SOHOKMATHS

Year 13 (in class test)

Differentiation (Skills based only)

you CHAN do it

Time: 36 minutes

Surname ______________________________ Other names ______________________________

☐ M2E Mr Chan/Ms Esteban Ruiz ☐ MaB Mr Chan/Mr Phillips

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill at the top of this page with your name, and tick the box with the class you belong to.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 4 questions in this question paper. The total mark for this paper is 30.
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Question	Marks	Score
1	7	
2	6	
3	9	
4	8	
Total:	30	

1. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

Question	Scheme	Marks
(a)	$p = 4\pi^2$ or $(2\pi)^2$	B1
		(1)
(b)	$x = (4y - \sin 2y)^2 \Rightarrow \frac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$	M1 A1
	Sub $y = \frac{\pi}{2} \Rightarrow \frac{dx}{dy} = 24\pi \quad (= 75.4)$ OR $\Rightarrow \frac{dy}{dx} = \frac{1}{24\pi} (= 0.013)$	M1
	Equation of tangent $y - \frac{\pi}{2} = \frac{1}{24\pi}\ x - 4\pi^2$	M1
	Using $y - \frac{\pi}{2} = \frac{1}{24\pi}\ x - 4\pi^2$ with $x = 0 \Rightarrow y = \frac{\pi}{3}$ cso	M1 A1
		(6)
	Alternative I for first two marks	
	$x = (4y - \sin 2y)^2 \Rightarrow x^{0.5} = 4y - \sin 2y$ $\Rightarrow 0.5x^{-0.5}\frac{dx}{dy} = 4 - 2\cos 2y$	M1A1
	Alternative II for first two marks	
	$x = \left(16y^2 - 8y\sin 2y + \sin^2 2y\right)$ $\Rightarrow 1 = 32y\frac{dy}{dx} - 8\sin 2y\frac{dy}{dx} - 16y\cos 2y\frac{dy}{dx} + 4\sin 2y\cos 2y\frac{dy}{dx}$ **Or** $1dx = 32y\,dy - 8\sin 2y\,dy - 16y\cos 2y\,dy + 4\sin 2y\cos 2y\,dy$	M1A1
		(7 marks)

Notes:

(a)

B1: $p = 4\pi^2$ or exact equivalent $2\pi^{\ 2}$. Also allow $x = 4\pi^2$

(b)

M1: Uses the chain rule of differentiation to get a form $A(4y - \sin 2y)(B \pm C\cos 2y)$, $A, B, C \neq 0$ on the right hand side.

Alternatively attempts to expand and then differentiate using product rule and chain rule to a form $x = \left(16y^2 - 8y\sin 2y + \sin^2 2y\right) \Rightarrow \frac{dx}{dy} = Py \pm Q\sin 2y \pm Ry\cos 2y \pm S\sin 2y\cos 2y \quad P, Q, R, S \neq 0$

A second method is to take the square root first. To score the method look for a differentiated expression of the form $Px^{-0.5} ... = 4 - Q\cos 2y$

A third method is to multiply out and use implicit differentiation. Look for the correct terms, condoning errors on just the constants.

Question notes *continued*

A1: $\frac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$ or $\frac{dy}{dx} = \frac{1}{2(4y - \sin 2y)(4 - 2\cos 2y)}$ with both sides correct. The lhs may be seen elsewhere if clearly linked to the rhs. In the alternative $\frac{dx}{dy} = 32y - 8\sin 2y - 16y\cos 2y + 4\sin 2y\cos 2y$

M1: Sub $y = \frac{\pi}{2}$ into their $\frac{dx}{dy}$ or inverted $\frac{dx}{dy}$. Evidence could be minimal, eg $y = \frac{\pi}{2} \Rightarrow \frac{dx}{dy} = ...$

It is not dependent upon the previous M1 but it must be a changed $x = (4y - \sin 2y)^2$

M1: Score for a correct method for finding the equation of the tangent at $\left('4\pi^2', \frac{\pi}{2}\right)$.

