SECTION B: MECHANICS

Answer ALL questions. Write your answers in the spaces provided.

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

1. A particle P of mass 0.5 kg is moving on a smooth horizontal plane.

The origin O is on the plane.

At time $t = 0$, P passes through O moving with velocity $(\mathbf{i} - \mathbf{j})\,\mathrm{m\,s^{-1}}$

At time t seconds, the resultant horizontal force acting on P is

$$[(3t - 1)\mathbf{i} + 2\mathbf{j}]\,\mathrm{N}$$

(a) Find the velocity of P at $t = 2$

(5)

(b) Find the distance of P from O at $t = 2$

(4)

Why this isn't SUVAT:

s

$u = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$v = ?$

$\boxed{a = \quad}$

$t = 2$

to use SUVAT we need acceleration (need 3 knowns to work out 4th unknown)

Given in Q

$\therefore \boxed{\underline{F}} = \boxed{m}\underline{a}$

(a)

$\begin{pmatrix} 3t-1 \\ 2 \end{pmatrix} = 0.5\underline{a}$

$\begin{pmatrix} 3t-1 \\ 2 \end{pmatrix} = \frac{1}{2}\underline{a} \Rightarrow \boxed{\underline{a} = \begin{pmatrix} 6t-2 \\ 4 \end{pmatrix}}$

SUVAT is only for constant acceleration. (Since acceleration changes as t changes acceleration isn't constant in this Q)

$\therefore$ use integration (differentiation

$\underline{a} = \begin{pmatrix} 6t-2 \\ 4 \end{pmatrix}$

$s \xrightarrow{\frac{d}{dt}} v \xrightarrow{\frac{d}{dt}} a$, $a \xrightarrow{\int dt} v \xrightarrow{\int dt} s$

$\therefore \underline{v} = \int \underline{a}\, dt$

Question 1 continued

$$\underline{v} = \int \underline{a} \, dt$$

$$= \int \begin{pmatrix} 6t-2 \\ 4 \end{pmatrix} dt$$

$$= \begin{pmatrix} 3t^2 - 2t \\ 4t \end{pmatrix} + \underline{c}$$

Find $\underline{c}$ using info from Q.

when $t=0$, $\underline{v} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3(0)^2 - 2(0) \\ 4(0) \end{pmatrix} + \underline{c}$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \underline{c}$$

$$\therefore \underline{v} = \begin{pmatrix} 3t^2 - 2t \\ 4t \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\underline{v} = \begin{pmatrix} 3t^2 - 2t + 1 \\ 4t - 1 \end{pmatrix}$$

$$\underline{v}\Big|_{t=2} = \begin{pmatrix} 3(2)^2 - 2(2) + 1 \\ 4(2) - 1 \end{pmatrix}$$

$$= \boxed{\begin{pmatrix} 9 \\ 7 \end{pmatrix} \text{ms}^{-1}}$$

Question 1 continued

(b)

$$\begin{array}{ccc} & s & \\ \int dt \uparrow & & \downarrow \frac{d}{dt} \\ & v & \\ \int dt \uparrow & & \downarrow \frac{d}{dt} \\ & a & \end{array}$$

$$\underline{s} = \int \underline{v} \, dt$$

$$= \int \begin{pmatrix} 3t^2 - 2t + 1 \\ 4t - 1 \end{pmatrix} dt$$

$$= \begin{pmatrix} t^3 - t^2 + t \\ 2t^2 - t \end{pmatrix} + \underline{d}$$

$$\boxed{\text{When } t = 0, \ \underline{s} = 0}$$

Since at origin when $t = 0$.

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0^3 - 0^2 + 0 \\ 2(0)^2 - 0 \end{pmatrix} + \underline{d}$$

$$\therefore \underline{d} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\therefore \quad \underline{s} = \begin{pmatrix} t^3 - t^2 + t \\ 2t^2 - t \end{pmatrix}$$

$$\underline{s}\Big|_{t=2} = \begin{pmatrix} (2)^3 - (2)^2 + 2 \\ 2(2)^2 - 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$$

$$\therefore \quad \text{distance} = \sqrt{6^2 + 6^2} = \sqrt{72} = \boxed{8.5 \text{ m}} \ (2sf)$$

2. A ladder AB, of weight W and length $2a$, has one end A resting on rough horizontal ground.

The other end B rests against a vertical wall.

A man of weight $6W$ stands on the ladder at a point C.

The coefficient of friction between the ladder and the ground is $\frac{1}{3}$

The ladder rests in limiting equilibrium at an angle θ to the ground, where

$\tan\theta = \frac{12}{5}$

The ladder is modelled as a uniform rod which lies in a vertical plane perpendicular to the wall.

