Mr Chan’s Roots of Polynomial Questions by Topic Pack

OCR June 2005

8 (a) The quadratic equation $x^2 - 2x + 4 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Show that $\alpha^2 + \beta^2 = -4$. [2]

(iii) Hence find a quadratic equation which has roots α^2 and β^2. [3]

(b) The cubic equation $x^3 - 12x^2 + ax - 48 = 0$ has roots p, $2p$ and $3p$.

(i) Find the value of p. [2]

(ii) Hence find the value of a. [2]

Let me go home

OCR Jan 2006

10 The roots of the equation

$$x^3 - 9x^2 + 27x - 29 = 0$$

are denoted by α, β and γ, where α is real and β and γ are complex.

(i) Write down the value of $\alpha + \beta + \gamma$. [1]

(ii) It is given that $\beta = p + iq$, where $q > 0$. Find the value of p, in terms of α. [4]

(iii) Write down the value of $\alpha\beta\gamma$. [1]

(iv) Find the value of q, in terms of α only. [5]

Let me go home

OCR June 2006

10 The cubic equation $x^3 - 2x^2 + 3x + 4 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

The cubic equation $x^3 + px^2 + 10x + q = 0$, where p and q are constants, has roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$.

(ii) Find the value of p. [3]

(iii) Find the value of q. [5]

Let me go home

OCR Jan 2007

7 The quadratic equation $x^2 + 5x + 10 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Show that $\alpha^2 + \beta^2 = 5$. [2]

(iii) Hence find a quadratic equation which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [4]

Let me go home

OCR June 2007

6 The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots α, β and γ.

(i) (a) Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$. [2]

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

(ii) (a) Use the substitution $x = \frac{1}{u}$ to find a cubic equation in u with integer coefficients. [2]

(b) Use your answer to part (ii)(a) to find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. [2]

Let me go home

OCR Jan 2008

3 The cubic equation $2x^3 - 3x^2 + 24x + 7 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \frac{1}{u}$ to find a cubic equation in u with integer coefficients. [2]

(ii) Hence, or otherwise, find the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$. [2]

Let me go home

OCR Jan 2008

9 (i) Show that $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$. [2]

(ii) The quadratic equation $x^2 - 5x + 7 = 0$ has roots α and β. Find a quadratic equation with roots α^3 and β^3. [6]

Let me go home

OCR June 2008 (Complex Numbers but also Roots of Polynomial)

6 The cubic equation $x^3 + ax^2 + bx + c = 0$, where a, b and c are real, has roots $(3 + i)$ and 2.

(i) Write down the other root of the equation. [1]

(ii) Find the values of a, b and c. [6]

Let me go home

OCR Jan 2009

8 (i) Show that $(\alpha - \beta)^2 \equiv (\alpha + \beta)^2 - 4\alpha\beta$. [2]

The quadratic equation $x^2 - 6kx + k^2 = 0$, where k is a positive constant, has roots α and β, with $\alpha > \beta$.

(ii) Show that $\alpha - \beta = 4\sqrt{2}k$. [4]

(iii) Hence find a quadratic equation with roots $\alpha + 1$ and $\beta - 1$. [4]

Let me go home

OCR June 2009

4 The roots of the quadratic equation $x^2 + x - 8 = 0$ are p and q. Find the value of $p + q + \frac{1}{p} + \frac{1}{q}$. [4]

5 The cubic equation $x^3 + 5x^2 + 7 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \sqrt{u}$ to find a cubic equation in u with integer coefficients. [3]

(ii) Hence find the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. [2]

Let me go home

OCR Jan 2010

2 The cubic equation $2x^3 + 3x - 3 = 0$ has roots α, β and γ.

(i) Use the substitution $x = u - 1$ to find a cubic equation in u with integer coefficients. [3]

(ii) Hence find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. [2]

Let me go home

OCR Jan 2010

6 One root of the cubic equation $x^3 + px^2 + 6x + q = 0$, where p and q are real, is the complex number $5 - i$.

(i) Find the real root of the cubic equation. [3]

(ii) Find the values of p and q. [4]

Let me go home

OCR June 2010

7 The quadratic equation $x^2 + 2kx + k = 0$, where k is a non-zero constant, has roots α and β. Find a quadratic equation with roots $\frac{\alpha + \beta}{\alpha}$ and $\frac{\alpha + \beta}{\beta}$. [7]

Let me go home

OCR Jan 2011

8 The quadratic equation $2x^2 - x + 3 = 0$ has roots α and β, and the quadratic equation $x^2 - px + q = 0$ has roots $\alpha + \frac{1}{\alpha}$ and $\beta + \frac{1}{\beta}$.