Allow for $y - \frac{\pi}{2} = \frac{1}{\text{their numerical } {}^{dx}\!/\!{}_{dy}}\ x - \text{their } 4\pi^2$

Allow for $\left(y - \frac{\pi}{2}\right) \times \text{their numerical } {}^{dx}\!/\!{}_{dy} = x - \text{their } 4\pi^2$

Even allow for $y - \frac{\pi}{2} = \frac{1}{\text{their numerical } {}^{dx}\!/\!{}_{dy}}\ x - p$

It is possible to score this by stating the equation $y = \frac{1}{24\pi}x + c$ as long as $\left('4\pi^2', \frac{\pi}{2}\right)$ is used in a subsequent line.

M1: Score for writing their equation in the form $y = mx + c$ and stating the value of 'c' or setting $x = 0$ in their $y - \frac{\pi}{2} = \frac{1}{24\pi}\ x - 4\pi^2$ and solving for y.

Alternatively using the gradient of the line segment AP = gradient of tangent.

Look for $\frac{\frac{\pi}{2} - y}{4\pi^2} = \frac{1}{24\pi} \Rightarrow y = ..$ Such a method scores the previous M mark as well.

At this stage all of the constants must be numerical. It is not dependent and it is possible to score this using the "incorrect" gradient.

A1: cso $y = \frac{\pi}{3}$. You do not have to see $\left(0, \frac{\pi}{3}\right)$

2.

$$f(x) = \frac{(2x+5)^2}{x-3} \quad x \neq 3$$

(a) Find $f'(x)$ in the form $\dfrac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are fully factorised quadratic expressions.

(4)

(b) Hence find the range of values of x for which $f(x)$ is increasing.

(2)

Question Number	Scheme	Marks
(i) (a)	$f'(x) = \dfrac{4(x-3)(2x+5)-(2x+5)^2}{(x-3)^2}$ or $\dfrac{(x-3)(8x+20)-(4x^2+20x+25)}{(x-3)^2}$	M1 A1
	$= \dfrac{(2x+5)(2x-17)}{(x-3)^2}$	M1 A1
(b)	Attempts both critical values or finds one "correct" end	M1
	$x < -2.5,\ x > 8.5$ (accept $x \leqslant -2.5,\ x \geqslant 8.5$)	A1
		(6)

(i)(a)

M1 Attempts the quotient rule and achieves

$$f'(x)=\frac{A(x-3)(2x+5)-B(2x+5)^2}{(x-3)^2} \quad A,B>0 \text{ condoning slips}$$

Alternatively uses the product rule and achieves

$$\frac{d}{dx}\left\{(x-3)^{-1}(2x+5)^2\right\}=\pm A(2x+5)^2(x-3)^{-2}+B(x-3)^{-1}(2x+5) \quad A,B>0$$

They may attempt to multiply out the $(2x+5)^2$ first which is fine as long as they reach a 3TQ.

A1 Score for correct unsimplified $f'(x)$

M1 Attempts to take out a factor of $(2x+5)$ or multiplies out and attempts to factorise the numerator.

The method must be seen $\frac{(x-3)4(2x+5)\pm(2x+5)^2}{...}=\frac{(2x+5)\{(x-3)4\pm(2x+5)\}}{...}$ condoning slips.

If the method is not seen it may be implied by a correct result for their fraction

This can be achieved from an incorrect quotient or product rule. E.g. $\frac{vu'+uv'}{v^2}$ or $\frac{vu'-uv'}{v}$

It can be scored by candidates who multiply out their numerators and then factorise by taking out a factor of $(2x+5)$

If the product rule is used it would be for writing as a single fraction and taking out, from the numerator, a common factor of $(2x+5)$.

