my Further Mechanics playlist, where I try to split question into types such as “sphere and spere”, ”sphere and wall” etc

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https://youtu.be/1VN0wDHhpss

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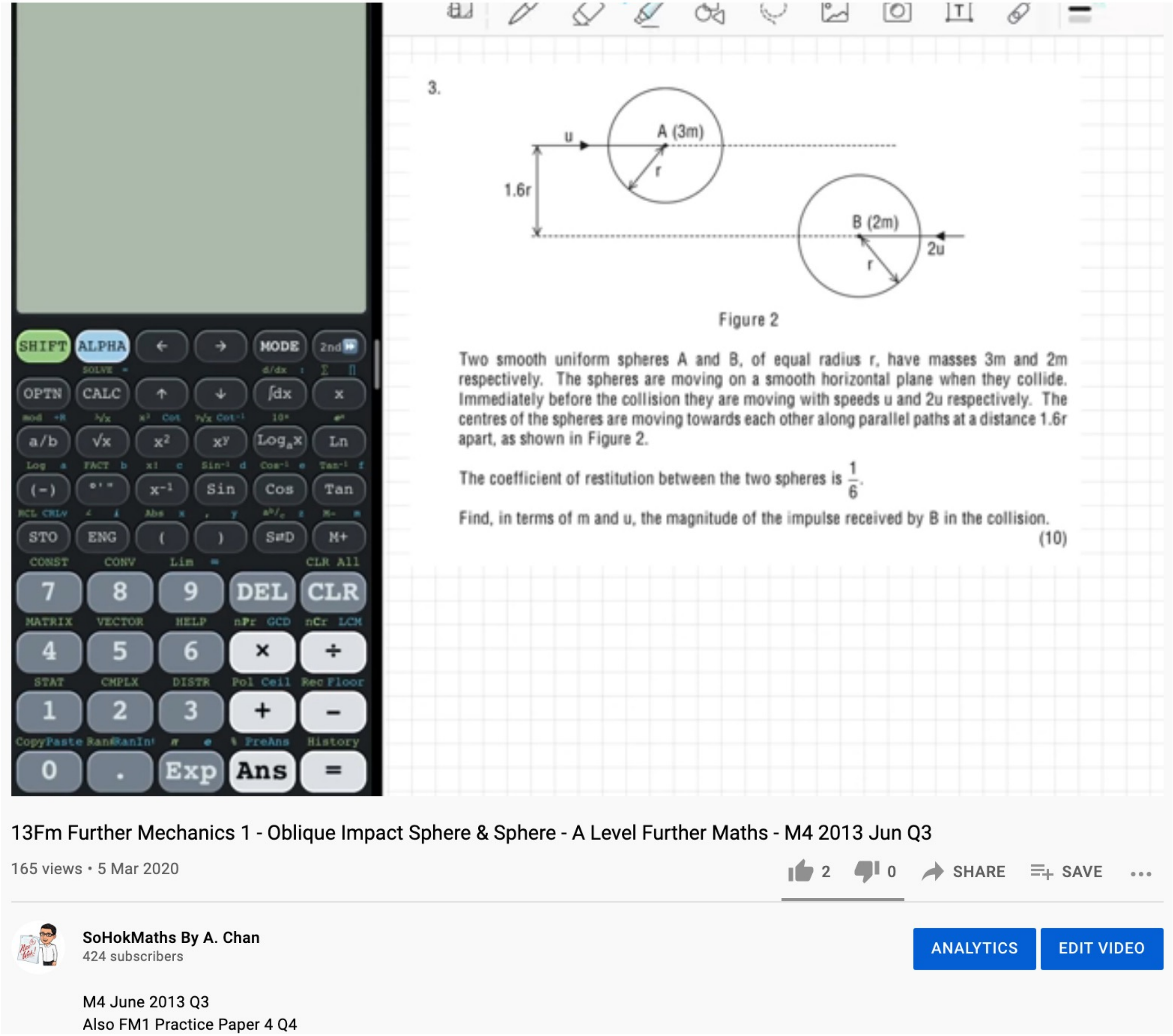

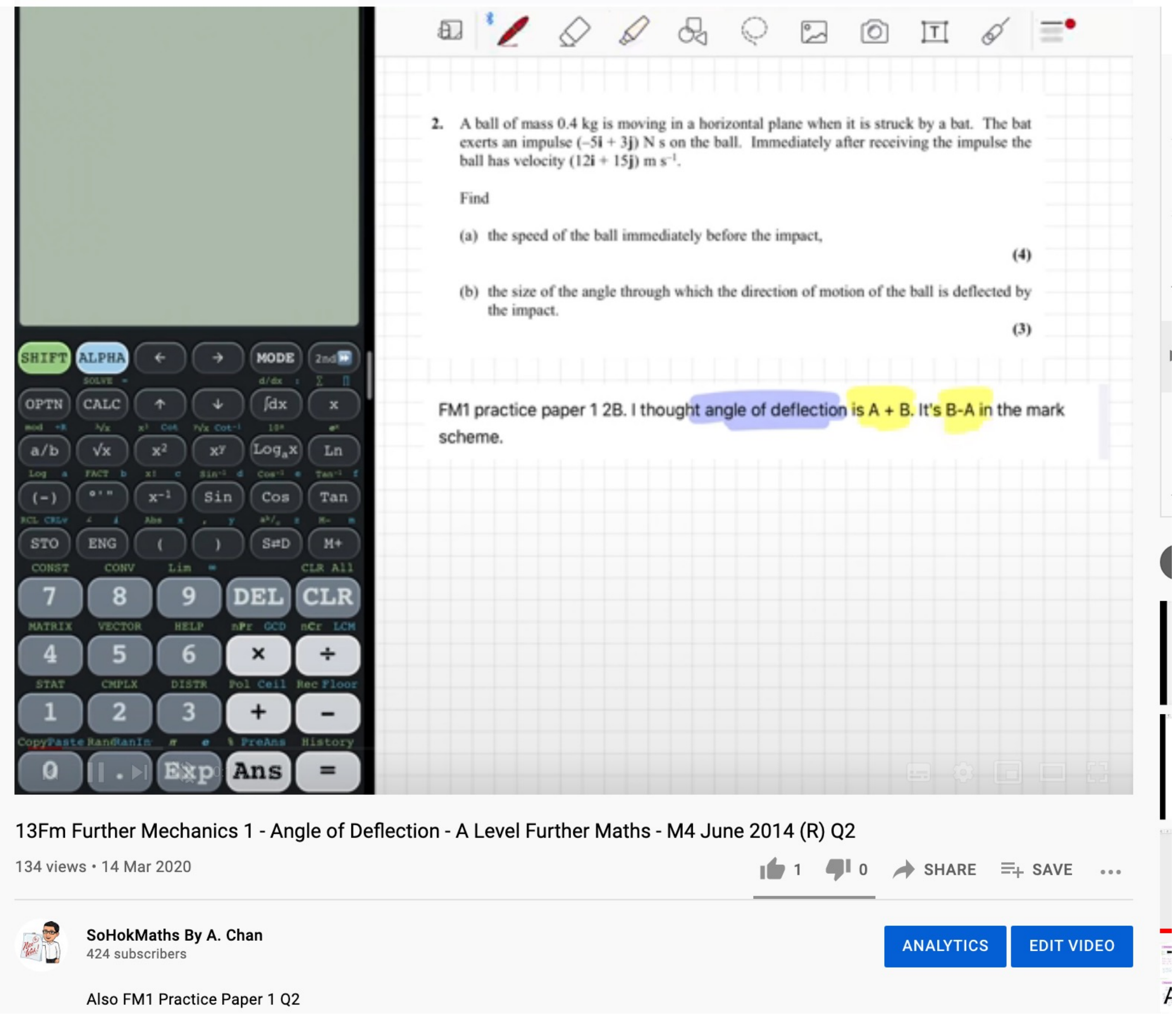

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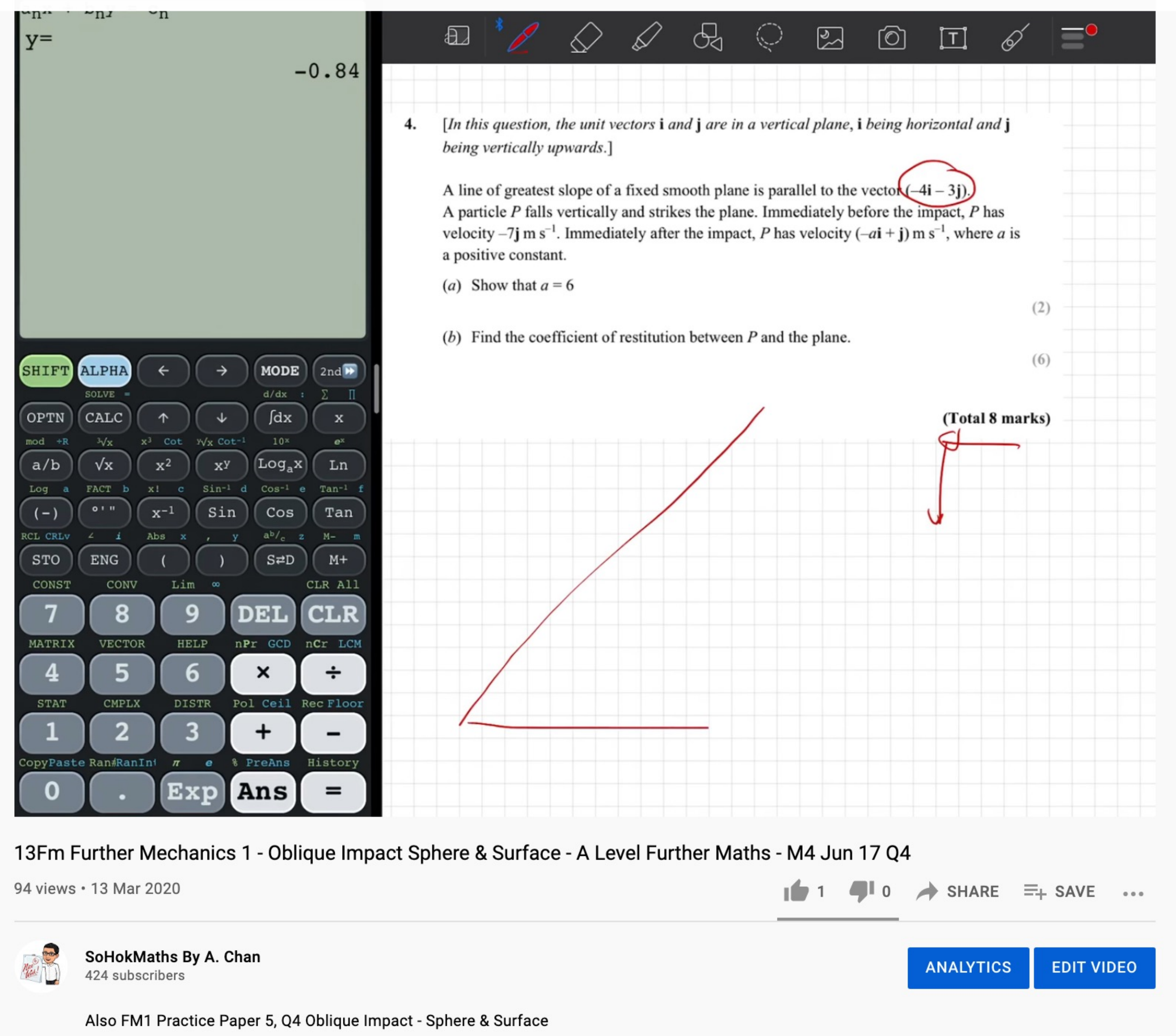


https://youtu.be/L7GeEH5_cz4

June 2007 M4

A smooth uniform sphere A has mass $2m$ kg and another smooth uniform sphere B, with the same radius as A, has mass m kg. The spheres are moving on a smooth horizontal plane when they collide. At the instant of collision the line joining the centres of the spheres is parallel to $\mathbf{j}$. Immediately **after** the collision, the velocity of A is $(3\mathbf{i} - \mathbf{j})$ m s^{-1} and the velocity of B is $(2\mathbf{i} + \mathbf{j})$ m s^{-1}. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(a) Find the velocities of the two spheres immediately before the collision.