The man is modelled as a particle and the vertical wall is modelled as smooth.

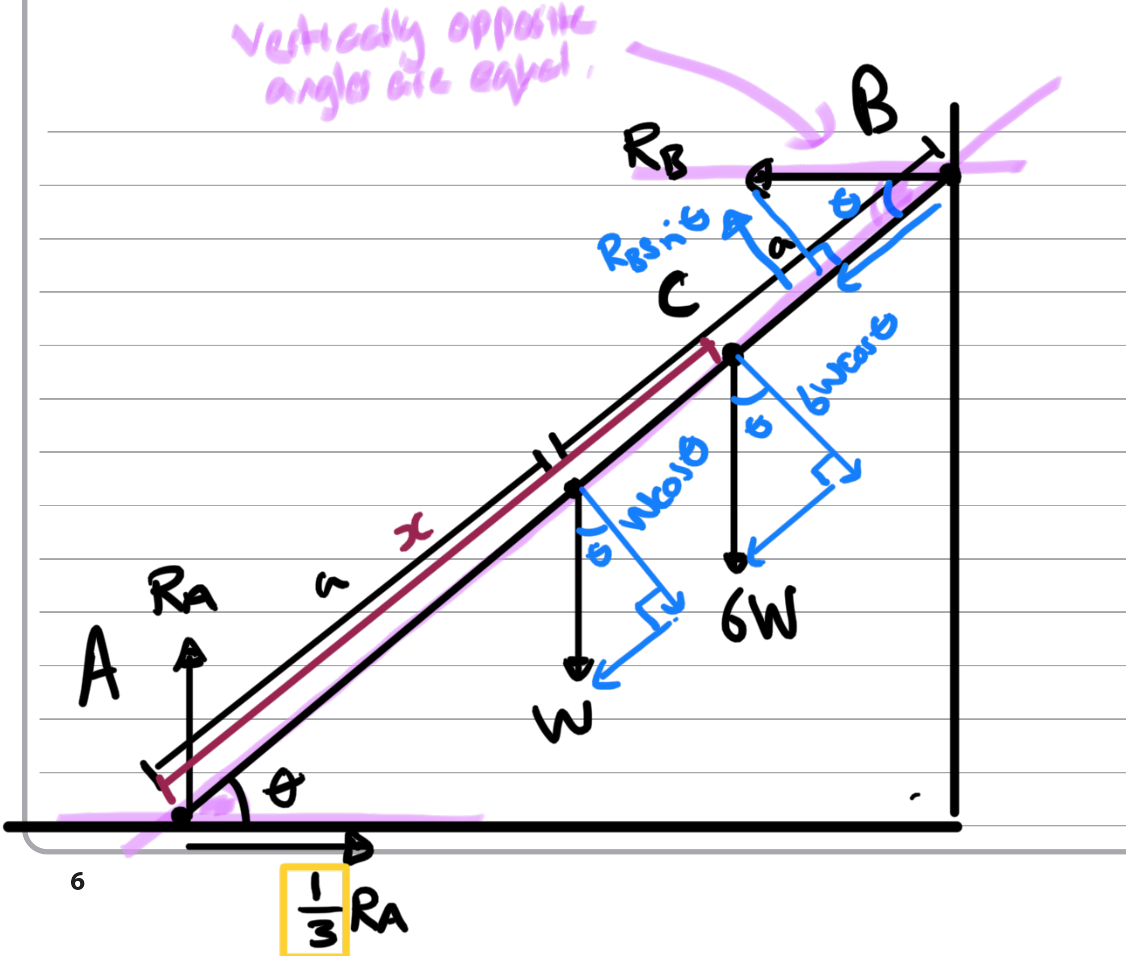
(a) Find, in terms of W, the magnitude of the normal reaction exerted by the wall on the ladder at B. **(4)**

(b) Find the length AC. **(4)**

(c) State the assumptions you have used to model the ladder as uniform and the ladder as a rod. **(2)**

In a refinement of the model, the vertical wall is considered to be rough. The ladder is still in limiting equilibrium.

