Dynamics on an Inclined Plane

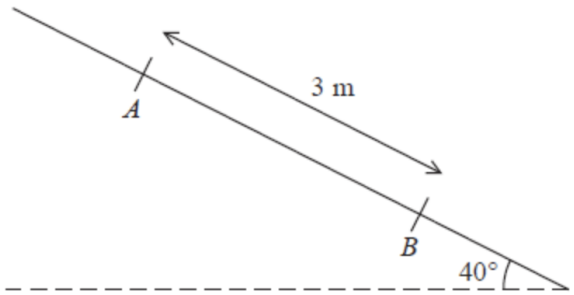
Question 1:

A small brick of mass 0.5 kg is placed on a rough plane which is inclined to the horizontal at an angle θ, where $\tan\theta = \frac{4}{3}$, and released from rest. The coefficient of friction between the brick and the plane is $\frac{1}{3}$.

Find the acceleration of the brick.

[9]

Question 2:



A rough plane is inclined at 40° to the horizontal. Two points A and B are 3 metres apart and lie on a line of greatest slope of the inclined plane, with A above B. A particle P of mass m kg is held at rest on the plane at A. The coefficient of friction between P and the plane is $\frac{1}{2}$. The particle is released.

(a) Find the acceleration of P down the plane.

[5]

(b) Find the speed of P at B.

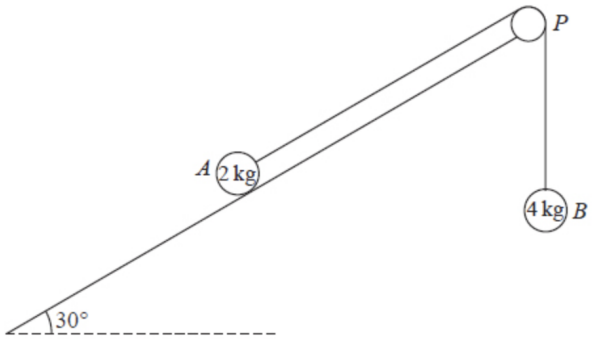
[2]

Question 3:

A lifeboat slides down a straight ramp inclined at an angle of 15° to the horizontal. The lifeboat has mass 800 kg and the length of the ramp is 50 m. The lifeboat is released from rest at the top of the ramp and is moving with a speed of 12.6 ms^{-1} when it reaches the end of the ramp. By modelling the lifeboat as a particle and the ramp as a rough inclined plane, find the coefficient of friction between the lifeboat and the ramp.

[9]

Question 4:



A fixed rough plane is inclined at 30° to the horizontal. A small smooth pulley P is fixed at the top of the plane. Two particles A and B, of mass 2 kg and 4 kg respectively, are attached to the ends of a light inextensible string which passes over the pulley P. The coefficient of friction between A and the plane is $\frac{1}{\sqrt{3}}$. Initially A is held at rest on the plane. The particles are released from rest with the string taut and A moves up the plane.

(a) Find the magnitude of the acceleration of the particles.

[7]

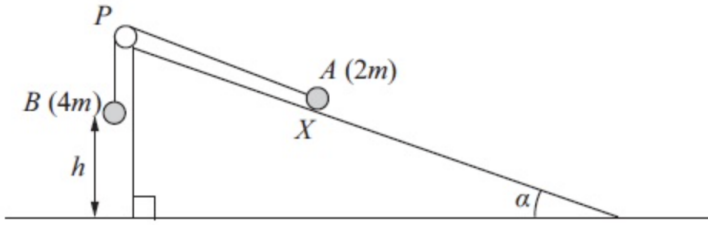
(b) Find the tension in the string immediately after the particles are released.

[2]

(c) Find the force exerted on the pulley by the string.

[2]

Question 5:



Particles A and B, of mass $2m$ and $4m$ respectively, are on a rough plane, connected by a light inextensible string. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between A and the plane is $\frac{1}{4}$. The string passes over a small smooth pulley P fixed at the top of the plane. The system is released from rest with the string taut, with A at the point X and with B at a height h above the ground. For the motion until B hits the ground,

(a) State one reason why the magnitudes of the accelerations of the two particles are the same.

[1]

(b) Find the acceleration of each particle.

[7]

(c) Find, in terms of m, the force exerted on the pulley by the string.