(7)

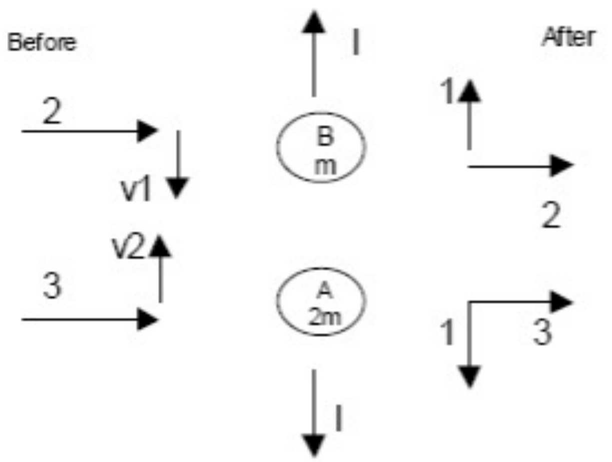
(b) Find the magnitude of the impulse in the collision.

(2)

(c) Find, to the nearest degree, the angle through which the direction of motion of A is deflected by the collision.

(4)

(Total 13 marks)

Question Number	Scheme		Marks
(a)			
	CLM: $2v_2 - v_1 = 1 - 2 = -1$		M1A1
	NIL: $1 + 1 = \frac{1}{2}(v_1 + v_2)$		M1A1
	$\therefore v_2 = 1, v_1 = 3$	*Dependent on both M's above*	DM1
	Horizontal components unchanged (i.e. 2 & 3)	*Independent of all other marks*	A1
	$\mathbf{v}_A = 3\mathbf{i} + \mathbf{j}$; $\mathbf{v}_B = 2\mathbf{i} - 3\mathbf{j}$		A1
			(7)
(b)			
	For B: $I = m(1-(-3)) = 4m$		M1A1
	(Or For A: $-I = 2m(-1 - 1)$ $\therefore I = 4m$)		(2)

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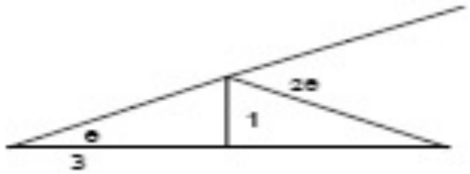
a)	M1 Conservation of momentum along the line of centres. Condone sign errors A1 equation correct M1 Impact law along the line of centres. *e* must be used correctly, but condone sign errors. A1 equation correct. The signs need to be consistent between the two equations M1 Solve the simultaneous equations for their v_1 and v_2. A1 **i** components correct – independent mark A1 v_A & v_B correct		
b)	M1 Impulse = change in momentum for one sphere. Condone order of subtraction. A1 Magnitude correct.		

(c)	$\begin{pmatrix}3\\1\end{pmatrix}\begin{pmatrix}3\\-1\end{pmatrix}=\sqrt{3^2+1^2}.\sqrt{3^2+(-1)^2}\cos\theta$	M1A1
	$\Rightarrow 8=10\cos\theta$	M1
	$\theta = 37^{\circ}$	A1
	Alternative:	(4)
	 M1	
	where $\tan\theta=\frac{1}{3}$ A1	
	required angle is 2θ M1A1	

c)	M1 Any complete method to find the trig ratio of a relevant angle. A1 $\cos\theta = \frac{4}{5}$, $\tan\frac{\theta}{2} = \frac{1}{3}$, ... Or M1 find angle of approach to the line of centres and angle after collision. A1 values correct. (both 71.56 .....) M1 solve for θ A1 37^0 (Q specifies nearest degree) Special case: candidates who act as if the line of centres is in the direction of **i**: CLM u+2v = 8 NIL v-u = 2 u=4/3, v=10/3 4/3i + j ; 10/3i − j Impulse 2m-4/3m = 2/3m $\frac{10+1}{\sqrt{10}\sqrt{\frac{109}{9}}} = \cos\theta$ θ = 1.70^0 Work is equivalent, so treat as a MR: M1A0M1A0M1A1A1 M1A1 M1A1M1A1		

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June 2008 M4

Two small smooth spheres A and B have equal radii. The mass of A is $2m$ kg and the mass of B is m kg. The spheres are moving on a smooth horizontal plane and they collide. Immediately before the collision the velocity of A is $(2\mathbf{i} - 2\mathbf{j})$ m s^{-1} and the velocity of B is $(-3\mathbf{i} - \mathbf{j})$ m s^{-1}. Immediately after the collision the velocity of A is $(\mathbf{i} - 3\mathbf{j})$ m s^{-1}.

Find the speed of B immediately after the collision.

(5)

(Total 5 marks)

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Question Number	Scheme	Marks
	$2m(2\mathbf{i}-2\mathbf{j})+m(-3\mathbf{i}-\mathbf{j})=2m(\mathbf{i}-3\mathbf{j})+m\mathbf{v}$	M1 A1
	$(\mathbf{i}-5\mathbf{j})=(2\mathbf{i}-6\mathbf{j})+\mathbf{v}$	
	$(-\mathbf{i}+\mathbf{j})=\mathbf{v}$	A1
	$\lvert\mathbf{v}\rvert=\sqrt{(-1)^2+1^2}=\sqrt{2}\text{ m s}^{-1}$ cwo	DM1 A1
		5

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June 2008 M4

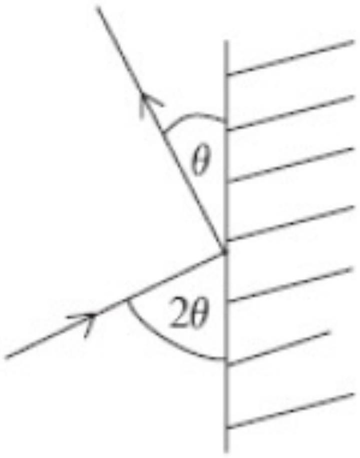

Figure 1

A small smooth ball B, moving on a horizontal plane, collides with a fixed vertical wall. Immediately before the collision the angle between the direction of motion of B and the wall is 2θ, where $0° < \theta < 45°$. Immediately after the collision the angle between the direction of motion of B and the wall is θ, as shown in Figure 1.

Given that the coefficient of restitution between B and the wall is $\frac{3}{8}$, find the value of $\tan \theta$.

(8)

(Total 8 marks)

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Question Number	Scheme	Marks
	$u\cos 2\theta = v\cos\theta$	M1 A1
	$\frac{3}{8}u\sin 2\theta = v\sin\theta$	M1 A1
	$3\tan 2\theta = 8\tan\theta$	M1
	$\dfrac{6\tan\theta}{1-\tan^2\theta} = 8\tan\theta$	M1
	$\tan^2\theta = \frac{1}{4} \quad (\tan\theta \neq 0)$	
	$\tan\theta = \frac{1}{2}$	M1 A1 **8**

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June 2009 M4

Two small smooth spheres A and B, of mass 2 kg and 1 kg respectively, are moving on a smooth horizontal plane when they collide. Immediately before the collision the velocity of A is $(\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of B is $-2\mathbf{i}$ m s^{-1}. Immediately after the collision the velocity of A is $\mathbf{j}$ m s^{-1}.

(a) Show that the velocity of B immediately after the collision is $2\mathbf{j}$ m s^{-1}.

(3)

(b) Find the impulse of B on A in the collision, giving your answer as a vector, and hence show that the line of centres is parallel to $\mathbf{i} + \mathbf{j}$.

(4)

(c) Find the coefficient of restitution between A and B.

(6)

(Total 13 marks)

Question Number	Scheme	Marks
(a)	CLM: $2(\mathbf{i}+2\mathbf{j}) + -2\mathbf{i} = 2\mathbf{j} + \mathbf{v}$ $\mathbf{v} = 2\mathbf{j}\ \text{m s}^{-1}$	M1 A1 A1 (3)
(b)	$\mathbf{I} = 2(\mathbf{j} - (\mathbf{i}+2\mathbf{j}))$ $= (-2\mathbf{i} - 2\mathbf{j})$ Ns Since $\mathbf{I}$ acts along l.o.c.c. , l.o.c.c is parallel to $\mathbf{i}+\mathbf{j}$	M1 A1 A1 B1 (4)
(c)	Before A: $(\mathbf{i}+2\mathbf{j}).\frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j}) = \frac{3}{\sqrt{2}}$ B: $-2\mathbf{j}.\frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j}) = \frac{-2}{\sqrt{2}}$ After A: $\mathbf{j}.\frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j}) = \frac{1}{\sqrt{2}}$ B: $2\mathbf{j}.\frac{1}{\sqrt{2}}(\mathbf{i}+\mathbf{j}) = \frac{2}{\sqrt{2}}$	M1 A3
	NIL: $e = \dfrac{\frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{\frac{3}{\sqrt{2}} - \frac{-2}{\sqrt{2}}} = \frac{1}{5}$	DM1 A1 (6) [13]

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June 2010 M4

Two smooth uniform spheres S and T have equal radii. The mass of S is 0.3 kg and the mass of T is 0.6 kg. The spheres are moving on a smooth horizontal plane and collide obliquely. Immediately before the collision the velocity of S is $\mathbf{u}_1$ m s^{-1} and the velocity of T is $\mathbf{u}_2$ m s^{-1}. The coefficient of restitution between the spheres is 0.5. Immediately after the collision the velocity of S is $(-\mathbf{i} + 2\mathbf{j})$ m s^{-1} and the velocity of T is $(\mathbf{i} + \mathbf{j})$ m s^{-1}. Given that when the spheres collide the line joining their centres is parallel to $\mathbf{i}$,

(a) find

(i) $\mathbf{u}_1$,

(ii) $\mathbf{u}_2$.

(6)

After the collision, T goes on to collide with a smooth vertical wall which is parallel to $\mathbf{j}$. Given that the coefficient of restitution between T and the wall is also 0.5, find

(b) the angle through which the direction of motion of T is deflected as a result of the collision with the wall,

(5)

(c) the loss in kinetic energy of T caused by the collision with the wall.

(3)

(Total 14 marks)

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(a)	$\uparrow 2$ $\quad\quad$ $\uparrow 1$		
	$1 \leftarrow$ $\quad\quad$ $\rightarrow 1$		
	S 0.3kg $\quad\quad$ T 0.6 kg		
	$2 \uparrow$ $\quad\quad$ $\uparrow 1$		
	$\rightarrow v$ $\quad\quad$ $w \leftarrow$		
	$0.3v - 0.6w = 0.3$	M1 A1	
	$v - 2w = 1$		
	$\frac{1}{2}(v + w) = 2$	M1 A1	
	$v + w = 4$		
	$w = 1, v = 3$		
	(i) $\mathbf{u}_1 = 3\mathbf{i} + 2\mathbf{j}$ (ii) $\mathbf{u}_2 = -\mathbf{i} + \mathbf{j}$	A1 A1	
			(6)
(b)	$\uparrow 1$		
	$v \leftarrow$		
	$v = 0.5$	B1	
	$1 \uparrow$		
	$\rightarrow 1$		
			
	$\tan\theta = 0.5$ $\quad\quad$ $\tan\theta =$ their v	M1	
	$\theta = 26.6$	A1	
	their $\theta + 45°$	M1	
	Defln angle $= 45 + 26.6 = 71.6°$	A1	
			(5)
(c)	KE Loss $= \frac{1}{2} \times 0.6 \times \{(1^2 + 1^2) - (1^2 + v^2)\}$	M1 A1	
	$= 0.225$ J	A1	
			(3)
			14

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June 2011 M4

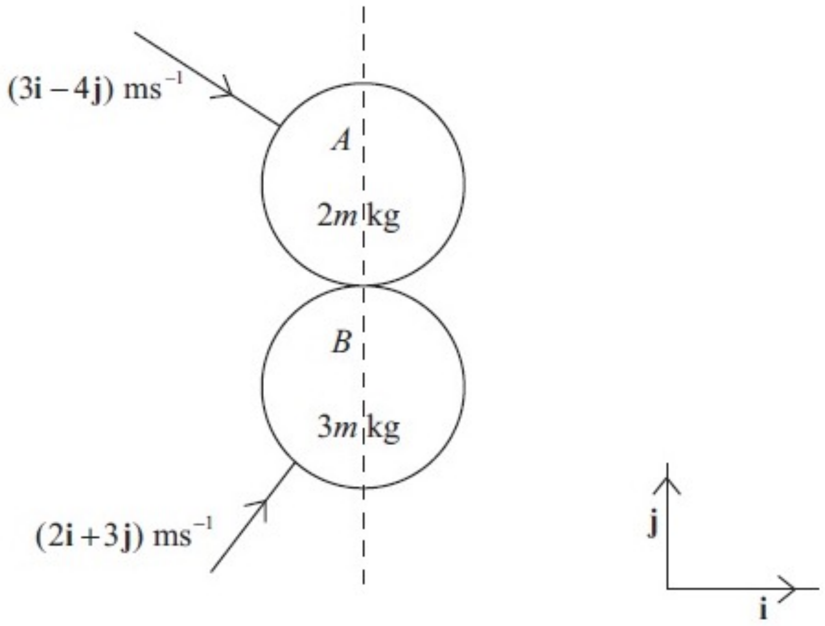

Figure 1

Two smooth uniform spheres A and A have masses $2m$ kg and $3m$ kg respectively and equal radii. The spheres are moving on a smooth horizontal surface. Initially, sphere A has velocity $(3\mathbf{i} - 4\mathbf{i})$ m s^{-1} and sphere B has velocity $(2\mathbf{i} - 3\mathbf{j})$ m s^{-1}. When the spheres collide, the line joining their centres is parallel to $\mathbf{j}$, as shown in Figure 1. The coefficient of restitution between the spheres is $\frac{3}{7}$. Find, in terms of m, the total kinetic energy lost in the collision.