(i) Show that $p = \frac{5}{6}$. [4]

(ii) Find the value of q. [5]

Let me go home

OCR June 2011

10 The cubic equation $x^3 + 3x^2 + 2 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \frac{1}{\sqrt{u}}$ to show that $4u^3 + 12u^2 + 9u - 1 = 0$. [5]

(ii) Hence find the values of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ and $\frac{1}{\alpha^2\beta^2} + \frac{1}{\beta^2\gamma^2} + \frac{1}{\gamma^2\alpha^2}$. [5]

Let me go home

Ocr June 2011

9 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is $16 - 30i$.

(i) Write down the other root of the quadratic equation. [1]

(ii) Find the values of a and b. [4]

(iii) Use an algebraic method to solve the quartic equation $y^4 + ay^2 + b = 0$. [7]

Let me go home

OCR JAN 2012

10 The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

The cubic equation $x^3 + ax^2 + bx + c = 0$ has roots α^2, β^2 and γ^2.

(ii) Show that $c = -\frac{4}{9}$ and find the values of a and b. [9]

Let me go home

OCR Jan 2013

4 The quadratic equation $x^2 + x + k = 0$ has roots α and β.

(i) Use the substitution $x = 2u + 1$ to obtain a quadratic equation in u. [2]

(ii) Hence, or otherwise, find the value of $\left(\frac{\alpha - 1}{2}\right)\left(\frac{\beta - 1}{2}\right)$ in terms of k. [2]

9 (i) Show that $(\alpha\beta + \beta\gamma + \gamma\alpha)^2 = \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$. [3]

(ii) It is given that α, β and γ are the roots of the cubic equation $x^3 + px^2 - 4x + 3 = 0$, where p is a constant. Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ in terms of p. [5]

Let me go home

Ocr June 2012

3 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is the complex number $4 - 3i$. Find the values of a and b. [4]

6 The quadratic equation $2x^2 + x + 5 = 0$ has roots α and β.

(i) Use the substitution $x = \frac{1}{u + 1}$ to obtain a quadratic equation in u with integer coefficients. [3]

(ii) Hence, or otherwise, find the value of $\left(\frac{1}{\alpha} - 1\right)\left(\frac{1}{\beta} - 1\right)$. [3]

Let me go home

Ocr June 2013

8 The cubic equation $kx^3 + 6x^2 + x - 3 = 0$, where k is a non-zero constant, has roots α, β and γ.

Find the value of $(\alpha + 1)(\beta + 1) + (\beta + 1)(\gamma + 1) + (\gamma + 1)(\alpha + 1)$ in terms of k. [6]

Let me go home

OCR June 2005

8 **(a)** The quadratic equation $x^2 - 2x + 4 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Show that $\alpha^2 + \beta^2 = -4$. [2]

(iii) Hence find a quadratic equation which has roots α^2 and β^2. [3]

(b) The cubic equation $x^3 - 12x^2 + ax - 48 = 0$ has roots p, $2p$ and $3p$.

(i) Find the value of p. [2]

(ii) Hence find the value of a. [2]

Let me go home

8.	(a) (i) $\alpha + \beta = 2 \quad \alpha\beta = 4$	B1B1	2	Values stated
	(ii) *EITHER* $\alpha^2 + \beta^2 = -4$	M1 A1	2	Use $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ Obtain given answer correctly
	OR	M1 A1		Find numeric values of roots, square and add Obtain given answer correctly
	(iii) $x^2 + 4x + 16 = 0$	B1		State or use $\alpha^2\beta^2 = 16$

		M1		Or use substitution $u = x^2$ Write down a quadratic equation of correct form or rearrange and square
		A1	3	Obtain $x^2 + 4x + 16 = 0$
	(b) (i) $p = 2$	M1		Use sum or product of roots to obtain $6p = 12$ Or $6p^3 = 48$
	(ii) $a = 44$	A1	2	Obtain $p = 2$
		M1		Attempt to find $\Sigma\alpha\beta$ numerically or in terms of p or substitute their 2, 4 or 6 in equation
		A1ft	2	Obtain $11p^2$
			11	

Let me go home

OCR Jan 2006

10 The roots of the equation

$$x^3 - 9x^2 + 27x - 29 = 0$$

are denoted by α, β and γ, where α is real and β and γ are complex.

