Year 12
October Post Half Term Review

This exam has 5 questions, for a total of 45 marks.

Topic List

Pure

- Indices and Surds
- Quadratics
- Polynomial graph sketching
- Linear transformation of graphs
- Circles and Lines (Coordinate Geometry)

Applied

- Kinematics (vertical motion)
- Representation of Data

Question:	1	2	3	4	5	Total
Marks	4	8	10	13	10	45
Score:						

- Advised to print in "A3-booklets", this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

Andrew Chan
7th November 2023

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.
[This line in exams usually means you can use your graphical calculator to do the whole question, use it to reject unwanted solutions/giving you a final answer before you start.]

Solve

$$2x - 3\sqrt{x} - 14 = 0$$

(4)

Question 1 continued

(Total for Question 1 is 4 marks)

2. Mr Chan kicks a football vertically upwards with a speed of u ms^{-1} from a point which is 0.9 m above the horizontal ground. The ball reaches a greatest height of 10.9 m **above the ground**.

You may assume that there is no air resistance acting on the ball during its motion.

(a) Show that $u = 14$.

(3)

(b) Determine the time between the ball being kicked and the ball hitting the ground.
*[Note Edexcel requirements here: whenever a numerical value of g is used, you **must** leave your final answers in 2 or 3 significant figures.]*

(4)

(c) In additional to the assumption given in the question, write down one further assumption that you have made in your solution.

(1)

Question 2 continued

(Total for Question 2 is 8 marks)

3. Given that

$$f(x) = (2x + 5)(x^2 - 5x + 4)$$

(a) Sketch the graph of $y = f(x)$

(2)

Given also that

$$g(x) = 2x^3 - 5x^2 - 17x + 48$$

(b) Use factor theorem to show that $x = -3$ is a root of $g(x) = 0$

(2)

(c) Hence show that $x = -3$ is the only real root of $g(x) = 0$

(4)

(d) Show that $y = g(x)$ is a translation of $y = f(x)$ by $\begin{pmatrix} 0 \\ k \end{pmatrix}$, stating the value of k.

(2)

Question 3 continued

(Total for Question 3 is 10 marks)

4.

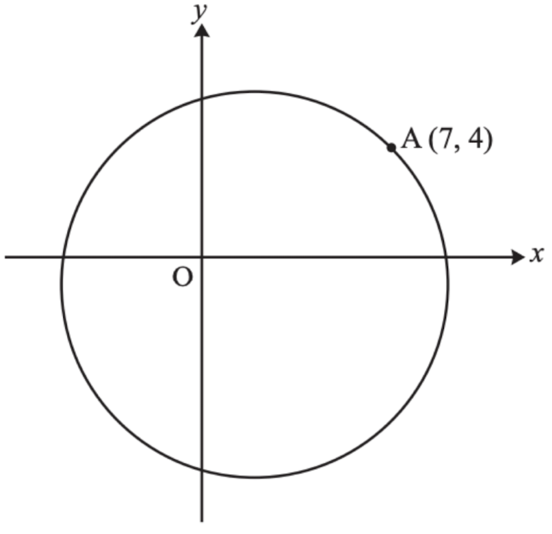

Figure 1

Figure 1 shows a sketch of the circle C with equation

$$(x-2)^2+(y+1)^2=50$$

The point $A(7,4)$ lies on the circle.

(a) State the radius and the coordinates of the centre of C.

(2)

(b) Show that, when the line $y=2x+k$ is a tangent to the circle, k satisfies the equation

$$k^2+10k-225=0$$

(5)

The line l has equation

$$y=2x-10$$

Given that l passes through A.

(c) Find the coordinates of the point B where l intersects the circle again.

(4)

(d) Hence show that the perpendicular distance from the centre of C to the line l is $\sqrt{5}$.

(2)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 13 marks)

5. Mr Dordoy recorded information about the time, t minutes, correct to the nearest minute, taken by 50 students to complete a race.

Time (minutes)	$t \leq 27$	$28 \leq t \leq 30$	$31 \leq t \leq 35$	$36 \leq t \leq 45$	$46 \leq t \leq 60$
Number of students	0	4	28	14	4

(a) Calculate estimates of the mean and standard deviation of the data.
[Exam technique: here you have to show at least one line of working out.]