(10)

(Total 10 marks)

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	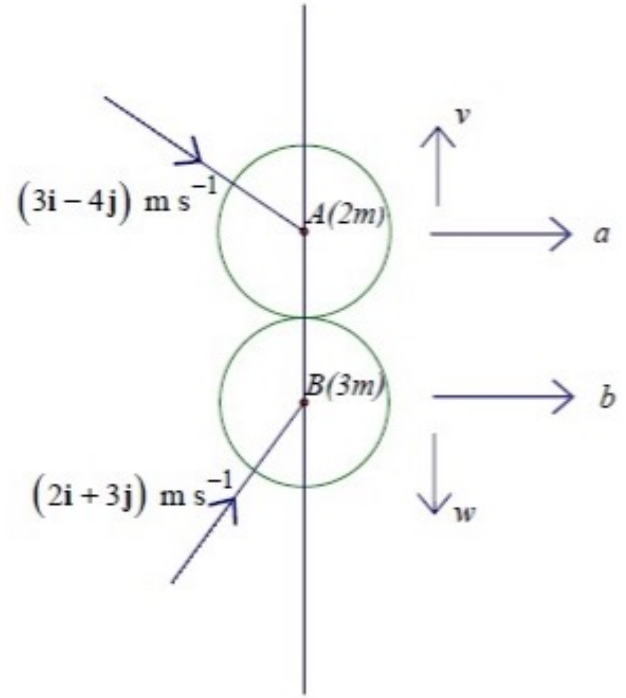		
	$\leftrightarrow a=3$ & $b=2$	B1	
	b Conservation of linear momentum: $-4\times2+3\times3 = 2v-3w(=1)$	M1A1	
	Restitution: $v+w=e\times7$ $(=3)$	M1A1	
	Solve the simultaneous equations	DM1	
	giving $v=2$ and $w=1$	A1	
	KE lost $=\frac{1}{2}\times2m\times((16+9)-(4-9))+\frac{1}{2}\times3m\times((9+4)-(1-4))$	M1A1	
	$=24m$ (J)	A1	
			10

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June 2011 M4

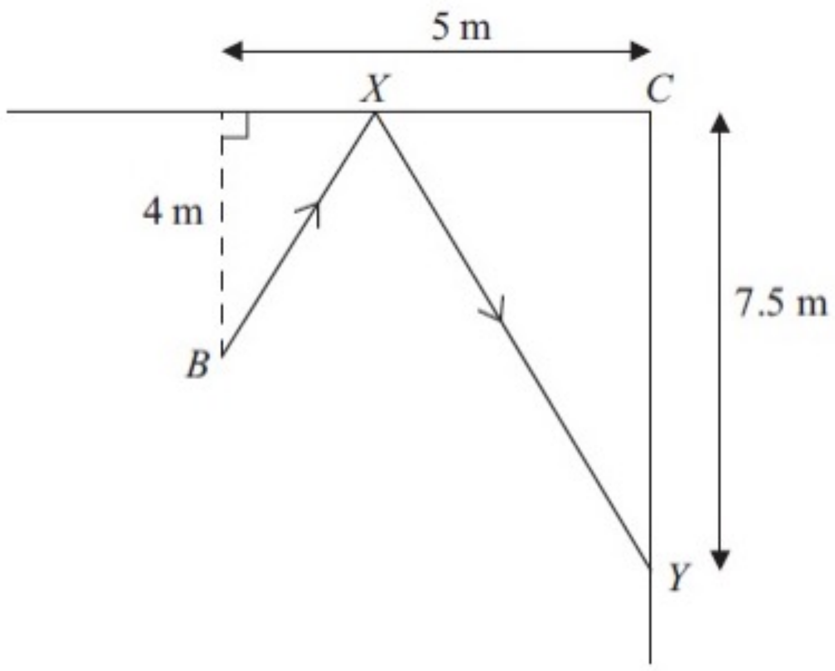

Figure 2

Figure 2 represents part of the smooth rectangular floor of a sports hall. A ball is at B, 4 m from one wall of the hall and 5 m from an adjacent wall. These two walls are smooth and meet at the corner C. The ball is kicked so that it travels along the floor, bounces off the first wall at the point X and hits the second wall at the point Y. The point Y is 7.5 m from the corner C.

The coefficient of restitution between the ball and the first wall is $\frac{3}{4}$.

Modelling the ball as a particle, find the distance CX.

(9)

(Total 9 marks)

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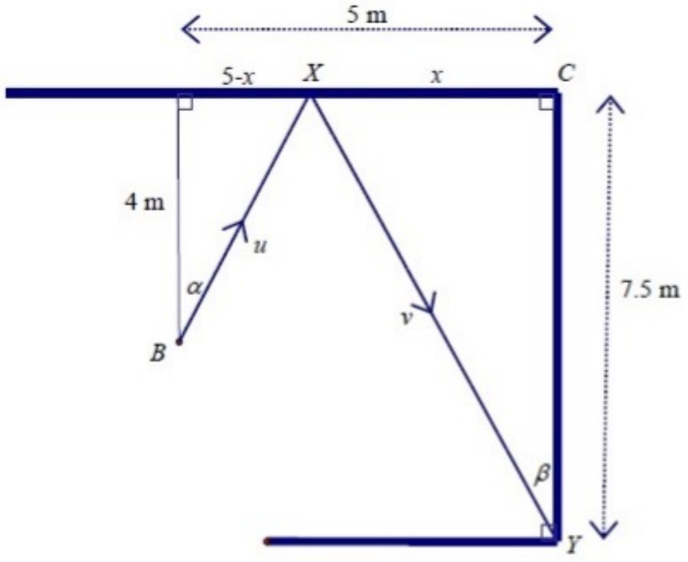
Question Number	Scheme	Marks
		
	At X: $\leftrightarrow$ $u\sin\alpha = v\sin\beta$	M1A1
	$\updownarrow$ $v\cos\beta = eu\cos\alpha$	M1A1
	$4v\cos\beta = 3u\cos\alpha$	
	Eliminate u & v by dividing: $\frac{\tan\alpha}{3} = \frac{\tan\beta}{4}$	M1
	Substitute for the trig ratios: $\frac{5-x}{3\times 4} = \frac{x}{4\times 7.5}$	DM1A1
	Solve for x: $37.5 - 7.5x = 3x$	DM1
	$x = 3.57$ (m) or better, $\frac{25}{7}$	A1
		9

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June 2012 M4

A smooth uniform sphere S, of mass m, is moving on a smooth horizontal plane when it collides obliquely with another smooth uniform sphere T, of the same radius as S but of mass $2m$, which is at rest on the plane. Immediately before the collision the velocity of S makes an angle a, where $\tan a = \frac{3}{4}$, with the line joining the centres of the spheres.

Immediately after the collision the speed of T is V. The coefficient of restitution between the spheres is $\frac{3}{4}$.

(a) Find, in terms of V, the **speed** of S

(i) immediately before the collision,

(ii) immediately after the collision.

(9)

(b) Find the angle through which the direction of motion of S is deflected as a result of the collision.

(4)

(Total 13 marks)

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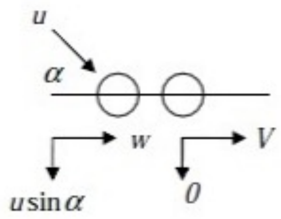
Question Number	Scheme	Marks	Notes
(a)			
	$mu\cos\alpha = mw + 2mV$	M1 A1	CLM parallel to the line of centres. $\frac{4}{5}u = w + 2V$. Need all terms but condone sign errors.
	$eu\cos\alpha = -w + V$	M1 A1	Impact law. Must be the right way round. $\frac{3}{4}\times\frac{4}{5}u = V - w$
	$u\cos\alpha(e+1) = 3V \Rightarrow$ (i) $u = \frac{15V}{7}$	M1 A1	Eliminate w and solve for u in terms of V or v.v. 2.14V or better
	$\Rightarrow w = -\frac{2V}{7}$	A1	Solve for w in terms of V. -0.286V or better
	(ii) speed of $S = \sqrt{(\frac{-2V}{7})^2 + (u\sin\alpha)^2} = \frac{V\sqrt{85}}{7}$	M1 A1 (9)	Use of Pythagoras with their $u\sin\alpha$ and w. $\sqrt{\left(\frac{-2V}{7}\right)^2 + \left(\frac{15V}{7}\times\frac{3}{5}\right)^2}$ $\sqrt{\frac{85}{49}}V$, accept 1.32V or better
(b)	$\tan\theta = \frac{\frac{9V}{7}}{\frac{2V}{7}} = \frac{9}{2}$	M1 A1	Direction of S after the collision. Condone $\frac{2}{9}$ 77.5° or 12.5° seen or implied
	defln angle = 180° - $(\theta + \alpha)$	DM1	Combine their θ and α to find the required angle. e.g. $12.5° + \tan^{-1}\left(\frac{4}{3}\right)$
	= 65.7° (3 sf)	A1 (4) **13**	Accept 66°

June 2009 M4

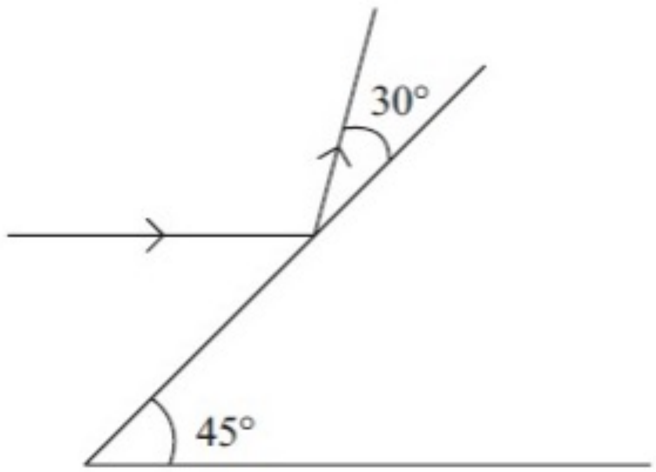


Figure 1

A fixed smooth plane is inclined to the horizontal at an angle of 45º. A particle *P* is moving horizontally and strikes the plane. Immediately before the impact, *P* is moving in a vertical plane containing a line of greatest slope of the inclined plane. Immediately after the impact, *P* is moving in a direction which makes an angle of 30º with the inclined plane, as shown in Figure 1.

Find the fraction of the kinetic energy of *P* which is lost in the impact.