(i) Write down the value of $\alpha + \beta + \gamma$. [1]

(ii) It is given that $\beta = p + \mathrm{i}q$, where $q > 0$. Find the value of p, in terms of α. [4]

(iii) Write down the value of $\alpha\beta\gamma$. [1]

(iv) Find the value of q, in terms of α only. [5]

Let me go home

10.	(i) $\alpha + \beta + \gamma = 9$	B1	1	
	(ii) $p = \dfrac{9 - \alpha}{2}$	B1 M1 A1 A1	4	State or use other root is *p* - i*q* Substitute into (i) Obtain $2p + \alpha = 9$ Obtain correct answer a.e.f.
	(iii) $\alpha\beta\gamma = 29$	B1	1	
	(iv) $\alpha(p^2 + q^2) = 29$ $q = \sqrt{\dfrac{29}{\alpha} - \dfrac{(9-\alpha)^2}{4}}$	M1 A1ft M1 M1 A1	5 **11**	Substitute into (iii) Obtain unsimplified expression with no i's Rearrange to obtain q or q^2 Substitute their expression for *p* a.e.f. Obtain correct answer a.e.f.
	(iv) Alternative method $2p\alpha + p^2 + q^2 = 27$ $q = \sqrt{27 - \dfrac{(9-\alpha)^2}{4} - \alpha(9 - \alpha)}$	M1 A1 M1 M1 A1		Substitute into $\alpha\beta + \beta\gamma + \gamma\alpha = 27$ Obtain unsimplified expression with no i's Rearrange to obtain q or q^2 Substitute their expression for *p* a.e.f. Obtain correct answer a.e.f.

Let me go home

OCR June 2006

10 The cubic equation $x^3 - 2x^2 + 3x + 4 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

The cubic equation $x^3 + px^2 + 10x + q = 0$, where p and q are constants, has roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$.

(ii) Find the value of p. [3]

(iii) Find the value of q. [5]

Let me go home

10	(i) $\alpha + \beta + \gamma = 2$ $\alpha\beta\gamma = -4$	B1 B1		Write down correct values
.	$\alpha\beta + \beta\gamma + \gamma\alpha = 3$	B1	3	
	(ii)	M1		Sum new roots
	$\alpha + 1 + \beta + 1 + \gamma + 1 = 5$	A1ft		Obtain numeric value using their (i)
	$p = -5$	A1ft	3	p is negative of their answer
	(iii)	M1*		Expand three brackets
		A1		$\alpha\beta\gamma + \alpha\beta + \beta\gamma + \gamma\alpha + \alpha + \beta + \gamma + 1$
		DM1		Use their (i) results
		A1ft		Obtain 2
	$q = -2$	A1ft	5	q is negative of their answer
			11	**Alternative for (ii) & (iii)**
		M2		Substitute $x = u - 1$ in given equation
		A1		Obtain correct unsimplified equation for u
		M1		Expand
		A2		Obtain $u^3 - 5u^2 + 10u - 2 = 0$
		A1 A1		State correct values of p and q.

Let me go home

OCR Jan 2007

7 The quadratic equation $x^2 + 5x + 10 = 0$ has roots α and β.

(i) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(ii) Show that $\alpha^2 + \beta^2 = 5$. [2]

(iii) Hence find a quadratic equation which has roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [4]

Let me go home

7.	(i) $\alpha + \beta = -5 \quad \alpha\beta = 10$	B1 B1	2	State correct values
	(ii) $\alpha^2 + \beta^2 = 5$	M1		Use $(\alpha + \beta)^2 - 2\alpha\beta$
		A1	2	Obtain given answer correctly, using value of -5
	(iii)	B1		Product of roots = 1
		M1		Attempt to find sum of roots
		A1		Obtain $\frac{5}{10}$ or equivalent
	$x^2 - \frac{1}{2}x + 1 = 0$	B1ft	4	Write down required quadratic
			8	equation, or any multiple.