A1 $\frac{(2x+5)(2x-17)}{(x-3)^2}$ but accept expressions such as $\frac{4(x+2.5)(x-8.5)}{(x-3)^2}$ or $\frac{(2x+5)(2x-17)}{(x-3)(x-3)}$

Note the final two marks in (i)(a) may be scored in (i)(b), ONLY IF the correct work is done on the complete fraction, not just the numerator

(i)(b)

M1 Achieves the two critical values from the quadratic numerator of their $f'(x)$

Alternatively finds one correct end for their $(2x+5)(2x-17)>0$ or $(2x+5)(2x-17)\geqslant 0$

So award for either $x<-2.5$ or $x>$ their 8.5 which may be scored from an intermediate line.

A1 $x<-2.5, x>8.5$ (accept $x\leqslant -2.5,\ x\geqslant 8.5$).

Ignore any references to "and" or "or" so condone $x<-2.5$ and $x>8.5$

Mark the final response. This is not isw

It may follow working such as $(2x+5)(2x-17)>0 \Rightarrow x>-\frac{5}{2}, x>\frac{17}{2}$. So $x<-2.5, x>8.5$

Accept alternative forms such as $(-\infty,-2.5]\cup[8.5,\infty)$

3.

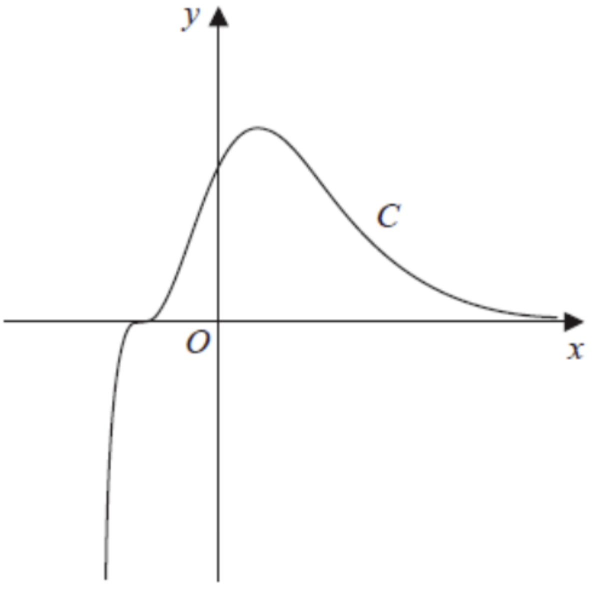

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$f(x) = (2x+1)^3 e^{-4x}$$

(a) Show that

$$f'(x) = A(2x+1)^2(1-4x)\mathrm{e}^{-4x}$$

where A is a constant to be found.

(4)

(b) Hence find the exact coordinates of the two stationary points on C.

(3)

The function g is defined by

$$g(x) = 8f(x-2)$$

(c) Find the coordinates of the maximum stationary point on the curve with equation $y = g(x)$.

(2)

Question Number	Scheme	Marks
(a)	$f'(x)=6(2x+1)^2e^{-4x}-4(2x+1)^3e^{-4x}$	M1 A1
	$=2(2x+1)^2e^{-4x}\{3-2(2x+1)\}$	dM1
	$=2(2x+1)^2(1-4x)e^{-4x}$	A1
		(4)
(b)	Sets $f'(x)=0\Rightarrow x=-\frac{1}{2},\frac{1}{4}$	B1
	Either $f\left("-\frac{1}{2}"\right)=\ldots$ or $f\left("\frac{1}{4}"\right)=\ldots$	M1
	Both $\left(-\frac{1}{2},0\right)$ and $\left(\frac{1}{4},\frac{27}{8e}\right)$	A1
		(3)
(c)	$\left(\frac{9}{4},\frac{27}{e}\right)$	B1ft B1ft
		(2)
		9 marks

Notes

(a)

M1 Attempts the product rule to achieve $P(2x+1)^2e^{-4x}\pm Q(2x+1)^3e^{-4x}$

May also be attempted by the quotient rule - equivalent form after e terms cancel.