(6)

(Total 6 marks)

Question Number	Scheme	Marks
	CLM along plane: $v\cos 30^\circ = u\cos 45^\circ$	M1 A1
	$v = u\sqrt{\frac{2}{3}}$	A1
	Fraction of KE Lost $= \dfrac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2} = \dfrac{\frac{1}{2}mu^2 - \frac{1}{2}m\frac{2}{3}u^2}{\frac{1}{2}mu^2} = \dfrac{1}{3}$	M1 M1 A1
		[6]

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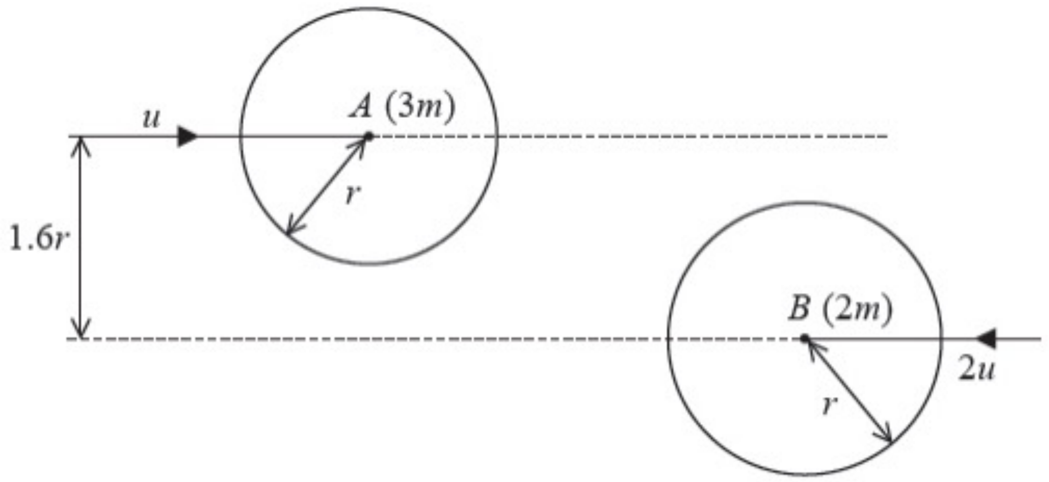

Figure 2

Two smooth uniform spheres A and B, of equal radius r, have masses $3m$ and $2m$ respectively. The spheres are moving on a smooth horizontal plane when they collide. Immediately before the collision they are moving with speeds u and $2u$ respectively. The centres of the spheres are moving towards each other along parallel paths at a distance $1.6r$ apart, as shown in Figure 2.

The coefficient of restitution between the two spheres is $\frac{1}{6}$.

Find, in terms of m and u, the magnitude of the impulse received by B in the collision.

(10)

(Total 10 marks)

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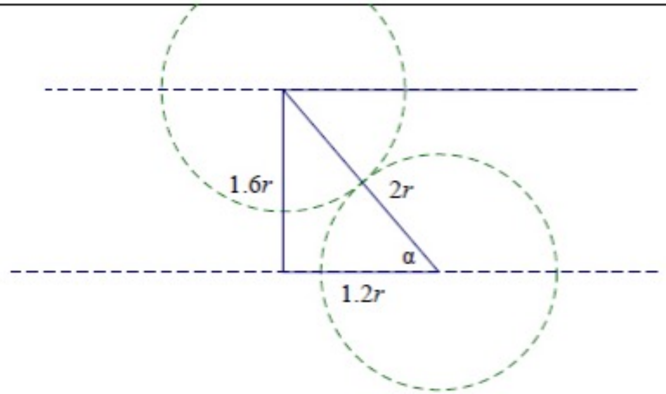
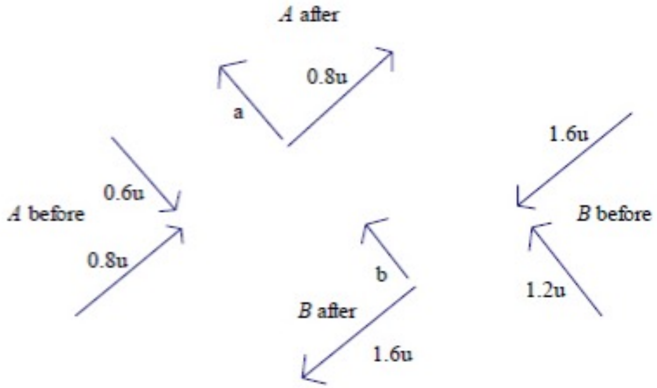
Question Number	Scheme	Marks	
			
	$0.6u$ or $u\cos\alpha$	B1	component of the initial velocity of *A* parallel to the line of centres on impact
	$1.2u$ or $2u\cos\alpha$	B1	component of the initial velocity of *B* parallel to the line of centres on impact
	$2m\times1.2u-3m\times0.6u=3ma+2mb$	M1	CLM parallel to the line of centres. Requires all the terms.
	$(3a+2b=0.6u)$	A1ft	Correct unsimplified for their $0.6u$ and $1.2u$
	$e(1.2u+0.6u)=a-b$	M1	Restitution parallel to the line of centres. Must be used the right way round.
	$(a-b=0.3u)$	A1ft	Correct unsimplified for their $0.6u$ and $1.2u$ If signs are inconsistent between the two equations, penalise here.
		DM1	Solve a pair of simultaneous eqns in *a* & *b* for one of *a* & *b*. Dependent on the two previous M marks.
	$a=0.24u$ or $b=-0.06u$	A1	In terms of *u* only
	$(1.2u-(-0.06u))\times2m=2.52mu$	M1	Find impulse on *A* or *B*. Unsimplified. For their *a* or *b*. Correct mass for the velocities used.
	or $(0.24u-(-0.6u))\times3m=2.52mu$	A1 **(10)**	$\frac{63}{25}$

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June 2013 M4

[*In this question* **i** *and* **j** *are perpendicular unit vectors in a horizontal plane*]

A small smooth ball of mass *m* kg is moving on a smooth horizontal plane and strikes a fixed smooth vertical wall. The plane and the wall intersect in a straight line which is parallel to the vector 2**i** + **j**. The velocity of the ball immediately before the impact is *b***i** m s^{-1}, where *b* is positive. The velocity of the ball immediately after the impact is *a*(**i** + **j**) m s^{-1}, where *a* is positive.

(a) Show that the impulse received by the ball when it strikes the wall is parallel to (− **i** + 2**j**).

(1)

Find

(b) the coefficient of restitution between the ball and the wall,

(8)

(c) the fraction of the kinetic energy of the ball that is lost due to the impact.

(3)

(Total 12 marks)

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Question Number	Scheme	Marks	
(a)	State that impulse acts perpendicular to the wall and demonstrate that (2**i** + **j**).(-**i** +2**j**) = 0	B1	Requires scalar product or gradient diagram.
(b)	Impulse momentum equation: $m(\mathbf{v}-\mathbf{u}) = m[(a\text{-}b)\mathbf{i} + a\mathbf{j}] = \lambda(\text{-}\mathbf{i} + 2\mathbf{j})$ $\Rightarrow a = -2(a-b)$, $3a = 2b$	M1 A2 A1	Requires all terms present and of the correct structure -1 each error
	OR Taking scalar products of velocities with (2**i** + **j**) $\begin{pmatrix} b \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 2b$ and $\begin{pmatrix} a \\ a \end{pmatrix} \bullet \begin{pmatrix} 2 \\ 1 \end{pmatrix} = 3a$ No change parallel to the wall so $2b = 3a$.	 M1 A1A1 A1	
	Scalar products with (–**i** +2**j**): $\begin{pmatrix} b \\ 0 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \end{pmatrix} = -b$ and $\begin{pmatrix} a \\ a \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 2 \end{pmatrix} = a$ Impact equation: $a=eb$ $e = \frac{2}{3}$	B1 M1A1 A1	

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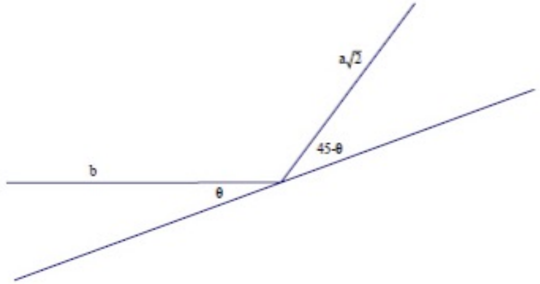
Question Number	Scheme	Marks	
(b) alt			
	$b\cos\theta = a\sqrt{2}\cos(45-\theta)$	M1 A2	Parallel to the wall. Condone trig confusion? -1 each error. Both angles in same variable?
	$b\cos\theta = a\cos\theta + a\sin\theta,\ \ 2b-2a=a$ $2b=3a$	A1	
	Use of $\tan\theta = \frac{1}{2}$	B1	When seen in (b). Implied by 26.6 or 18.4
	$a\sqrt{2}\sin(45-\theta) = eb\sin\theta$	M1	Perpendicular to the wall. Condone consistent trig confusion?
	$a\cos\theta = (a+eb)\sin\theta,\ \ 2a = a+eb$	A1	$e=\sqrt{\frac{10a^2}{b^2}-4}$
	$e=\frac{2}{3}$	A1	0.67 or better
(c)	Fraction of KE lost $=\frac{b^2-2a^2}{b^2}$	M1A1	
	$=\frac{1-2\times\frac{4}{9}}{1}=\frac{1}{9}$	A1	
		(12)	

June 2013 M4 (Replaced)

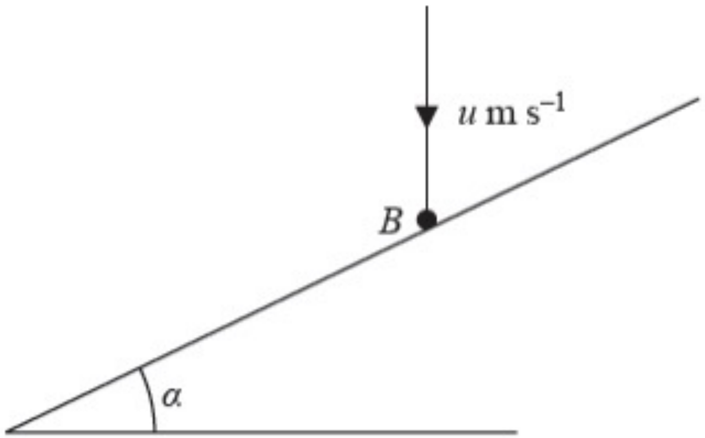


Figure 1

A smooth fixed plane is inclined at an angle α to the horizontal. A smooth ball B falls vertically and hits the plane. Immediately before the impact the speed of B is u m s^{-1}, as shown in Figure 1. Immediately after the impact the direction of motion of B is horizontal. The coefficient of restitution between B and the plane is $\frac{1}{3}$.

Find the size of angle α.

(6)

(Total 6 marks)

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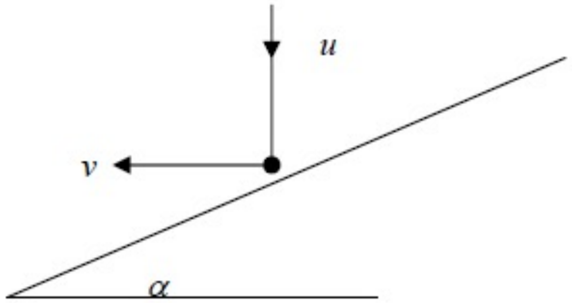
Question Number	Scheme	Marks	
			
	CLM: $u\sin\alpha = v\cos\alpha$	M1 A1	Must be in correct direction but condone trig confusion
	Impact: $\frac{1}{3}u\cos\alpha = v\sin\alpha$	M1 A1	Condone consistent trig confusion
	$\frac{1}{3}\times\frac{1}{\tan\alpha} = \tan\alpha$	M1	
	$\tan\alpha = \frac{1}{\sqrt{3}}$		
	$\alpha = 30°$ (or $\frac{\pi}{6}$ or 0.52 rad)	A1 **(6)** **[6]**	

[3. Table of contents](#)

June 2013 M4 (Replaced)

A smooth uniform sphere *A*, of mass $5m$ and radius r, is at rest on a smooth horizontal plane. A second smooth uniform sphere *B*, of mass $3m$ and radius r, is moving in a straight line on the plane with speed u m s^{-1} and strikes *A*. Immediately before the impact the direction of motion of *B* makes an angle of 60° with the line of centres of the spheres. The direction of motion of *B* is turned through an angle of 30° by the impact.