(3)

(b) In a histogram illustrating the data, the height of the block for the $31 \leq t \leq 35$ class is 5.6 cm. Find the height of the block for the $28 \leq t \leq 30$ class.
[There is no need to sketch the histogram.]

(2)

The data in the table are used to estimate the median time.

(c) State, with a reason whether the estimated median time is more than 33 minutes, less than 33 minutes or equal to 33 minutes.

(2)

(d) Hence use linear interpolation to estimate the median.

(1)

It was found that the winner's time had been incorrectly recorded and that it was actually less than 27 minutes 30 seconds. State whether each of the following will increase, decrease or remain the same:

(e) (i) the mean,

(ii) the standard deviation,

(iii) the median,

(iv) the interquartile range,

(2)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

Question 5 continued

(Total for Question 5 is 10 marks)

Total for paper is 45 marks

Year 12 Post-Half Term Mini Test (Review) (Marking Scheme)

November 2, 2023

Question 1

Question Number	Scheme	Marks
(a)	$2x-3\sqrt{x}-5=9 \Rightarrow 2x-3\sqrt{x}-14=0$ and treats as quadratic equation	M1
	$\Rightarrow (2\sqrt{x}-7)(\sqrt{x}+2)=0 \Rightarrow (\sqrt{x}=)\ \frac{7}{2},(-2)$	A1
	$\Rightarrow x=\left(\frac{7}{2}\right)^2=\frac{49}{4}$	dM1 A1
		(4)

(a) Note calculators in this question are not acceptable so answers on their own or roots without working score 0 marks.

M1 **Way One**: Sets $2x-3\sqrt{x}-5=9 \Rightarrow 2x-3\sqrt{x}-14=0$ and attempts to solve a 3TQ quadratic in $\sqrt{x}$ or sets eg $u=\sqrt{x}$ and attempts to solve a 3TQ in u $(2u^2-3u-14=0)$. See general guidance for solving a quadratic. Condone use of other variables including $x=\sqrt{x}$.

Condone slips in their rearrangement to achieving a 3TQ quadratic and proceeding to find a value but the method must be sound. To score they must either

- show the factorised form of their quadratic eg $(2u-7)(u+2)$
- show embedded values in the quadratic formula
- show their method completing the square

Way Two: Sets $2x-14=3\sqrt{x}$ oe and attempts to square leading to a 3TQ quadratic on one side of an equation. Condone slips when multiplying out eg $(2x-14)^2$ and rearranging their equation but their method must be sound.
Note: $2x-14=3\sqrt{x} \Rightarrow 4x^2-196=9x$ would be M0.

A1 **Way One**: $(\sqrt{x}=)\ \frac{7}{2}$ or eg $(u=)\ \frac{7}{2}$ Ignore any reference to the -2. Condone $x=\frac{7}{2}$

Note the roots do not imply M1A1

Way Two: $4x^2-65x+196=0$ oe (the terms should be collected on one side of the equation, but condone lack of $=0$)

dM1 **Way One**: Attempts to find one value for x. Condone 4 or squaring -2

Way Two: Attempts to find one value for x by solving their quadratic (see general guidance for solving a quadratic). To score they must either

- show the factorised form of their quadratic eg $(4x-49)(x-4)$
- show embedded values in the quadratic formula
- show their method completing the square

A1 $x=\frac{49}{4}$ or 12.25 or $12\frac{1}{4}$ only. If 4 is found it must be rejected