A1 $f'(x)=6(2x+1)^2e^{-4x}-4(2x+1)^3e^{-4x}$ which may be unsimplified

dM1 Correctly takes out a common factor of $(2x+1)^2e^{-4x}$ from their expression with an intermediate step before the final answer. Allow if there are minor slips in the $(2x+1)^2e^{-4x}$ as a factor (such as $(2x+)^2e^{-4x}$) if recovered - look for the correct remaining terms in the bracket { }. Allow going from an expanded cubic to a factorised form for this mark:

$e^{-4x}\left(2-24x^2-32x^3\right)\to 2(2x+1)^2(1-4x)e^{-4x}$.

A1 Achieves $2(2x+1)^2(1-4x)e^{-4x}$ with no incorrect algebra. Accept with the brackets in either order.

(b)

B1 $x=-\frac{1}{2},\frac{1}{4}$ o.e. Both required.

M1 Attempts to substitute one of $x=\pm\frac{1}{2},\pm\frac{1}{4}$ into f(x). If substitution not seen may be implied by either of $\left(-\frac{1}{2},0\right)$ or $\left(\frac{1}{4},\frac{27}{8e}\right)$ o.e (accept awrt 1.24 for this mark) or by either of $\left(\frac{1}{2},\frac{8}{e^2}\right)$ (awrt 1.08) or $\left(-\frac{1}{4},\frac{e}{8}\right)$ (awrt 0.340) o.e.

A1 For $\left(-\frac{1}{2},0\right)$ and $\left(\frac{1}{4},\frac{27}{8e}\right)$ o.e. must be exact but isw after exact coordinates given.

Allow as $x=\ldots, y=\ldots$ as long as clearly paired.
Allow the M and A marks if seen in part (c) - mark (b) and (c) together.

4.

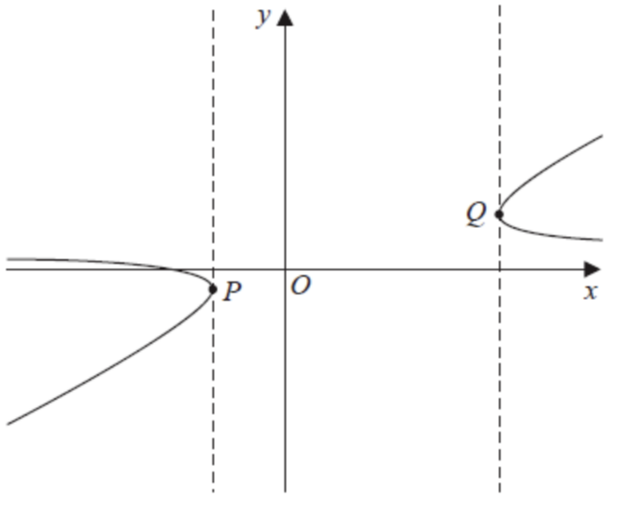

Figure 2

Figure 2 shows a sketch of the curve with equation

$$x = \frac{2y^2 + 6}{3y - 3}$$

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ giving your answer as a fully simplified fraction.

(4)

The tangents at points P and Q on the curve are parallel to the y-axis, as shown in Figure 2.

(b) Use the answer to part (a) to find the equations of these two tangents.

(4)

Question Number	Scheme	Marks
(a)	$x=\dfrac{2y^2+6}{3y-3}\Rightarrow\left(\dfrac{dx}{dy}=\right)\dfrac{4y(3y-3)-3(2y^2+6)}{(3y-3)^2}$	M1 A1
	$\dfrac{dx}{dy}=\dfrac{6y^2-12y-18}{9(y-1)^2}=\dfrac{2y^2-4y-6}{3(y-1)^2}$ o.e	dM1, A1
		(4)
(b)	P and Q are where $\dfrac{dx}{dy}=0$ or where $2y^2-4y-6=0$	B1
	Solves $2y^2-4y-6=0\Rightarrow 2(y-3)(y+1)=0\Rightarrow y=3,-1$	M1
	Subs $y=-1$ and 3 in $x=\dfrac{2y^2+6}{3y-3}\Rightarrow x=..$	dM1
	Achieves $x=-\dfrac{4}{3}$ and $x=4$	A1cso
		(4) **8 marks**

Notes

(a)