Find

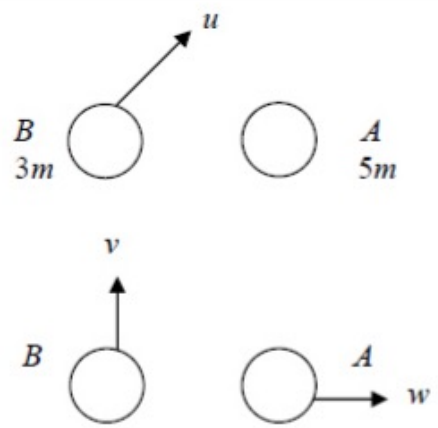
(a) the speed of *B* immediately after the impact,

(3)

(b) the coefficient of restitution between the spheres.

(6)

(Total 9 marks)

Question Number	Scheme	Marks	
(a)			
	After impact B moves perpendicular to the line of centres	B1	can be implied by appropriate use of θ in an equation, or seen on the diagram
	Perp. to line of centres: $v = u\sin 60 = u\frac{\sqrt{3}}{2}$	M1A1	
		(3)	
(b)	Parallel to line of centres:		
	Con of Mom $3mu\cos 60 + 5m\times 0 = 3m\times 0 + 5mw$	M1A1	
	N.L.R. $eu\cos 60 = w$	M1A1	
	$\frac{1}{2}eu = w$ & $\frac{3}{2}u = 5w$		
	$\rightarrow \frac{1}{2}eu = \frac{3}{10}u$	DM1	Dependent on the two previous M marks
	$e = \frac{3}{5}$	A1	
		(6) **[9]**	

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June 2014 M4

A small ball is moving on a smooth horizontal plane when it collides obliquely with a smooth plane vertical wall. The coefficient of restitution between the ball and the wall is $^{1}/_{3}$. The speed of the ball immediately after the collision is half the speed of the ball immediately before the collision.

Find the angle through which the path of the ball is deflected by the collision.

(8)

(Total 8 marks)

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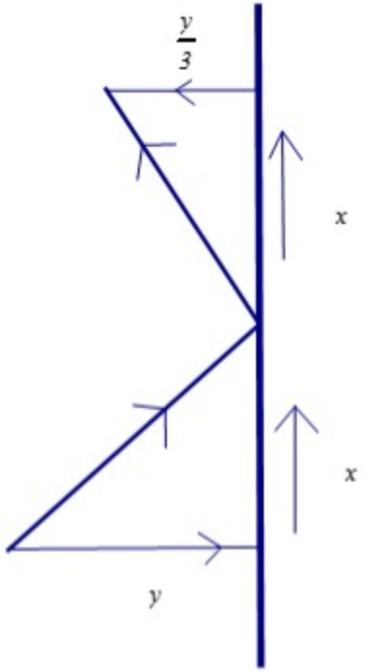
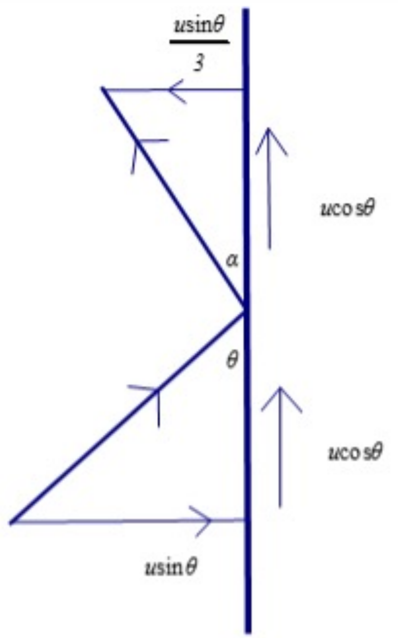
Question Number	Scheme	Marks	Notes
alt1			
	Speed perpendicular to wall after collision = $\frac{y}{3}$	B1	
	Speed parallel to the wall is unchanged	B1	
	$\frac{1}{2}\left(x^2+y^2\right)=x^2+\frac{1}{9}y^2$	M1 A1	Use the speeds to form an equation in x & y (or equivalent) Correct unsimplified
	$9\left(x^2+y^2\right)=2\left(9x^2+y^2\right)$, $9x^2=7y^2$, $x=\frac{\sqrt{7}}{3}y$	A1	Correct ratio for x & y (any equivalent form)
	direction deflected by $\tan^{-1}\frac{y}{x}+\tan^{-1}\frac{y}{3x}$	M1 A1	To find the correct angle Correct in x & y
	$=\tan^{-1}\sqrt{\frac{27}{5}}+\tan^{-1}\sqrt{\frac{3}{5}}=104.5°$ (104)	A1 [8]	

3. Table of contents

Question Number	Scheme	Marks	Notes
alt2			
	Speed perpendicular to wall after collision = $\frac{u\sin\theta}{3}$	B1	
	Speed parallel to the wall is unchanged	B1	
	$\frac{u^2}{4} = \frac{u^2}{9}\sin^2\theta + u^2\cos^2\theta$	M1	Use the speeds to form an equation in u & θ (or equivalent)
		A1	Correct unsimplified
	$27\cos^2\theta = 5\sin^2\theta$, $\tan^2\theta = \frac{27}{5}$	A1	Correct trig ratio for θ (or equivalent)
	deflected by $\theta+\alpha$, $\tan(\theta+\alpha) = \frac{\tan\theta + \frac{1}{3}\tan\theta}{1 - \frac{1}{3}\tan^2\theta}$ $\left(= -\sqrt{15}\right)$	M1	To find the correct angle
		A1	Correct in θ (or equivalent)
	$\theta + \alpha = 104.5°$ (104)	A1 [8]	

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June 2014 M4

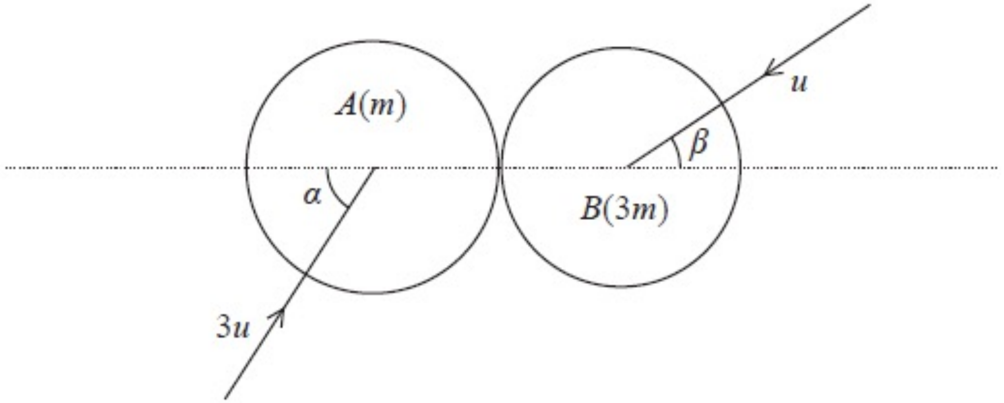

Figure 1

Two smooth uniform spheres *A* and *B* have equal radii. The mass of *A* is m and the mass of *B* is $3m$. The spheres are moving on a smooth horizontal plane when they collide obliquely. Immediately before the collision, *A* is moving with speed $3u$ at angle α to the line of centres and *B* is moving with speed u at angle β to the line of centres, as shown in Figure 1. The coefficient of restitution between the two spheres is $^1/_5$. It is given that $\cos\alpha = {}^1/_3$ and $\cos\beta = {}^2/_3$ and that α and β are both acute angles.

(a) Find the magnitude of the impulse on *A* due to the collision in terms of m and u.

(8)

(b) Express the kinetic energy lost by *A* in the collision as a fraction of its initial kinetic energy.

(4)

(Total 12 marks)

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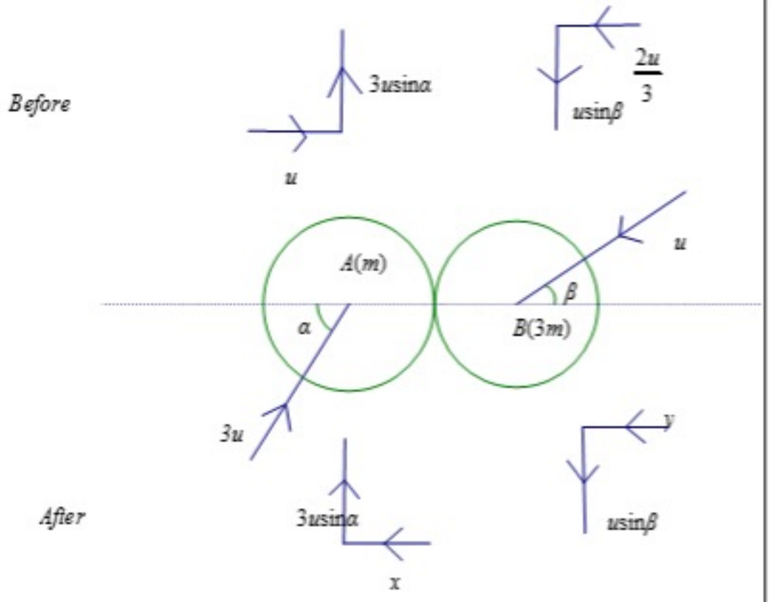
Question Number	Scheme	Marks	Notes
a			
	CLM: $mx+3my=3m\times u\cos\beta-m\times 3u\cos\alpha=mu$	M1	Terms of correct structure but condone sign errors
	$(x+3y=u)$	A1	
	NEL: $x-y=\frac{1}{5}(3u\cos\alpha+u\cos\beta)\left(=\frac{1}{5}\left(u+\frac{2}{3}u\right)=\frac{1}{3}u\right)$	M1	equation of correct structure but condone sign errors
		A1	
	$x=\frac{u}{2}$, or $y=\frac{u}{6}$	DM 1	Dependent on the two previous M marks. Solve for x or y
		A1	Correct for their x or y
	Magnitude of the impulse on A = $mu-\left(m\times-\frac{u}{2}\right)=\frac{3mu}{2}$	M1	
		A1	Must be positive
		[8]	

b	Component of velocity perpendicular to the line of centres before = component after = $3u\sin\alpha = 3u \times \frac{\sqrt{8}}{3} = \sqrt{8}u$	B1	
	KE lost = $\frac{m}{2}\left(9u^2 - \left(8u^2 + \frac{1}{4}u^2\right)\right)\left[= \frac{3}{8}mu^2\right]$	M1 A1	Change in KE. Does not need to be a fraction at this stage. Does not need to include the (cancelling) component perpendicular to the line of centre. Correct unsimplified
	Fraction lost = $\frac{3/8}{9/2} = \frac{3}{8} \times \frac{2}{9} = \frac{1}{12}$	A1 [4]	

June 2014 M4 (Replaced)

A small smooth ball of mass m is falling vertically when it strikes a fixed smooth plane which is inclined to the horizontal at an angle α, where $0° < \alpha < 45°$. Immediately before striking the plane the ball has speed u. Immediately after striking the plane the ball moves in a direction which makes an angle of 45° with the plane. The coefficient of restitution between the ball and the plane is e. Find, in terms of m, u and e, the magnitude of the impulse of the plane on the ball.

(11)

(Total 11 marks)

Question Number	Scheme		Marks
	$v\cos 45^\circ = u\sin\alpha$	parallel	M1 A1
	$v\sin 45^\circ = eu\cos\alpha$	perpendicular	M1 A1
	$e = \tan\alpha$	or square & add	M1 A1
	$I = m(v\cos 45^\circ + u\cos\alpha)$	impulse	M1 A1
	$= mu(\sin\alpha + \cos\alpha)$	in terms of $u,\ \alpha$	M1
	$= \dfrac{mu(1+e)}{\sqrt{1+e^2}}$	in terms of $u,\ e$	M1 A1
			11

June 2014 (Replaced)

A smooth uniform sphere S is moving on a smooth horizontal plane when it collides obliquely with an identical sphere T which is at rest on the plane. Immediately before the collision S is moving with speed U in a direction which makes an angle of 60° with the line joining the centres of the spheres. The coefficient of restitution between the spheres is e.

(a) Find, in terms of e and U where necessary,

(i) the speed and direction of motion of S immediately after the collision,

(ii) the speed and direction of motion of T immediately after the collision.

(12)

The angle through which the direction of motion of S is deflected is δ°.

Find

(i) the value of e for which δ takes the largest possible value,

(ii) the value of δ in this case.