(d) State, giving a reason, how this would affect your answer to part (a). **(2)**



$\tan\theta = \frac{12}{5}$

between ladder and ground

$\boxed{\mu = \frac{1}{3}}$

Question 2 continued

a) $R(\uparrow)$: $R_A = W + 6W$

$R_A = 7W$ ①

$R(\rightarrow)$: $\frac{1}{3}R_A = R_B$ ②

<u>Sub ① into ②:</u>

$\frac{1}{3}R_A = R_B$

$\frac{1}{3}(7W) = R_B$

$\therefore \boxed{R_B = \frac{7}{3}W}$

we could of labelled middle to C "x" so total distance would be "a + x" but creates more work

Distance AC which we are trying to find.

b) $M(A)$: $(2a \times R_B \sin\theta) = (a \times W\cos\theta) + (x \times 6W\cos\theta)$

$\div \cos\theta \qquad \div \cos\theta \qquad \div \cos\theta$

So we get everything in terms of $\tan\theta$ which we are given in the Q.

$(2a \times R_B \times \tan\theta) = (a \times W) + (x \times 6W)$

$(2a \times R_B \times \frac{12}{5}) = (a \times W) + (x \times 6W)$

from (a)

$(2a \times \frac{7}{3}\cancel{W} \times \frac{12}{5}) = (a \times \cancel{W}) + (x \times 6\cancel{W})$

$2 \times \frac{7}{3} \times \frac{12}{5} \longrightarrow \frac{56}{5}a = a + 6x$

$$\frac{56}{5}a = a + 6x$$

$$\frac{51}{5}a = 6x$$

$\frac{51}{5} \div 6$

$$\therefore \boxed{x = 1.7a}$$

(c) Uniform = weight in the middle of ladder

rod = ladder doesn't bend. Couldn't do moments calculation in same way if it bent.

(d) If the wall was rough there would be a vertical force acting upwards from point B.

$\Rightarrow$ (Resolving vertically) R_A would be reduced

So this doesn't need to be as large to keep ladder in equilibrium.

$$R_A + \mu_B R_B = w + 6w$$

$\Rightarrow$ (Resolving horizontally) friction at A is also reduced

smaller since R_A smaller from above.

$$\boxed{\tfrac{1}{3} R_A} = R_B$$

$\Rightarrow$ R_B is also reduced.

3.

$\tan\alpha = \frac{3}{4}$

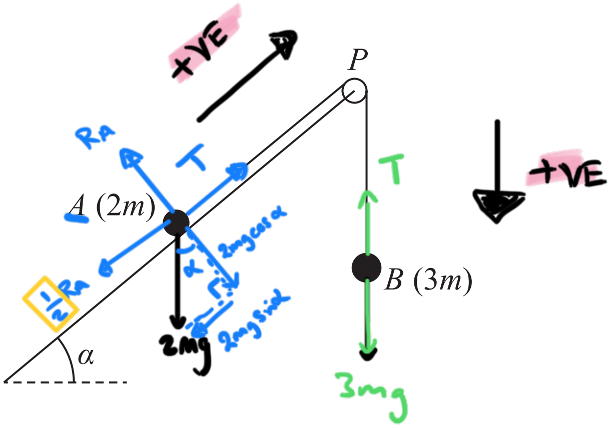


Figure 1

One end of a light inextensible string is attached to a particle A of mass $2m$. The other end of the string is attached to a particle B of mass $3m$. The string passes over a small, smooth, light pulley P which is fixed at the top of a rough inclined plane. The plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$

Particle A is held at rest on the plane with the string taut and B hanging freely below P, as shown in Figure 1. The section of the string AP is parallel to a line of greatest slope of the plane.

The coefficient of friction between A and the plane is $\frac{1}{2}$

Particle A is released and begins to move up the plane.

For the motion before A reaches the pulley,

(a) (i) write down an equation of motion for A,

(ii) write down an equation of motion for B,

(4)

(b) find, in terms of g, the acceleration of A,

(5)

(c) find the magnitude of the force exerted on the pulley by the string.

"What is the pulley feeling due to the string"

(4)

(d) State how you have used the information that P is a smooth pulley.

(1)

$F = ma$

(a) i) $\underline{\underline{A}}$ $\quad T - \boxed{\frac{1}{2}R_A} - 2mg\sin\alpha = 2ma$ (1)

forces acting on A.

Could also call this F for friction...
But since moving $F = F_{max} = \mu R$.

ii) $\underline{\underline{B}}$ $\quad 3mg - T = 3ma$ (2)

forces acting on B.

