

STEP 1 Mathematics

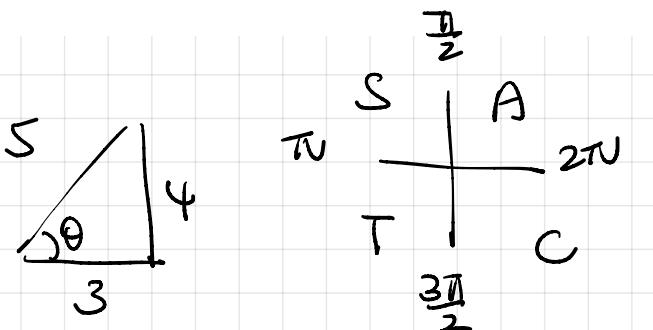
2005 Question 4

Trigonometry

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- 4 (a) Given that $\cos \theta = \frac{3}{5}$ and that $\frac{3\pi}{2} \leq \theta \leq 2\pi$, show that $\sin 2\theta = -\frac{24}{25}$, and evaluate $\cos 3\theta$.
- (b) Prove the identity $\tan 3\theta \equiv \frac{3\tan \theta - \tan^3 \theta}{1 - 3\tan^2 \theta}$.

Hence evaluate $\tan \theta$, given that $\tan 3\theta = \frac{11}{2}$ and that $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$.



$$\cos \theta = \frac{3}{5}$$

$$\sin \theta = -\frac{4}{5}$$

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta \\ &= 2 \left(-\frac{4}{5}\right) \left(\frac{3}{5}\right) \\ &= -\frac{24}{25}\end{aligned}$$

$$\begin{aligned}\cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ &= \left(\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^2 \\ &= \frac{9}{25} - \frac{16}{25} \\ &= -\frac{7}{25}\end{aligned}$$

$$\cos 3\theta = \cos(2\theta + \theta)$$

$$\begin{aligned}&= \cos 2\theta \cos \theta - \sin 2\theta \sin \theta \\ &= \left(-\frac{7}{25}\right) \left(\frac{3}{5}\right) - \left(-\frac{24}{25}\right) \left(-\frac{4}{5}\right)\end{aligned}$$

$$= \frac{-21}{125} - \frac{96}{125}$$

$$\begin{array}{r} 24 \\ 1 \times 4 \\ \hline 96 \end{array}$$

$$\cos 3\theta = -\frac{117}{125} \quad //$$

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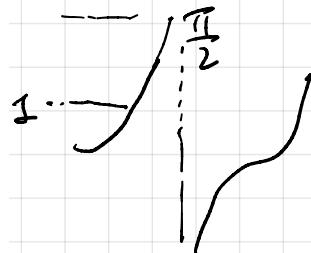
$$\tan 2\theta = \frac{\tan \theta + \tan \theta}{1 - \tan^2 \theta} = \frac{2\tan \theta}{1 - \tan^2 \theta}$$

$$\tan 3\theta = \tan(2\theta + \theta) = \frac{\frac{2\tan \theta}{1 - \tan^2 \theta} + \tan \theta}{1 - \frac{2\tan \theta}{1 - \tan^2 \theta} \tan \theta}$$

$$= \frac{[2\tan \theta + \tan \theta(1 - \tan^2 \theta)] / (1 - \tan^2 \theta)}{((1 - \tan^2 \theta) - 2\tan^2 \theta) / (1 - \tan^2 \theta)}$$

$$\tan 3\theta \equiv \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

$$\tan\left(\frac{\pi}{4}\right) = 1$$



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$$\tan\theta > 1$$

↙
need to reject answers.

$$\frac{11}{2} = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$$

$$11 - 33\tan^2\theta = 6\tan\theta - 2\tan^3\theta$$

$$\text{Let } \tan\theta = y$$

$$2y^3 - 33y^2 - 6y + 11 = 0$$

$$\text{Try } y = \frac{1}{2} \quad f\left(\frac{1}{2}\right) = 0$$

$y = 0$	$y = \frac{1}{2}$
$y = -1$	$y = -1$
$y = 2$	$y = -2$
$y = 3$	$y = -3$

$$2\left(\frac{1}{8}\right) - 33\left(\frac{1}{4}\right) - 6\left(\frac{1}{2}\right) + 11 = 0$$

$$\frac{1}{4} - \frac{33}{4} - \frac{12}{4} + \frac{44}{4} = 0$$

$(2y-1)$ is factor.

$$(2y-1)(ay^2 + by + c) = 2y^3 - 3y^2 - 6y + 11$$

$$2a=2, \quad a=1, \quad (y^3)$$

$$2b-a=-33 \quad (y^2)$$

$$2b-1=-33$$

$$2b=-32$$

$$b=-16$$

$$-c=11 \quad (y^0)$$

$$c=-11$$

$$2c-b=-6 \quad (y^1)$$

$$2(-11)-(-16)$$

$$= -22+16$$

$$= -6$$

$$2HS=RHS \quad \checkmark$$

$$y = \tan \theta$$

$$(2y-1)(y^2 - 16y - 11) = 0$$

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$$\tan \theta \geq 1$$

$$2y - 1 = 0$$

$$y = \frac{1}{2}$$

(reqd)

$$y^2 - 16y + 11 = 0$$

$$\frac{16 \pm \sqrt{16^2 - 4(1)(-11)}}{2} = y$$

$$\begin{array}{r} 16 \\ \times 16 \\ \hline 160 \\ 936 \\ \hline 256 \end{array}$$

$$\frac{16 \pm \sqrt{300}}{2} = y$$

$$\frac{16 \pm 10\sqrt{3}}{2} = y$$

$$16 \pm 5\sqrt{3} = y$$

$$y = 8 + \sqrt{75} \quad y = 8 - \sqrt{75}$$

$$y = 16 \dots$$

$$y = 21$$

(reqd)

$$\sqrt{64} < \sqrt{75} < \sqrt{81}$$

$8 < \sqrt{75} < 9$

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$$\tan \theta = 8 + \sqrt{75}$$

OR

$$\tan \theta = 8 + 5\sqrt{3}$$