(3)

(Total 15 marks)

Question Number	Scheme		Marks
(a)	$mv_1 + mv_2 = mu\cos 60°$	Momentum	M1 A1
	$-v_1 + v_2 = eu\cos 60°$	Impact law	M1 A1
	$v_1 = \dfrac{u(1-e)}{4}$		M1 A1
	Speed of $S=\sqrt{\dfrac{u^2(1-e)^2}{16}+\dfrac{3u^2}{4}}$	Solve for v_1 and find speed	A1
	$=\dfrac{u}{4}\sqrt{e^2-2e+13}$		
	$\tan\theta = \dfrac{u\sqrt{3}}{2v_1} = \dfrac{2\sqrt{3}}{(1-e)}$	Use components to find dirn	M1 A1
	S moves at arctan $\dfrac{2\sqrt{3}}{(1-e)}$ to the line of centres		M1 A1
	$v_2 = \dfrac{u(1+e)}{4}$	v_2 in terms of u, e	B1 (12)
(b)	T has speed $\dfrac{u(1+e)}{4}$ along the line of centres	Conclusion	
	θ is a max when $e = 1$ then $\theta = 90°$		M1 A1
	then deflection angle is $90° - 60° = 30°$ $\delta = 30$		A1 (3) **15**

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June 2015 M4

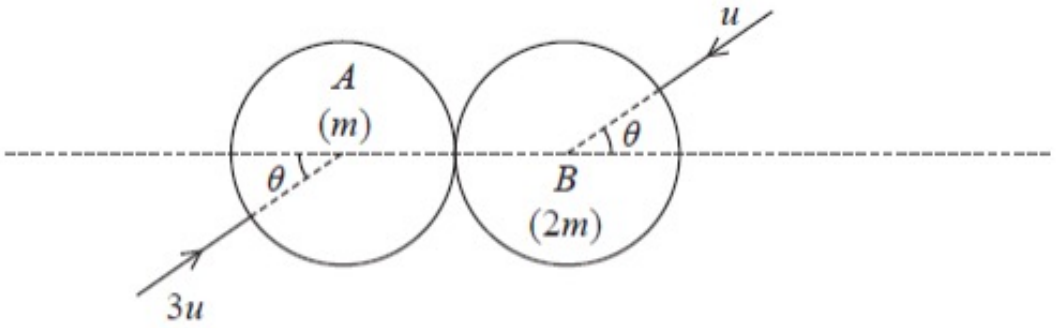

Figure 1

Two smooth uniform spheres *A* and *B* with equal radii have masses *m* and 2*m* respectively. The spheres are moving in opposite directions on a smooth horizontal surface and collide obliquely. Immediately before the collision, *A* has speed 3*u* with its direction of motion at an angle θ to the line of centres, and *B* has speed *u* with its direction of motion at an angle θ to the line of centres, as shown in Figure 1. The coefficient of restitution between the spheres is $\frac{1}{8}$

Immediately after the collision, the speed of *A* is twice the speed of *B*.

Find the size of the angle θ.

(12)

(Total for question = 12 marks)

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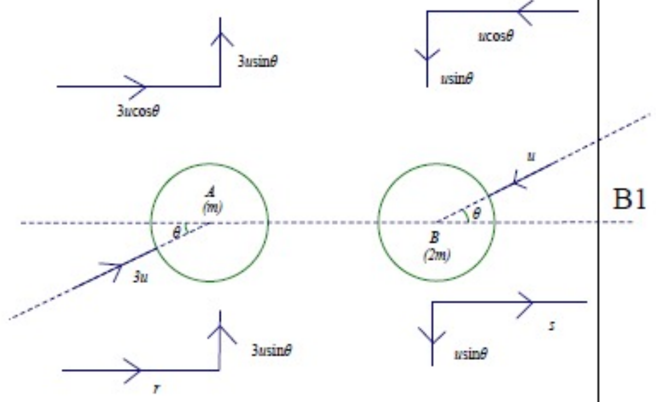
Question Number	Scheme	Marks	Notes
		B1	After collision $u\sin\theta$ and $3u\sin\theta$ perpendicular to l of c Seen or implied
	CLM : $r+2s=3u\cos\theta-2u\cos\theta(=u\cos\theta)$	M1	Requires all four terms but condone sign errors and consistent sin/cos confusion.. Must be dimensionally consistent
		A1	Correct unsimplified equation
	Impact: $s-r=e\times 4u\cos\theta\left(=\dfrac{u\cos\theta}{2}\right)$	M1	Must be the right way round, but condone sign errors and consistent sin/cos confusion
		A1	Correct unsimplified equation. Signs consistent with CLM equation.
	$\Rightarrow r=0, s=\dfrac{u\cos\theta}{2}$	DM1	Solve the simultaneous equations to find the horizontal components of velocities. Dependent on the two preceding M marks

		A1	Both correct
	After the collision: $(3u\sin\theta)^2+r^2=4\left((u\sin\theta)^2+s^2\right)$	M1	Use $v_A=2v_B$. Condone 2 in place of 4.
		A1ft	Correct unsimplified equation (in r and s)
	$9u^2\sin^2\theta=4u^2\sin^2\theta+4.\frac{u^2}{4}\cos^2\theta$	A1	Obtain an equation in θ (correct only)
	$\tan^2\theta=\frac{1}{5}$, $\theta=24.1(°)$ (0.421 radians)	DM1	Solve for θ. Dependent on the previous M1
		A1	Correct to 3 sf or better
alt	For those who prefer everything with trig:		
	$v_A\sin\alpha=3u\sin\theta$, $v_B\sin\beta=u\sin\theta$	B1	Perpendicular to the l.o.c.
	$m.3u\cos\theta-2m.u\cos\theta=mv_A\cos\alpha+2mv_B\cos\beta$	M1	CLM
	$(u\cos\theta=v_A\cos\alpha+2v_B\cos\beta)$	A1	
	$\frac{1}{8}\times(3u\cos\theta+u\cos\theta)=v_B\cos\beta-v_A\cos\alpha$	M1	Impact law
	$\left(\frac{u}{2}\cos\theta=v_B\cos\beta-v_A\cos\alpha\right)$	A 1	

	$\frac{u}{2}\cos\theta = v_B\cos\beta$, $0 = v_A\cos\alpha\left(\Rightarrow \sin\alpha = 1\right)$	DM1	Simultaneous equations
		A1	
	$v_A\sin\alpha = v_A = 2v_B = 3u\sin\theta$	M1	Use $v_A = 2v_B$ to find β
	$v_B\sin\beta = u\sin\theta \Rightarrow \frac{3u\sin\theta}{2}\sin\beta = u\sin\theta$	A1	Equation without v_A and v_B
	$\sin\beta = \frac{2}{3}$	A1	
	$2v_B = 3u\sin\theta$ & $\frac{u}{2}\cos\theta = v_B\cos\beta$ $\Rightarrow 6\tan\theta = \frac{2}{\cos\beta}\left(= 2\times\frac{3}{\sqrt{5}}\right)$	M1	Solve for θ
	$\tan\theta = \frac{1}{\sqrt{5}}$, $\theta = 24.1(°)$ (0.421 radians)	A1	
		[12]	

June 2007 M4

A small ball is moving on a horizontal plane when it strikes a smooth vertical wall. The coefficient of restitution between the ball and the wall is e. Immediately before the impact the direction of motion of the ball makes an angle of 60° with the wall. Immediately after the impact the direction of motion of the ball makes an angle of 30° with the wall.

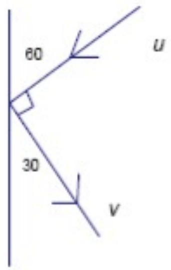
(a) Find the fraction of the kinetic energy of the ball which is lost in the impact.

(6)

(b) Find the value of e.

(4)

(Total 10 marks)

Question Number	Scheme	Marks
(a)	 $u\cos 60^0 = v\cos 30^0$ $u = v\sqrt{3}$ KE lost $= \frac{1}{2}m(u^2 - v^2)$ Fraction of KE lost $= 1 - \left(\frac{v}{u}\right)^2$ $= 1 - \frac{1}{3} = \frac{2}{3}$ or at least 3sf ending in 7 or $\frac{3}{4}(1 - e^2)$	M1A1 A1 M1 DM1 A1 (6)
(b)	$e = \frac{v\sin 30°}{u\sin 60°}$ $= \frac{v}{u} \cdot \frac{1}{\sqrt{3}}$ $= \frac{1}{3}$	M1A1 DM1 A1 (4)
a)	M1 Resolve parallel to the wall *Alt: reasonable attempt at equation connecting two variables* A1 Correct as above or equivalent *equation correct* A1 u in terms of v or v.v. - not necessarily simplified. *or ration of the two variables correct* M1 expression for KE lost DM1 expression in one variable for fraction of KE lost – could be u/v as above A1 cao	*The first three marks can be awarded in (b) if not seen in (a)*
b)	M1 Use NIL perpendicular to the wall and form equation in e A1 Correct unsimplified expression as above or $eu\sin 60^o = v\sin 30^o$ or equivalent DM1 Substitute values for trig functions or use relationship from (a) and rearrange to e = ... A1 cao accept decimals to at least 3sf	*The first two marks can be awarded in (a)*

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June 2016 M4

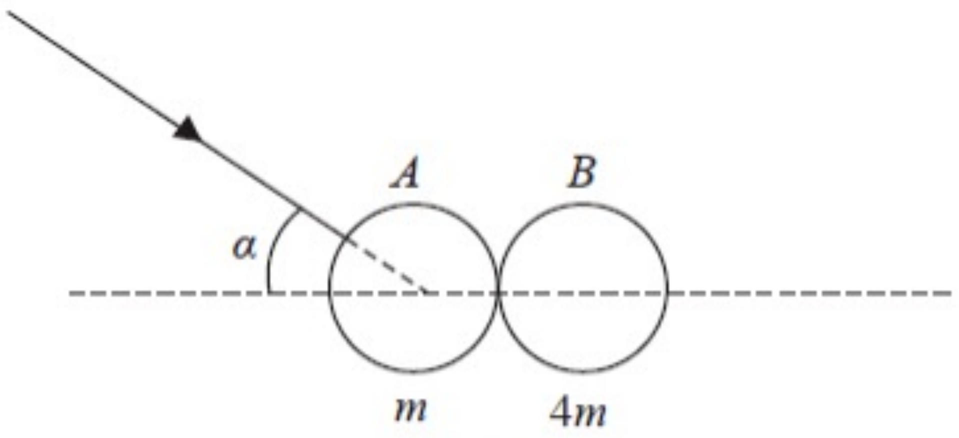

Figure 1

A smooth uniform sphere A of mass m is moving on a smooth horizontal plane when it collides with a second smooth uniform sphere B, which is at rest on the plane. The sphere B has mass $4m$ and the same radius as A. Immediately before the collision the direction of motion of A makes an angle α with the line of centres of the spheres, as shown in Figure 1. The direction of motion of A is turned through an angle of 90° by the collision and the coefficient of restitution between the spheres is $\frac{1}{2}$

Find the value of $\tan \alpha$

(8)

(Total for question = 8 marks)

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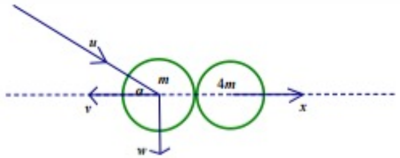
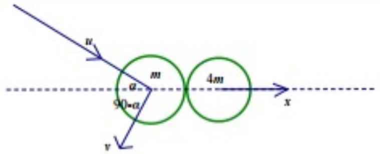
Q	Scheme	Marks	Notes
			