M1 Attempts the quotient rule. Condone slips on the coefficients - look for $\dfrac{Ay(3y-3)-B(2y^2+6)}{(3y-3)^2}$

$A,B>0$. Allow a product rule attempt:

$x=(2y^2+6)(3y-3)^{-1}\Rightarrow\left(\dfrac{dx}{dy}=\right)Ay(3y-3)^{-1}+(2y^2+6)\times -B(3y-3)^{-2}$

A1 Correct differentiation which may be unsimplified. Allow if the $\dfrac{dx}{dy}$ is missing or called $\dfrac{dy}{dx}$ for this mark. By product rule $4y(3y-3)^{-1}+(2y^2+6)\times -3(3y-3)^{-2}$ Condone missing brackets if recovered.

dM1 Requires an attempt to get a single fraction with some attempt to simplify.
For the quotient rule look for a simplification of the numerator with like terms collected giving a 3TQ.
Attempts via the product rule will require a correct method to put as a single fraction.

A1 $\left(\dfrac{dx}{dy}=\right)\dfrac{2y^2-4y-6}{3y^2-6y+3}$ or exact simplified equivalent such as $\dfrac{2(y-3)(y+1)}{3(y-1)^2}$ isw after a correct simplified answer. Common factor 3 must have been cancelled. Must be seen in part (a). A0 if called $\dfrac{dy}{dx}$ but allow A1 if LHS is not stated.

Attempts at $\dfrac{dy}{dx}$ can score the first 3 marks if correct. Allow use of x in place of y for the Ms.

(b)

B1 Indicates P and Q are where $\dfrac{dx}{dy}=0$ or where their $2y^2-4y-6=0$ (which may be the denominator of $\dfrac{dy}{dx}$ if they found this instead).

M1 Solves their 3TQ from an attempt at $\dfrac{dx}{dy}=0$ (or denominator of their $\dfrac{dy}{dx}$=0), usual rules.

dM1 Substitutes both their solutions to $2y^2-4y-6=0$ into $x=\frac{2y^2+6}{3y-3}$. Condone slips if the attempt is clear. At least one should be correct if no method is shown. A1cso Achieves $x=-\frac{4}{3}$ and $x=4$ only. Must be equations not just values but isw after correct equations seen as long as no contrary work is shown (such as giving horizontal lines). Accept equivalents. Must have come from a correct derivative - though allow from an isw form if a numerical factor was lost in the numerator. Must be exact. Answers from no working score 0/4 as the question instructs use of part (a), so must see the attempt at setting $\frac{dx}{dy}=0$		
Alt (a)	$x=\frac{2y^2+6}{3y-3}\Rightarrow 3xy-3x=2y^2+6\Rightarrow 3x+3y\frac{dx}{dy}-3\frac{dx}{dy}=4y$ $\frac{dx}{dy}=\frac{4y-3x}{3(y-1)}$	M1 A1 dM1, A1 **(4)**
(b) First 2 marks.	States that P and Q are where $\frac{dx}{dy}=0$ or where $4y-3x=0$ $\Rightarrow \frac{4}{3}y=\frac{2y^2+6}{3y-3}\Rightarrow 4y^2-4y=2y^2+6\Rightarrow$ as main scheme	B1 M1
Alt II (a)	$x=\frac{2y^2+6}{3y-3}=\frac{2y}{3}+\frac{2}{3}+\frac{8}{3(y-1)}\Rightarrow\frac{dx}{dy}=\frac{2}{3}-\frac{8}{3(y-1)^2}$ $\frac{dx}{dy}=\frac{2(y-1)^2-8}{3(y-1)^2}=\frac{2y^2-4y-6}{3(y-1)^2}$ oe	M1 A1 dM1, A1 **(4)**
Notes		
(a) M1 Attempts long division or other method to achieve $Ay+B+\frac{C}{3y-3}$ oe and differentiates. A1 Correct differentiation. dM1 Attempts to get a single fraction and simplifies numerator to 3TQ or uses difference of squares to factorise. A1 Correct answer.		

Total for paper is 30 marks