Question 2

(a)	$v^2 = u^2 + 2as,\ v = 0, a = \pm g, s = \pm 10$ $0 = u^2 + 2(\pm 9\cdot 8)(\mp 10)$ $u = (\mp)14 \qquad (\mathrm{ms}^{-1})$	M1 A1 A1 **[3]**	 g opposing s Convincing (dependent on M1)
(b)	$s = ut + \frac{1}{2}at^2$, $s = \pm 0\cdot 9$, $u = \pm 14, a = \pm g$ $\pm 0\cdot 9 = (\mp 14)t + \frac{1}{2}(\pm 9\cdot 8)t^2$ $4\cdot 9t^2 - 14t - 0\cdot 9 = 0$ (oe) Solving their quadratic $\left(t = \frac{14\pm\sqrt{196-4(4\cdot 9)(-0\cdot 9)}}{9\cdot 8}\right)$ $t = 2\cdot 9$ s / 2.92s only must be 2/3 sf// 	M1 A1 m1 A1 **[4]**	correct no. terms, condone g **and** s opposing u Calculator gives $t = \frac{10+\sqrt{109}}{7}$ cao
(c)	Any sensible assumption. e.g. Ball modelled as a particle. Acceleration due to gravity is constant. rotational effects are included.	B1 **[1]**	

if two stages : M1 for both 4 and 6 with correct values, condone only sign errors.

A1 ⇒ substituted

m1 ⇒ solving both 3TQ

A1 ⇒ 2.9/2.92 seen

Question 3

11		a)	graph of cubic correct way up	B1	**B0** if stops at x-axis	must not have any ruled sections; no curving back; condone slight 'flicking out' at ends but not approaching another turning point; allow max on y-axis or in 1st or 2nd quadrants; condone some 'doubling' or 'feathering' (deleted work still may show in scans)
			crossing x-axis at $-5/2$, 1 and 4		on graph or nearby; may be in coordinate form; **M1** for $x^2 - 5x + 4 = (x - 4)(x - 1)$ or for roots 4 and 1 found mark intent for intersections with both axes	allow if no graph, but marked on x-axis condone intercepts for x and / or y given as reversed coordinates
			crossing y-axis at 20	B1	or $x = 0$, $y = 20$ seen if consistent with graph drawn	allow if no graph, but eg B0 for graph with intn on y-axis nowhere near their indicated 20

Question			Answer	Marks	Guidance	
11		b)	$g(-3) = 2 \times (-3)^3 - 5 \times 9 - 17 \times -3 + 48$ $= -54 - 45 + 51 + 48$ $= 0$	B1	condone $(-3)^3$ instead of -27 etc, but next step of working must be shown correctly or **B1** for correct division of $g(x)$ by $(x + 3)$ with remainder 0 and the conclusion immediately following this (or explicitly connected to it) that $g(-3) = 0$ or that -3 is a root of $g(x)$ oe	B0 for just $x + 3$ is a factor or for $x + 3$ is a root
			$(x + 3)$ used or stated as factor	M1		
		c)	correctly finding other factor as $2x^2 - 11x + 16$	B2	accept $b = -11$ found **M1** for correct division of cubic by $(x + 3)$ as far as obtaining $2x^2 - 11x$ (may be in grid) or for two correct terms of $2x^2 - 11x + 16$ obtained by inspection	
			$121 - 128$ isw or -7	A1	for correct substitution into $b^2 - 4ac$ and obtaining negative (may be seen in formula); no ft from wrong factor	must be correctly simplified to at least the $121 - 128$ stage
			conclusion no real roots from quadratic factor/equation, so -3 is only real root of $g(x)$	A1	dep on previous A1; must refer back to original request, just 'no real roots' is not sufft they need to mention -3 or say 'so just one real root' or 'no more real roots'	
11		d)	[$f(x) =$] $2x^3 - 5x^2 - 17x + 20$ with correct working	B2	**B1** if no working or **M1** for correct working condone inclusion of $+ k$ even if labelled as $f(x)$ instead of $g(x)$	if no working in (iii), check whether the relevant work has already been done in (i). If it has, tick it on the copy in the image zone and allow the mark, but only if $f(x)$ appears/is used in (iii).