	Along line of centres:		
	Con of mom: $mu\cos\alpha = 4mx - mv$	M1	$mu\cos\alpha = 4mx - mv\cos\beta$ or $mu\cos\alpha = 4mx - mv\sin\alpha$ Need to see all 3 terms, but condone sign errors & trig. confusion
	$(u\cos\alpha = 4x - v)$	A1	$(u\cos\alpha = 4x - v\cos\beta)$ $(u\cos\alpha = 4x - v\sin\alpha)$
	NLR: $\frac{1}{2}u\cos\alpha = x + v$	M1	$\frac{1}{2}u\cos\alpha = x + v\cos\beta$ $\frac{1}{2}u\cos\alpha = x + v\sin\alpha$ Must be used the right way round, but condone sign errors & consistent trig. confusion
	$(2u\cos\alpha = 4x + 4v)$	A1	$(2u\cos\alpha = 4x + 4v\cos\beta)$ $(2u\cos\alpha = 4x + 4v\sin\alpha)$
	$(5v = u\cos\alpha)$		$(5v\tan\alpha = u)$ $(u\cos\alpha = 5v\cos\beta)$
	Perp to line of centres: no change to velocity so vel $= w = u\sin\alpha$	B1 (A1)	$v\cos\alpha = u\sin\alpha$ $(v = u\tan\alpha)$
	Deflected through $90°$ $\left(\tan\alpha = \frac{v}{w}\right)$	B1	$90°$ used correctly. E.g. use of $90 - \alpha$ in an equation $(\tan\alpha \times \tan\beta = 1)$
	$\tan\alpha = \frac{\frac{1}{5}u\cos\alpha}{u\sin\alpha}$	M1	$5u\tan^2\alpha = u$ Form equation in α
	$\tan^2\alpha = \frac{1}{5}$ $\tan\alpha = \sqrt{\frac{1}{5}}$ or 0.4472....	A1	(0.45 or better)
		[8]	

June 2016 M4

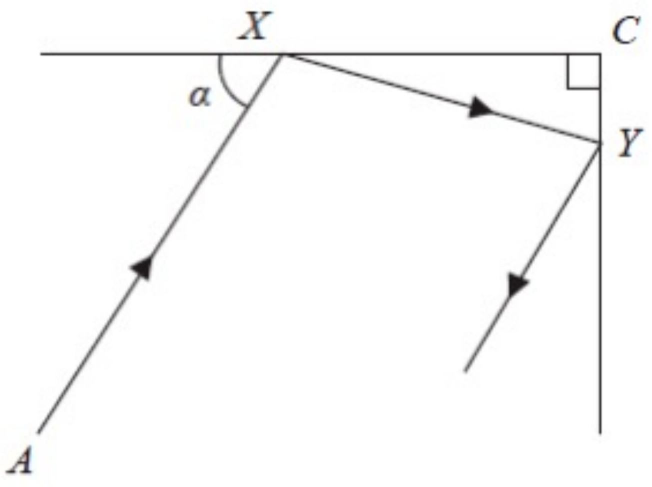


Figure 2

A small spherical ball P is at rest at the point A on a smooth horizontal floor. The ball is struck and travels along the floor until it hits a fixed smooth vertical wall at the point X. The angle between AX and this wall is α, where α is acute. A second fixed smooth vertical wall is perpendicular to the first wall and meets it in a vertical line through the point C on the floor. The ball rebounds from the first wall and hits the second wall at the point Y. After P rebounds from the second wall, P is travelling in a direction parallel to XA, as shown in Figure 2. The coefficient of restitution between the ball and the first wall is e. The coefficient of restitution between the ball and the second wall is ke.

Find the value of k.

(9)

(Total for question = 9 marks)

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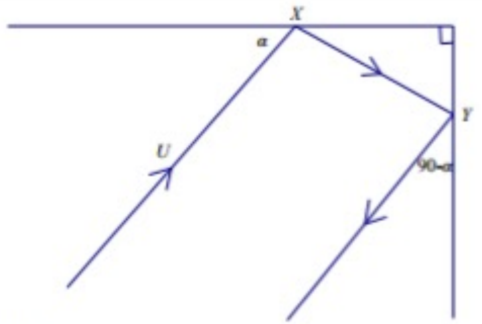
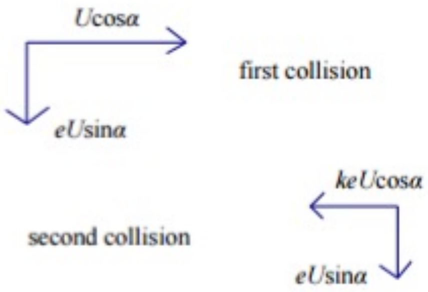
Q	Scheme	Marks	Notes
			
	First impact:		
	Component parallel to wall: $=U\cos\alpha$	B1	
	Perp to wall: NLR: $eU\sin\alpha$	M1	Correct use of impact law Condone trig. confusion
		A1	
	Second impact:		
	parallel to wall vel after $=eU\sin\alpha$	B1	In terms of U and α
	Perp to wall $ke\times U\cos\alpha$	B1	In terms of U and α
	Direction at $(90-\alpha)$ to the wall	B1	Seen or implied
	$\Rightarrow \tan(90-\alpha)=\dfrac{keU\cos\alpha}{Ue\sin\alpha}$ or $\tan\alpha=\dfrac{eU\sin\alpha}{keU\cos\alpha}$	M1	
	$\cot\alpha=k\cot\alpha$ or $\tan\alpha=\dfrac{1}{k}\tan\alpha$	A1	Equation in k and α
	$k=1$	A1	From correct work only
		[9]	
	NB:A candidate who makes a false assumption about an angle α in triangle CXY can score a maximum B1B1B1 B0B0 B1 M1 A0 (6/8)		

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June 2017 M4

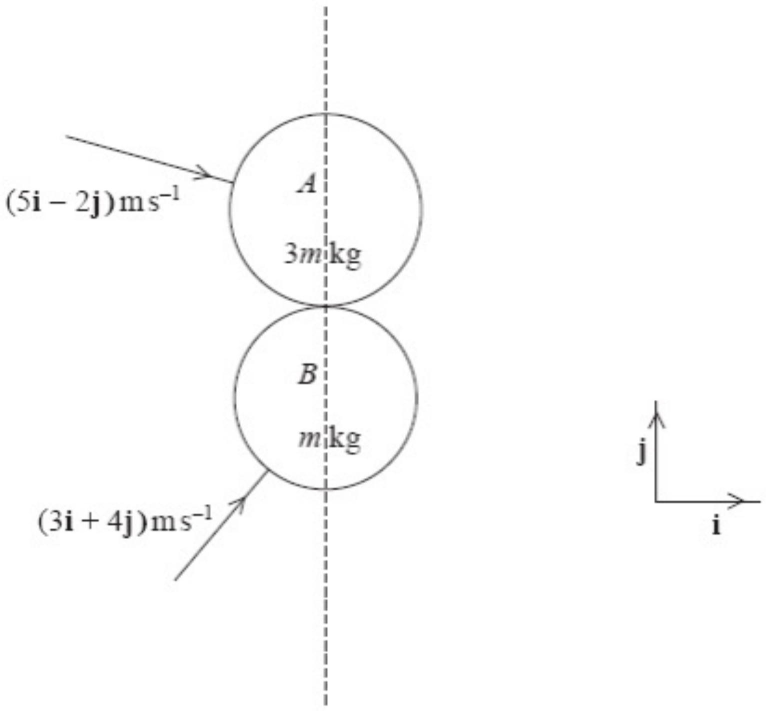

Figure 1

Two smooth uniform spheres *A* and *B* have masses 3*m*kg and *m*kg respectively and equal radii. The spheres are moving on a smooth horizontal surface. Initially, sphere *A* has velocity $(5\mathbf{i} - 2\mathbf{j})\text{ms}^{-1}$ and sphere *B* has velocity $(3\mathbf{i} + 4\mathbf{j})\text{ms}^{-1}$. When the spheres collide, the line joining their centres is parallel to **j**, as shown in Figure 1.
The coefficient of restitution between the two spheres is *e*.
The kinetic energy of sphere *B* immediately after the collision is 85% of its kinetic energy immediately before the collision.

Find

(a) the velocity of each sphere immediately after the collision,

(9)

(b) the value of *e*.

(3)

(Total for question = 12 marks)

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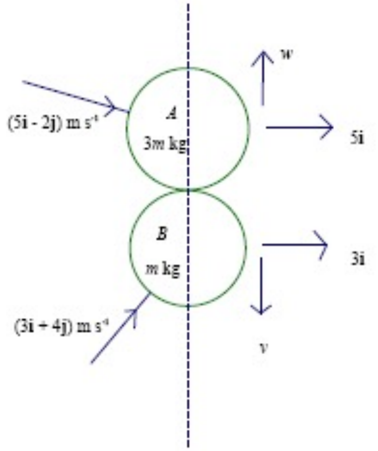
			
a	For A, component perpendicular to loc = 5	B1	
	For B, component perpendicular to loc = 3	B1	
	$\frac{1}{2}m\times 25\times\frac{85}{100}=\frac{1}{2}m\left(3^3+v^2\right)$	M1	Equation for kinetic energy of B For their "3"
	$\frac{85}{4}=9+v^2,\ \ v^2=\frac{49}{4}$	A1	
	$-6m+4m=3mw-mv$ $\left(=3mw-3.5m\right)$	M1	CLM parallel to loc. No missing/additional terms Condone sign error(s)
		A1ft	Correct unsimplified equation for CLM (with their values if substituted)
	$w=0.5$		
	Select correct root and state velocities:	DM1	
	$\mathbf{v}_B=\left(3\mathbf{i}-3.5\mathbf{j}\right)$ (m s^{-1})	A1	One correct
	$\mathbf{v}_A=\left(5\mathbf{i}+0.5\mathbf{j}\right)$ (m s^{-1})	A1	Both correct
		(9)	

b	$v+w=e(2+4)$	M1	Impact law parallel to loc. Used the right way round. Condone sign error(s)
	$0.5+3.5=6e$	A1ft	Correct unsimplified or with their values
	$e=\frac{2}{3}$	A1	
		(3)	
		[12]	

June 2017 M4

[In this question, the unit vectors **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.]*

A line of greatest slope of a fixed smooth plane is parallel to the vector $(-4\mathbf{i} - 3\mathbf{j})$. A particle P falls vertically and strikes the plane. Immediately before the impact, P has velocity $-7\mathbf{j}\,\text{ms}^{-1}$. Immediately after the impact, P has velocity $(-a\mathbf{i} + \mathbf{j})\,\text{ms}^{-1}$, where a is a positive constant.

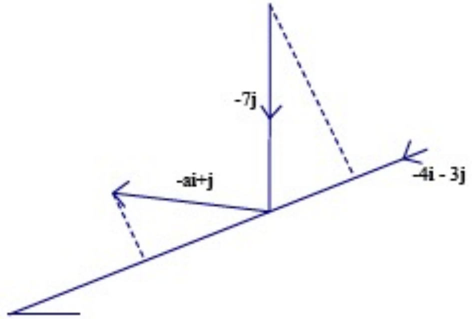
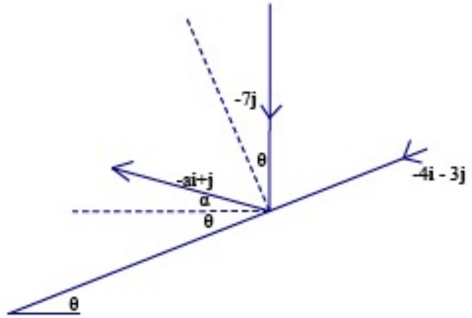
(a) Show that $a = 6$

(2)

(b) Find the coefficient of restitution between P and the plane.