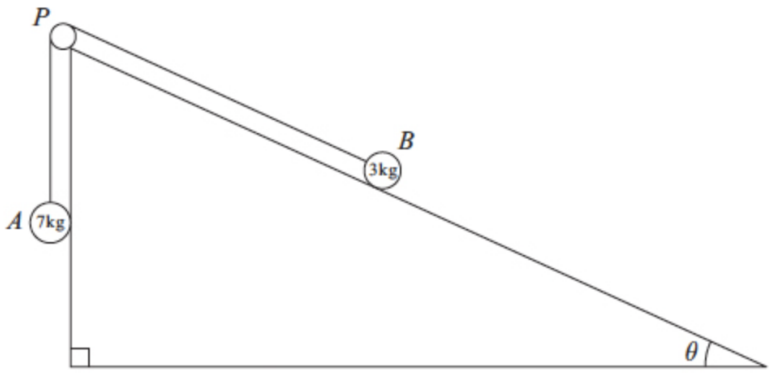
[4]

Particle B does not rebound when it hits the ground. A continues moving up the plane towards P.

(d) Given that A comes to rest at the point Y, without reaching P, find the distance XY in terms of h.

[6]

Question 6:



Two particles A and B, of mass 7 kg and 3 kg respectively, are attached to the ends of a light inextensible string. Initially B is held at rest on a rough fixed plane inclined at angle θ to the horizontal, where $\tan\theta = \frac{5}{12}$. The part of the string from B to P is parallel to a line of greatest slope of the plane. The string passes over a small smooth pulley, P, fixed at the top of the plane. The particle A hangs freely below P, as shown in above. The coefficient of friction between B and the plane is $\frac{2}{3}$. The particles are released from rest with the string taut and B moves up the plane.

(a) Find the magnitude of the acceleration of B immediately after release.

[10]

(b) Find the speed of B when it has moved 1 m up the plane.

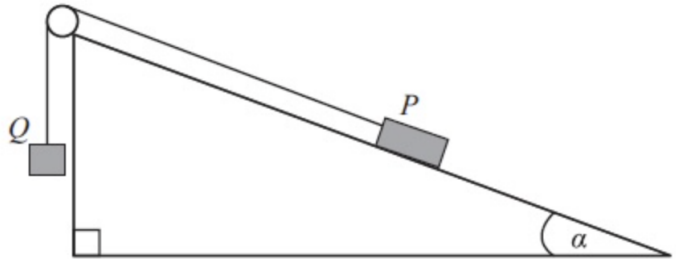
[2]

When B has moved 1 m up the plane the string breaks.

(c) Given that in the subsequent motion B does not reach P, find the time between the instants when the string breaks and when B comes to instantaneous rest.

[4]

Question 7:



Two particles P and Q have masses 0.3 kg and m kg respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a fixed rough plane. The plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$. The coefficient of friction between P and the plane is $\frac{1}{2}$. The string lies in a vertical plane through a line of greatest slope of the inclined plane. The particle P is held at rest on the inclined plane and the particle Q hangs freely below the pulley with the string taut. The system is released from rest and Q accelerates vertically downwards at 1.4 ms^{-2}.

(a) The magnitude of the normal reaction of the inclined plane on P.

[2]

(b) The value of m.

[8]

When the particles have been moving for 0.5 s, the string breaks.

(c) Assuming that P does not reach the pulley, find the further time that elapses until P comes to instantaneous rest.

[6]

Numerical Answers:

(1) $a = 0.6g$ ms^{-2}

(2) (a) $a = 2.55$ ms^{-2}
(b) $v = 3.91$ ms^{-1}

(3) $\mu = 0.10$

(4) (a) $a = \frac{1}{3}g$ ms^{-2}
(b) $T = \frac{8}{3}g$ N
(c) $F = \frac{8}{3}\sqrt{3}g$ N

(5) (a) The string is inextensible.
(b) $a = 0.4g$ ms^{-2}
(c) $F = \frac{48}{25}\sqrt{5}mg$ N
(d) $XY = 1.5h$

(6) (a) $a = 0.4g$ ms^{-2}
(b) $v = 2.8$ ms^{-1}
(c) $t = \frac{2}{7}$ s

(7) (a) $R = 0.24g$ N
(b) $m = 0.4$
(c) $t = \frac{1}{14}$ s