Question 3 continued

(b) Rearrange ②: $3mg - T = 3ma$

$$T = 3mg - 3ma$$

Sub ② into ①:

$$T - \frac{1}{2}R_A - 2mg\sin\alpha = 2ma$$

$$(3mg - 3ma) - \frac{1}{2}R_A - 2mg\sin\alpha = 2ma$$

Resolve A $R(\nwarrow)$: $\boxed{R_A = 2mg\cos\alpha}$

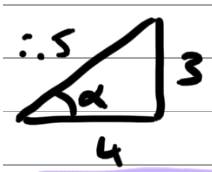
$$\therefore (3mg - 3ma) - \frac{1}{2}(2mg\cos\alpha) - 2mg\sin\alpha = 2ma$$

$$3\cancel{m}g - 3\cancel{m}a - \cancel{m}g\cos\alpha - 2\cancel{m}g\sin\alpha = 2\cancel{m}a$$

$$3g - 3a - g\cos\alpha - 2g\sin\alpha = 2a$$

$\tan\alpha = \frac{3}{4}$

$\therefore$



$\therefore \sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

$$3g - g\left(\frac{4}{5}\right) - 2g\left(\frac{3}{5}\right) = 5a$$

$$3g - \frac{4}{5}g - \frac{6}{5}g = 5a$$

$$\frac{5}{5}g = 5a$$

$$g = 5a$$

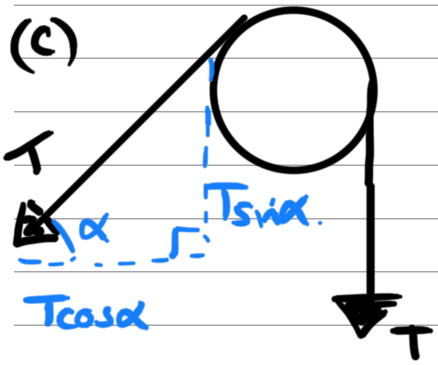
$$\boxed{a = \frac{1}{5}g}$$

(c) PTO.

Question 3 continued

This is a force diagram for what the pulley is feeling

(c)



from(b) from(a)

Sub $a = \frac{1}{5}g$ into ②:

$$T = 3mg - 3ma$$

$$T = 3mg - 3m\left(\frac{1}{5}g\right)$$

$$\boxed{T = \frac{12}{5}mg}$$

Resolving forces pulley is feeling:

$$\therefore \underline{F} = \begin{pmatrix} -T\cos\alpha \\ -T\sin\alpha - T \end{pmatrix}$$

→ +VE
↑ +VE

$$= \begin{pmatrix} -\left(\frac{12}{5}mg\right)\left(\frac{4}{5}\right) \\ -\left(\frac{12}{5}mg\right)\left(\frac{3}{5}\right) - \left(\frac{12}{5}mg\right) \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{48}{25}mg \\ -\frac{36}{25}mg - \frac{12}{5}mg \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{48}{25}mg \\ -\frac{96}{25}mg \end{pmatrix}$$

$$\therefore |\underline{F}| = \sqrt{\left(\frac{48}{25}mg\right)^2 + \left(\frac{96}{25}mg\right)^2}$$

Question 3 continued

$$= \sqrt{\frac{2304}{625} m^2 g^2 + \frac{9216}{625} m^2 g^2}$$

$$= \sqrt{\frac{2304}{125} m^2 g^2}$$

$$= \sqrt{\frac{2304}{125}}\, mg$$

$$= \frac{48\sqrt{5}}{25} mg$$

$$= \boxed{42.1\,m} \quad (3\text{ sf})$$

d) Smooth Pulley = Tension is the same on either side of the pulley.

If pulley was rough one side of the pulley would likely be taking more of the weight than the other side ($\therefore$ tension is not the same)

(Total for Question 3 is 14 marks)

[*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

4. At $t = 0$, a small ball B is projected from a fixed point O with velocity $(5\mathbf{i} + 8\mathbf{j})$ m s−1

The position vector of a point on the path of B is $(x\mathbf{i} + y\mathbf{j})$ m relative to O.

The ball is modelled as a particle moving freely under gravity.

The acceleration due to gravity is modelled as having magnitude 10 m s^{-2}

(a) Show that

$$y = 1.6x - 0.2x^2$$

(4)