(6)

(Total for question = 8 marks)

Q	Scheme	Marks	Notes
a			
	Components parallel to the plane unchanged: $\left(\begin{pmatrix}0\\-7\end{pmatrix}\cdot\frac{1}{5}\begin{pmatrix}-4\\-3\end{pmatrix}=\begin{pmatrix}-a\\1\end{pmatrix}\cdot\frac{1}{5}\begin{pmatrix}-4\\-3\end{pmatrix}\right)$ $\Rightarrow 21=4a-3$	M1	Use of scalar product. Do not need to see $\frac{1}{5}$
	$a=6$	A1	***Given Answer***
		(2)	
a alt			
	Components parallel to the plane: $7\sin\theta = v\cos(\theta+\alpha)$		$\theta+\alpha=46.3...°$
	$7\sin\theta = v(\cos\theta\cos\alpha-\sin\theta\sin\alpha)$, $\Rightarrow 7\tan\theta = a-\tan\theta$	M1	Equate components and form an equation in a and θ
	$8\tan\theta = a = 6$	A1	
		(2)	

b	Component of -7**j** parallel to the plane $=\frac{21}{\lvert 4\mathbf{i}+3\mathbf{j}\rvert}$	M1	Scalar product of -7**j** and unit vector parallel to plane
	$=4.2$	A1	
	$\sqrt{49-4.2^2}=\sqrt{31.36}$	M1	Use Pythagoras to find components perpendicular to the plane
	$\sqrt{36+1-4.2^2}=\sqrt{19.36}$	A1	Both correct
	$\sqrt{19.36}=e\times\sqrt{31.36}$	DM1	Use if impact law Dependent on preceding M mark
	$e=0.786$	A1	
		(6)	

Alt b	Component of -7**j** perpendicular to the plane $=\frac{1}{5}\begin{pmatrix}0\\-7\end{pmatrix}\cdot\begin{pmatrix}-3\\4\end{pmatrix}$	M1A1	
	Component of $-a\mathbf{i}+\mathbf{j}$ perpendicular to the plane $=\frac{1}{5}\begin{pmatrix}-6\\1\end{pmatrix}\cdot\begin{pmatrix}-3\\4\end{pmatrix}$	M1A1	
	Impact law: $e=\frac{\frac{1}{5}\times 22}{\frac{1}{5}\times 28}=\frac{22}{28}=\frac{11}{14}(=0.786)$	DM1 A1	
		(6)	

		(6)	
Alt b	Components perpendicular to the plane: $e\times 7\cos\theta=v\sin(\theta+\alpha)$	M1	
	$e\times 7\cos\theta=v(\sin\theta\cos\alpha+\cos\theta\sin\alpha)$	A1	$\theta+\alpha=46.3...°$, $v=\sqrt{37}$
	Substitute for α: $7e=6\tan\theta+1$	M1A1	
	Solve for e: $7e=6\times\frac{3}{4}+1=\frac{11}{2}$, $e=\frac{11}{14}$	DM1 A1	
		(6)	
Alt b	Components perpendicular to the plane: $e\times 7\cos\theta=v\sin(\theta+\alpha)$	M1	
	$e\times 7\cos\theta=v(\sin\theta\cos\alpha+\cos\theta\sin\alpha)$	A1	
	Divide and substitute for α: $e\cot\theta=\tan(\theta+\alpha)=\frac{\tan\theta+\tan\alpha}{1-\tan\theta\tan\alpha}$	M1	
	$=\frac{\frac{3}{4}+\frac{1}{6}}{1-\frac{3}{4\times 6}}=\frac{3\times 6+4}{4\times 6-3}$	A1	
	Solve for e: $e=\frac{22}{21}\times\frac{3}{4}=\frac{11}{14}$	DM1 A1	
		(6)	

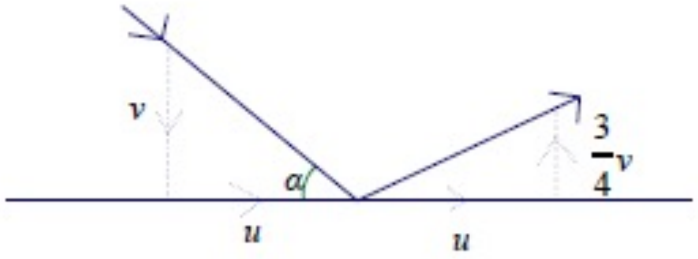
Q	Scheme	Marks	Notes
Alt b	Parallel: $7\sin\theta = \sqrt{37}\cos(\theta+\alpha)$ Perpendicular: $e7\cos\theta = \sqrt{37}\sin(\theta+\alpha)$	M1A1	Pair of equations
	$49\sin^2\theta = 37\cos^2(\theta+\alpha)$ $\Rightarrow \sin^2(\theta+\alpha) = 1 - \frac{49}{37}\sin^2\theta$	M1	Square and substitute to eliminate $\theta+\alpha$
	$49e^2\cos^2\theta = 37\sin^2(\theta+\alpha) = 37 - 49\sin^2\theta$	A1	
	$e^2 = \dfrac{37 - 49\times\frac{9}{25}}{49\times\frac{16}{25}} = \dfrac{121}{196}, \quad e = \dfrac{11}{14}$	M1A1	Substitute for θ to obtain e.
		(6)	
		[8]	

June 2018 M4

A small ball B, moving on a smooth horizontal plane, collides with a fixed smooth vertical wall. Immediately before the collision the angle between the direction of motion of B and the wall is α. The coefficient of restitution between B and the wall is $\frac{3}{4}$. The kinetic energy of B immediately after the collision is 60% of its kinetic energy immediately before the collision.

Find, in degrees, the size of angle α.

(Total for question = 8 marks)

Q	Scheme	Marks
		
	Velocity before & after: parallel to wall : u and u	B1
	Perpendicular to the wall : v and $\frac{3}{4}v$ Allow with ev	B1
	Kinetic energy: $\frac{1}{2}m\left(\frac{9}{16}v^2+u^2\right)=0.6\times\frac{1}{2}m\left(v^2+u^2\right)$	M1A2
	$\frac{90}{16}v^2+10u^2=6v^2+6u^2$	
	$4u^2=\frac{6}{16}v^2 \quad u^2=\frac{3}{32}v^2$	
	$\tan\alpha=\frac{v}{u}=\sqrt{\frac{32}{3}}$	M1A1
	$\alpha=73°$ (or better 72.976.....)	A1
		(8)

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Alt		
	Velocity before & after: parallel to wall : $u\cos\alpha$ and $u\cos\alpha$	B1
	Perpendicular to the wall : $u\sin\alpha$ and $\frac{3}{4}u\sin\alpha$	B1
	Kinetic energy: $\frac{1}{2}m\left(\frac{9}{16}(u\sin\alpha)^2+(u\cos\alpha)^2\right)=0.6\times\frac{1}{2}m\left((u\sin\alpha)^2+(u\cos\alpha)^2\right)$	M1A2
	$\frac{9}{16}\sin^2\alpha+\cos^2\alpha=\frac{3}{5}=\frac{9}{16}+\frac{7}{16}\cos^2\alpha$	M1
	$\cos^2\alpha=\frac{3}{35}$, $\alpha=\cos^{-1}\sqrt{\frac{3}{35}}=73.0°$ (1.27 radians)	A1,A1
		[8]

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June 2019 M4

A small smooth ball is moving on a horizontal plane when it strikes a fixed smooth vertical wall. The floor and the wall intersect in a straight line which is parallel to the vector $\mathbf{i}$. Immediately before the impact, the ball has velocity $(4\mathbf{i} - 3\mathbf{j})$ m s^{-1}. Immediately after the impact, the ball has velocity $(a\mathbf{i} + b\mathbf{j})$ m s^{-1}. The coefficient of restitution between the ball and the wall is e.

The kinetic energy of the ball immediately after the impact is 80% of its kinetic energy immediately before the impact.

Find the value of e.

(Total for question = 6 marks)

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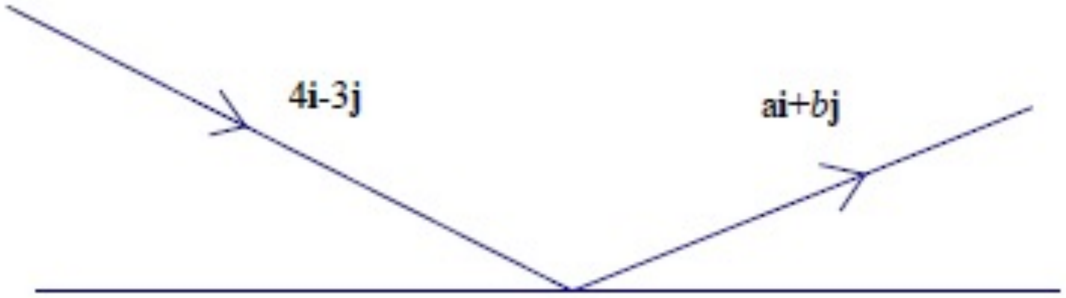
Question Number	Scheme	Marks
		
	Parallel to the wall: $4 = a$	B1
	Perpendicular to the wall: $b = 3e$	B1
	KE before & after: $\frac{4}{5}\times\frac{1}{2}m\left(4^2+3^2\right)=\frac{1}{2}m\left(a^2+b^2\right)$	M1A1
	Substitute and solve for e	M1
	$e=\frac{2}{3}$ (0.67 or better)	A1
		[6]
		Total 6

June 2019 M4

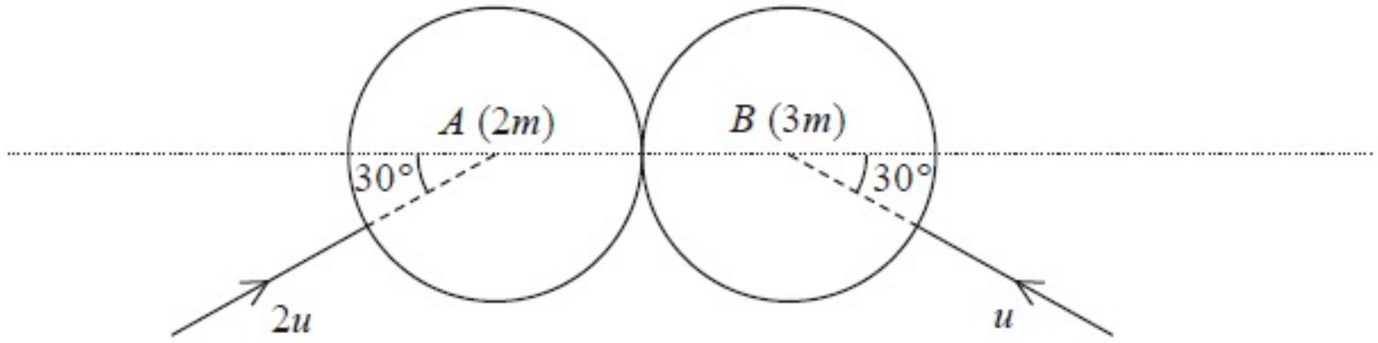


Figure 2

Two smooth uniform spheres, A and B, with equal radii, have masses $2m$ and $3m$ respectively. The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision, A is moving with speed $2u$ at 30° to the line of centres, and B is moving with speed u at 30° to the line of centres, as shown in Figure 2.

The direction of motion of A immediately after the collision is perpendicular to the direction of motion of A immediately before the collision. The direction of motion of B is deflected through an angle θ as a result of the collision.

The coefficient of restitution between A and B is e.

Find

(a) the speed of A immediately after the collision,

(2)

(b) the size of angle θ,

(8)

(c) the value of e.

(3)

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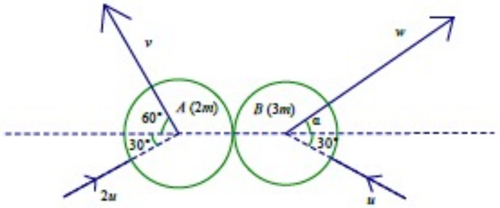
(Total for question = 13 marks)

Question Number	Scheme	Marks
a		
	For A ↕ : $2u\sin 30° = v\sin 60°$	M1
	$v = \frac{2}{\sqrt{3}}u \left(= \frac{2\sqrt{3}}{3}u\right)$	A1
		[2]
b	CLM : $2m \times 2u\cos 30° - 3m \times u\cos 30° = 3m \times w\cos\alpha - 2m \times v\cos 60°$	M1A1
	$\left(u\cos 30° + v = 3w\cos\alpha\right)$	
	$w\cos\alpha = \frac{1}{3}\left(\frac{\sqrt{3}}{2}u + \frac{2\sqrt{3}}{3}u\right) = \frac{7\sqrt{3}}{18}u$	A1
	For B ↕ : $u\sin 30° = w\sin\alpha = \frac{u}{2}$	M1A1
	$\Rightarrow \tan\alpha = \frac{9}{7\sqrt{3}}$	M1
	$\Rightarrow$ deflected through $\theta = 150° - \alpha = 113.4°$ $(113°)$ (1.98 radians)	M1A1
		[8]
c	Impact law: $w\cos\alpha + v\cos 60° = e\left(3u\cos 30°\right)$	M1A1
	$\left(\frac{7\sqrt{3}}{18}u + \frac{2\sqrt{3}}{3}u \times \frac{1}{2} = e \times \frac{3\sqrt{3}}{2}u\right)$	
	$e = \frac{13}{27}(= 0.48)$	A1
		[3]
		Total 13

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