Question			Answer	Marks	Guidance	
			$k = 28$ or $g(x)$ is translation of $f(x)$ by $\begin{pmatrix} 0 \\ 28 \end{pmatrix}$	B1 [3]	**B0** for just $g(x) = f(x) + 28$	B1 for $k = 28$ even if stated after no /wrong $f(x)$ obtained

Question 4

12	(i) a)		radius $\sqrt{50}$ isw wrong conversion to $5\sqrt{2}$ centre $(2, -1)$	B1 B1 [2]	**B1** for $5\sqrt{2}$	
12	(ii) c)					NB examiners must use annotation in this part; a tick where each mark is earned is sufficient
			$(x-2)^2 + (2x-9)^2 = 50$	M1	for subst from line into circle eqn; condone one error	eg condone omission of '=50' or having -11 instead of -9
			$5x^2 - 40x + 35$ [= 0]	M1	for simplifying to solvable form; condone one further error	
			$x = 7$ or 1	A1	condone omission of 7 and just using 1	
			B = $(1, -8)$	B1		
	d)		midpt of AB = $\left(\frac{7 + their 1}{2}, \frac{4 + their -8}{2}\right)$ or $(4, -2)$	M1	or length of AB found ft ($\sqrt{180}$ if correct) <u>and</u> Pythagoras used with ½ AB and r	Must use the coordinates of B since 'hence': so M0 for eqn of line through centre perp to AB and intersection with AB used to find mid point of AB or M0 for equation of AB and formula for dist of pt from line used
			distance = $\sqrt{5}$ correctly obtained (answer given)	A1 [6]		

Question			Answer	Marks	Guidance	
12	(iii) b)		$(x-2)^2 + (2x + k + 1)^2 = 50$	M1	condone one error, eg omission of +1, but k must be included	
			$5x^2 + 4kx + k^2 + 2k - 45$ [= 0]	M1	condone one error; accept constant term $(k+1)^2 - 46$; must be rearranged to '=0' stage unless they go on to complete the square **M0** if wrong eqn used – no ft from original error, only condone one error from working with correct eqn	eg allow M1 for $5x^2 + 4kx + k^2 - 45$ [= 0]
			$b^2 - 4ac = 0$ oe soi	M1	may be earned near end allow for this condition quoted, even if then applied to wrong equation. It is sometimes earned at beginning	0 for just 'discriminant = 0' unless implied by later work
			$(4k)^2 - 4 \times 5 \times (k^2 + 2k - 45)$	M1	for correct substitution ft into $b^2 - 4ac$, dep on first M1 earned; brackets / signs must be correct	can be earned in formula (ignore rest of formula)
			correct simplification to given answer $k^2 + 10k - 225 = 0$.	A1 [5]	NB mark working not answer	

Question 5

14 (i)	Method is either: Just $4 \div 3$ or $\frac{4}{3}$ or: Use of ratio of correct frequencies AND ratio of widths (correct or 4 and 2)			
b)	$5.6 \times \frac{4}{28} \times \frac{5}{3}$ or $0.8 \times \frac{5}{3}$ or $(5.6 \div \frac{28}{5}) \times \frac{4}{3}$ or $\frac{4}{3}$ or $4 \div 3$ oe $= 1\frac{1}{3}$ or $\frac{4}{3}$ or 1.33 (3 sf) oe	M2 A1 3	M1 for $5.6 \times \frac{4}{28} \times \frac{4}{2}$ or $0.8 \times \frac{4}{2}$ or $(5.6 \div \frac{28}{4}) \times \frac{4}{2}$ or 0.8×2 oe (= 1.6) No wking, ans 1.3: M2A0 Ans 1.6: Check wking but probably M1M0A0	Correct calc'n using 5.6, 28, 4, 5, 3 oe: M2 Correct calc'n using 5.6, 28, 4, 4, 2 oe: M1 ie fully correct method: M2 or: incorrect class widths, otherwise correct method: M1 $\frac{4}{3}$ correctly obtained (or no wking) then further incorrect: M1M0A0 Use of ratio of widths OR freqs but not both: M0 eg $5.6 \times \frac{4}{28}$ (= 0.8) or $5.6 \times \frac{3}{5}$ (= 3.36): M0 $\frac{4}{2} = 2$: M0M0A0
ii c)	25 or 26 or 25.5 Med is 21st (or 22nd or 21.5th) in 31-35 class or "25 – 4" Can be implied by calc'n Med > 33 or "more than" d) 34.25 //	B1 B1 B1 3	or 25 & 26 or med in last ≈ 7 in class or 33 ≈ 14th in class or 33 ≈ 18th in whole set Can be implied by diagram indep	May be implied, eg by 21 or 22 or 21.5 Calc'ns need not be correct but need to contain relevant figures for gaining B1B1 The "≈" sign means ± 2 <u>Alternative Method</u>: 33 ≈ 18th value B1 More values above 33 than below oe B1 Med > 33 B1 Ignore comment on skew NB Use EITHER the main method OR the <u>Alternative Method</u> (above), not a mixture of the two. Choose the method that gives most marks.
iii a)	≥ 3 mid-pts attempted $\Sigma fx \div 50$ attempted (= $\frac{1819}{50}$) = 36.38 or 36.4 (3 sf) Σfx^2 attempted (= 68055.5) $\sqrt{\frac{68055.5}{50} - (\frac{1819}{50})^2}$ or $\sqrt{1361.11 - 36.38^2}$ (= $\sqrt{37.6056}$) = 6.13 (3 sfs) Alt for variance: $\Sigma f(x - \bar{x})^2$ (= 1880.28) M1 $\sqrt{\frac{1880.28}{50}}$ M1 = 6.13 (3 sf) A1	M1 A1 A1 6	seen or implied ≥ 3 terms. or 36 with correct working ≥ 3 terms. completely correct method except midpts & ft their mean, dep not √(neg)	Not nec'y correct values (29, 33, 40.5, 53) Allow on boundaries. Not class widths Allow on boundaries. Not class widths (3364, 30492, 22963.5, 11236) Allow class widths for this mark only NB mark is not just for "– mean²", unlike q5(iii) $\Sigma(fx)^2$: M0M0A0 If no wking for Σfx^2, check using their x and f If no wking or unclear wking: full mks for each correct ans for incorrect ans: $35.8 \le \mu \le 36.9$ M0M1A0 $6.0 \le sd \le 6.25$ M1M0A0
iv	(a) Decrease (b) Increase (c) Same (d) Same	B1 B1 4	Ignore other, eg "slightly" or "probably"	Ignore any comments or reasons, even if incorrect
Total		**16**		