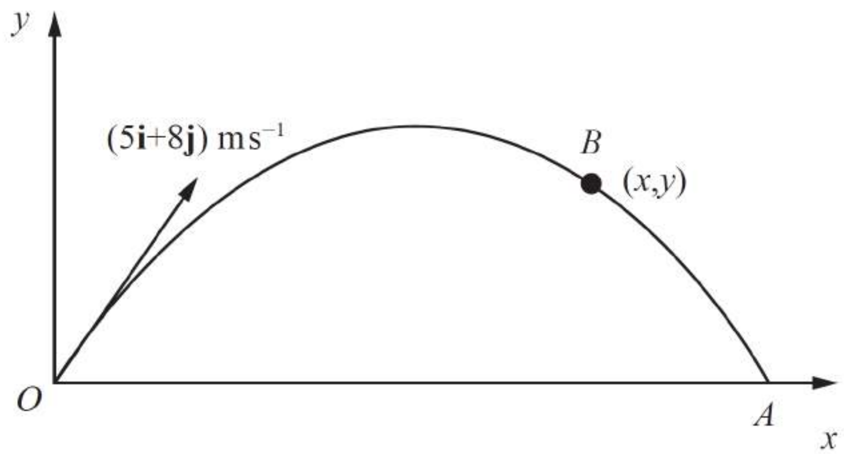


Figure 2

The ball passes through a point A which is on the same horizontal level as O, as shown in Figure 2.

(b) Using part (a), find the distance OA.

(2)

(c) Find the speed and the direction of motion of B as it passes through the point on the path where $x = 6$, giving your answers to 2 significant figures.

(6)

In reality, the acceleration due to gravity is less than 10 m s^{-2}

(d) State, giving a reason, how using a more accurate value for g would affect your answer to part (b).

(2)

(e) Suggest a possible improvement, apart from using a more accurate value for g, which could be made to the model.

(1)

Question 4 continued

+VE →

↑ +VE

(a) horizontal :

$s = x$
$\boxed{u = 5}$
$v =$
$a = 0$
$\boxed{t = T}$

usually 0 as no resistive forces in horizontal direction

vertical :

$s = y$
$\boxed{u = 8}$
$v =$
$a = -10$
$\boxed{t = T}$

told to take as 10 in Q. Opposite direction to what we chose as positive

Takes same time to get to position (x, y).
t is always what links our horizontal and vertical SUVAT calculations

$s = ut + \frac{1}{2}at^2$

$x = 5T + 0$

$x = 5T$

note: can also use distance = speed × time for horizontal (as $a = 0$)

$T = \frac{x}{5}$ ①

$s = ut + \frac{1}{2}at^2$

$y = 8T + \frac{1}{2}(-10)T^2$

$y = 8T - 5T^2$ ②

Sub ① into ② : $y = 8T - 5T^2$

$y = 8\left(\frac{x}{5}\right) - 5\left(\frac{x}{5}\right)^2$

$$\boxed{y = 1.6x - 0.2x^2}$$

As required.

Question 4 continued

(b) horizontal:

$s = 6$
$u = 5$
$v = ?$
$a = 0$
$t =$

$v^2 = u^2 + 2as$
$v^2 = 5^2 + 2(0)(6)$
$v^2 = 5^2$
$v = \pm 5$

horizontal velocity

vertical:

$\boxed{s = 2.4}$ ← can find this using part (a) then use SUVAT.

$u = 8$
$v = ?$
$a = -10$
$t =$

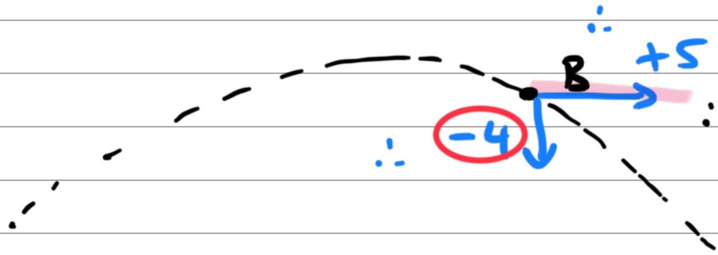
$y = 1.6x - 0.2x^2$
$y = 1.6(6) - 0.2(6)^2$
$y = 2.4$

$v^2 = u^2 + 2as$
$v^2 = 8^2 + 2(-10)(2.4)$
$v^2 = 16$
$v = \pm 4$

vertical velocity

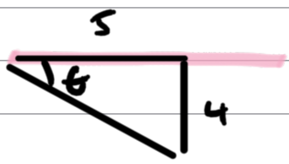
from diagram ball is moving downwards at B.

∴



$\therefore \underline{v} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$

$|\underline{v}| = \sqrt{5^2 + (-4)^2} = \boxed{6.4 \text{ ms}^{-1}}$ (2sf)



$\tan\theta = \frac{4}{5}$

$\theta = \tan^{-1}\left(\frac{4}{5}\right) = 39°$ (2sf)

∴ $\boxed{\text{Direction is } 39° \text{ below horizontal.}}$

Question 4 continued

(d) If gravity is reduced then ball will fall to ground slower

∴ OA will be further

∴ x found in part (b) will be larger

e) Take in to account air resistance (slowing the ball down)