Question 1:

The binomial series expansion of $(1+ax)^{2/3}$, $|ax|<1$, up to and including the term in x^2 is

$$1+\frac{1}{2}x+kx^2$$

where a and k are constants.

(a) Find the value of a.

[2]

(b) Find the value of k, giving your answer in its simplest form.

[2]

(c) Hence, find the coefficient of x^2 in the series expansion of $(4-9x)(1+ax)^{2/3}$, $|ax|<1$.

[2]

Question 2:

(a) Use the binomial expansion to show that

$$\sqrt{\frac{1+x}{1-x}} \approx a+bx+cx^2 \qquad |x|<1$$

where a, b, c are constants to be determined.

[6]

(b) Substitute $x=\frac{1}{26}$ into the expansion in part (a) to obtain an approximation to $\sqrt{3}$. Give your answer in the form $\frac{a}{b}$ where a, b are integers.

[3]

Question 3:

$$f(x)=\frac{27x^2+32x+16}{(3x+2)^2(1-x)}$$

(a) Express $f(x)$ in partial fractions.

[4]

(b) Hence, or otherwise, find the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2. Simplify each term.

[6]

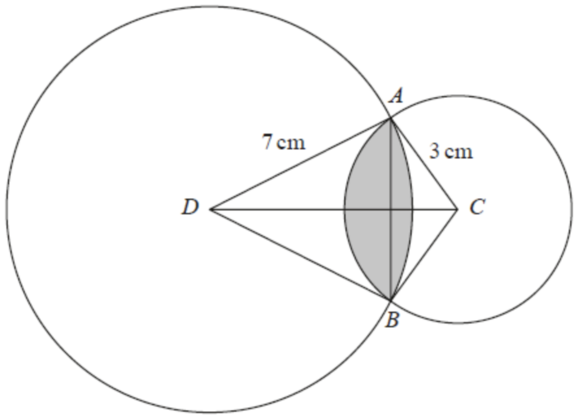
(c) State the range of values of x for which this expansion is valid.

[1]

(d) Find the percentage error made in using the series expansion in part (b) to estimate the value of $f(0.2)$. Give your answer to 2 significant figures.

[4]

Question 4:



The diagram shows a circle centred at D with radius 7 cm, and another circle centred at C with radius 3 cm. They intersect at the points A and B, and the distance CD is 9 cm.

(a) Find $\angle ADC$ in radians.

[2]

(b) Find $\angle ACD$ in radians.

[2]

The region common to both circles is shown shaded in the diagram.

(c) Find the perimeter of the shaded region.

[3]

(d) Find the area of the shaded region.

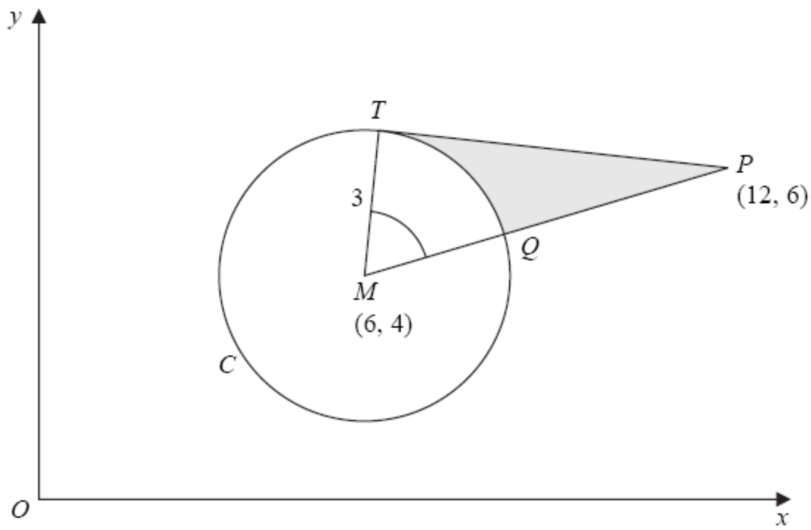
[4]

Question 5:

The circle C has centre $M(6, 4)$ and radius 3.

(a) Write down the equation of the circle.

[2]



The point T lies on the circle and the tangent at T passes through the point $P(12, 6)$. The line MP cuts the circle at Q.

(b) Find $\angle TMQ$ in radians. Give your answer to 4 decimal places.

[4]

The shaded region TPQ is bounded by the straight lines TP, QP and the arc TQ.

(c) Find the area of the shaded region TPQ. Give your answer to 3 decimal places.

[5]

Question 6:

$$g(x) = \arctan(x) \qquad x \in \mathbb{R}$$

(a) Sketch the graph of $y = g(x)$. On your sketch, clearly label the coordinates of any intersections with the axes, and the equations of any asymptotes.

[2]

(b) Showing each step of your working out, find the value of x for which $3g\left(\frac{1}{x} - 2\right) - \pi = 0$.
Express your answer in the form $a + b\sqrt{3}$ where a, b are constants to be determined.

[5]

Question 7:

Solve the equation

$$\sin^{-1}(3x) = \cos^{-1}(4x)$$

[6]

Question 8:

The acute angles x and y, and the constant m, are such that

$$\tan x = m \qquad \tan y = \frac{1}{8}m + 5 \qquad 79\sec^2 x + 64\sec^2 y = 2223$$

(a) Find the value of m.

[4]

(b) Find the exact value of $\sin x$.

[2]

(c) Find the exact value of $\cot y$.

[2]

Question 9:

Solve the equation

$$2\cot^2(3\theta) = 7\,\text{cosec}(3\theta) - 5 \qquad 0 \le \theta \le 2\pi$$

[10]

Numerical Answers:

(1) (a) $a = \frac{3}{4}$
 (b) $k = -\frac{1}{16}$
 (c) $-\frac{19}{4}$

(2) (a) $a = 1$, $b = 1$, $c = \frac{1}{2}$
 (b) $\frac{7025}{4056}$

(3) (a) $\frac{4}{(3x+2)^2} + \frac{3}{1-x}$
 (b) $4 + \frac{39}{4}x^2$
 (c) $|x| < \frac{2}{3}$ (which is the same as writing $-\frac{2}{3} < x < \frac{2}{3}$)
 (d) 1.1%

(4) (a) $\angle ADC = 0.283$
 (b) $\angle ACD = 0.709$
 (c) 8.21 cm
 (d) 2.66 cm^2

(5) (a) $(x-6)^2 + (y-4)^2 = 9$
 (b) $\angle TMQ = 1.0766$
 (c) 3.507

(6) $x = \frac{1}{5}$ only

(7) (a) Sketch
 (b) $x = 2 - \sqrt{3}$

(8) (a) $m = 2$ only

(b) $\sin x = \frac{2}{\sqrt{5}}$

(c) $\cot y = \frac{4}{21}$

(9) $\theta = 0.113, 0.934, 2.21, 3.03, 4.30, 5.12$