B1 (two correct)
B1 (four correct)

(Q4, June 2011)

5. Mr Dordoy recorded information about the time, t minutes, correct to the nearest minute, taken by 50 students to complete a race.

27.5 3 30.5 5 35.5

Time (minutes)	$t \leq 27$	$28 \leq t \leq 30$	$31 \leq t \leq 35$	$36 \leq t \leq 45$	$46 \leq t \leq 60$
Number of students	0	4	28	14	4

F.D. 4/3 5.6

(a) Calculate estimates of the mean and standard deviation of the data.

[Exam technique: here you have to show at least one line of working out.]

(3)

Deg Norm2 d/c a+bi

1-Variable

$\bar{x}$ =36.38

Σx =1819

Σx^2 =68055.5

σx =6.13234049

sx =6.19459935

n =50

$$\text{mean} = \frac{1819}{50} = 36.38$$

$$S.D. = \sqrt{\frac{68055.5}{50} - 36.38^2}$$

$$= 6.1323\ldots$$

(b) In a histogram illustrating the data, the height of the block for the $31 \leq t \leq 35$ class is 5.6 cm. Find the height of the block for the $28 \leq t \leq 30$ class.

[There is no need to sketch the histogram.]

1.33.... cm

(2)

The data in the table are used to estimate the median time.

(c) State, with a reason whether the estimated median time is more than 33 minutes, less than 33 minutes or equal to 33 minutes.

25th term ⇒ 21 terms into the class (33 is the mid-pt)

⇒ 21 is 75% of the freq.

(d) Hence use linear interpolation to estimate the median.

⇒ 34.25

(1)

30.5 33 35.5

50% 50%

It was found that the winner's time had been incorrectly recorded and that it was actually less than 27 minutes 30 seconds. State whether each of the following will increase, decrease or remain the same:

(e) (i) the mean, ↓

(ii) the standard deviation, ↑

(iii) the median, —

(iv) the interquartile range, —

(2)

Q:

1, 2, 3, 4, 5

$\bar{x} = 3$ $S.D. = \sqrt{2}$

add two numbers such that

a) S.D. ↑ 0, 6

b) S.D. ↓ 3, 3

c) S.D. — unchanged. $3+\sqrt{2}, 3-\sqrt{2}$

but mean is the same .....