Mini Test 03 - Circles, Binomial expansion, Graphs and Transformations / Graphs

Question 1

A circle C has equation

$$x^2 + y^2 + 4x - 10y - 21 = 0$$

- The point $P(5, 4)$ lies on C.

Find the equation of the tangent to C at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

[7]

$$x^2 + 4x + y^2 - 10y - 21 = 0$$

$$(x+2)^2 - 4 + (y-5)^2 - 25 - 21 = 0$$

$$(x+2)^2 + (y-5)^2 = 50$$

centre $(-2, 5)$ radius $\sqrt{50}$
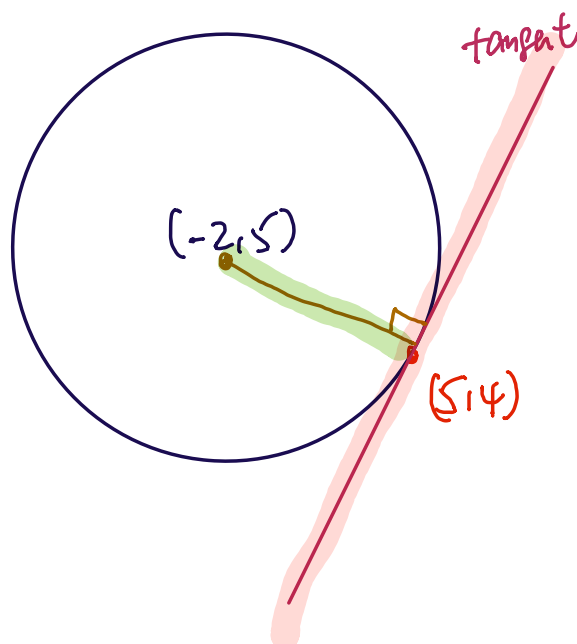
m radius : $\dfrac{5-4}{-2-5} = \dfrac{1}{-7}$

m tangent $= 7$

$y = 7x + c$ $(5, 4)$

$4 = 7(5) + c$

$c = -31$

$\boxed{y = 7x - 31}$

Question 2

(a) Find the first four terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{1}{4}x\right)^6 \qquad (a+b)^n$$

[4]

(b) Given that x is small, so terms in x^4 and higher powers of x may be ignored, show

$$\left(2 - \frac{1}{4}x\right)^6 + \left(2 + \frac{1}{4}x\right)^6 = a + bx^2$$

where a and b are constants to be found.

[3]

(a)

$C_0^6 = 1$	$C_1^6 = 6$	$C_2^6 = 15$	$C_3^6 = 20$
$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$
$\left(\frac{-1}{4}x\right)^0 = 1$	$\left(\frac{-1}{4}x\right)^1 = \frac{-1}{4}x$	$\left(\frac{-1}{4}x\right)^2 = \frac{1}{16}x^2$	$\left(\frac{-1}{4}x\right)^3 = \frac{-1}{64}x^3$

$$64 + -48x + 15x^2 - \frac{5}{2}x^3$$

(b)

$$\left(2 + \frac{1}{4}x\right)^6 = 64 + 48x + 15x^2 + \frac{5}{2}x^3$$

$$\left(2 - \frac{1}{4}x\right)^6 + \left(2 + \frac{1}{4}x\right)^6 = 128 + 30x^2 \quad //$$

$$a = 128 \quad b = 30 \quad //$$

Question 3

[handwritten: $y = f(x) = x^2$ ∪]

[handwritten: $f(-x) = (-x)^2 = x^2$ ∪]

[handwritten right: $y = f(x) = (x-2)^2$]
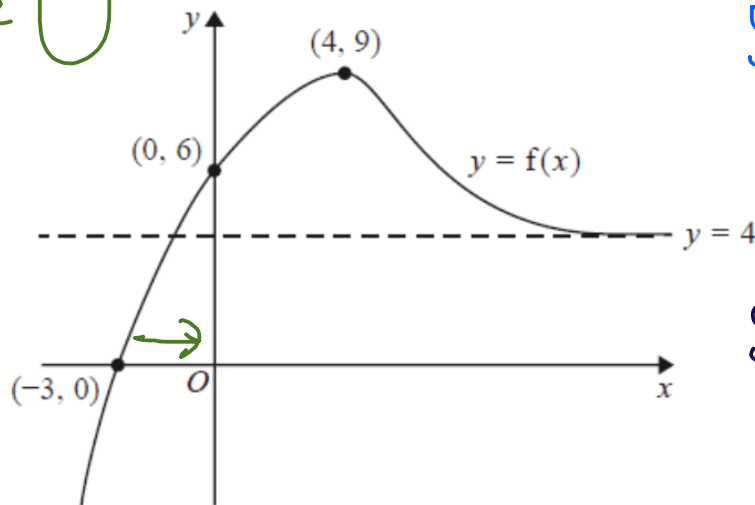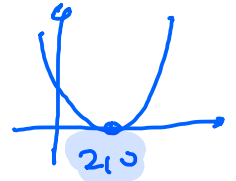
[handwritten: $y = f(\tfrac{1}{4}x) = (\tfrac{1}{4}x - 2)^2$; $\tfrac{1}{4}x - 2 = 0$; $\tfrac{1}{4}x = 2$; $x = 8$]

Figure 1

The curve C with equation $y = f(x)$ is shown in Figure 1.
The curve C

- has a single turning point, a maximum at $(4, 9)$

- crosses the coordinate axes at only two places, , $(-3, 0)$ and $(0, 6)$ *[handwritten: ↑ reflection]*

- has a single asymptote with equation $y = 4$

(a) State the equation of the asymptote to the curve with equation $y = f(-x)$. *[handwritten: $y = 4$]* **[1]**

(b) State the coordinates of the turning point on the curve with equation $y = f\left(\frac{1}{4}x\right)$. *[handwritten: $(16, 9)$]* **[1]**

Given that the line with equation $y = k$, where k is constant, intersects C at exactly one point,

(c) State the possible values for k. *[handwritten: $k = 9$ OR $k \leq 4$]* **[2]**

The curve C is transformed to a new curve that passes through the origin. *[handwritten: $a = 6$]*

(d) (i) Given that the new curve has equation $y = f(x) - a$, state the value of the constant a.

   (ii) Write down an equation for another single transformation of C that also passes through the origin. **[2]**

[handwritten: $f(x+3)$ three to the right]