Let me go home

OCR June 2007

6 The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots α, β and γ.

(i) **(a)** Write down the values of $\alpha + \beta + \gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$. [2]

(b) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

(ii) **(a)** Use the substitution $x = \frac{1}{u}$ to find a cubic equation in u with integer coefficients. [2]

(b) Use your answer to part **(ii) (a)** to find the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$. [2]

Let me go home

6	(i) (a) $\alpha + \beta + \gamma = 3, \alpha\beta + \beta\gamma + \gamma\alpha = 2$	B1 B1	2	State correct values
	(b) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)$	M1		State or imply the result and use their values
	$= 9 - 4 = 5$	A1 ft	2	Obtain correct answer
	(ii) (a) $\frac{3}{u^3} - \frac{9}{u^2} + \frac{6}{u} + 2 = 0$	M1		Use given substitution to obtain an equation
	$2u^3 + 6u^2 - 9u + 3 = 0$	A1	2	Obtain correct answer
	(b) $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -3$	M1		Required expression is related to new cubic stated or implied
		A1ft	2	-(their "b" / their "a")
			8	

Let me go home

OCR Jan 2008

3 The cubic equation $2x^3 - 3x^2 + 24x + 7 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \frac{1}{u}$ to find a cubic equation in u with integer coefficients. [2]

(ii) Hence, or otherwise, find the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$. [2]

3	(i) $7u^3 + 24u^2 - 3u + 2 = 0$	M1 A1	 2	Use given substitution Obtain correct equation a.e.f.
	(ii) *EITHER* correct value is $-\frac{3}{7}$	M1 A1ft	 2	Required expression related to new cubic Their c / their a
	OR correct value is $-\frac{3}{7}$	M1 A1	 **4**	Use $\dfrac{\alpha+\beta+\gamma}{\alpha\beta\gamma}$ or equivalent Obtain correct answer

Let me go home

OCR Jan 2008

9 **(i)** Show that $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$. [2]

(ii) The quadratic equation $x^2 - 5x + 7 = 0$ has roots α and β. Find a quadratic equation with roots α^3 and β^3. [6]

9	(i) $\alpha^3 + 3\alpha^2\beta + 3\alpha\beta^2 + \beta^3$	M1 A1	2	Correct binomial expansion seen Obtain given answer with no errors seen
	(ii) *Either* $\alpha + \beta = 5, \alpha\beta = 7$	B1 B1		State or use correct values
	$\alpha^3 + \beta^3 = 20$	M1 A1		Find numeric value for $\alpha^3 + \beta^3$ Obtain correct answer
		M1	6	Use new sum and product correctly in quadratic expression
	$x^2 - 20x + 343 = 0$	A1ft	**8**	Obtain correct equation
	Or	M1 A1		Substitute $x = u^{\frac{1}{3}}$ Obtain correct answer
	$u^{\frac{2}{3}} - 5u^{\frac{1}{3}} + 7 = 0$	M2		Complete method for removing fractional powers
	$u^3 - 20u + 343 = 0$	A2		Obtain correct answer

Let me go home

OCR June 2008 (Complex Numbers but also Roots of Polynomial)

6 The cubic equation $x^3 + ax^2 + bx + c = 0$, where a, b and c are real, has roots $(3 + \mathrm{i})$ and 2.

(i) Write down the other root of the equation. [1]

(ii) Find the values of a, b and c. [6]

Let me go home

6	(i)	3 – i	B1	Conjugate stated
			1	
	(ii)	*EITHER*	M1	Use sum of roots
			A1	Obtain correct answer
			M1	Use sum of pairs of roots
			A1	Obtain correct answer
			M1	Use product of roots
		$a = -8,\ b = 22,\ c = -20$	A1	Obtain correct answers
			6	
		OR	M1	Attempt to find a quadratic factor
			A1	Obtain correct factor
			M1	Expand linear and quadratic factors
		$a = -8, b = 22, c = -20$	A1A1A1	Obtain correct answers
		OR		
			M1	Substitute 1 imaginary & the real root into eqn
			M1	Equate real and imaginary parts
			M1	Attempt to solve 3 eqns.
		$a = -8, b = 22, c = -20$	A1A1A1	Obtain correct answers

OCR June 2008

8 The quadratic equation $x^2 + kx + 2k = 0$, where k is a non-zero constant, has roots α and β. Find a quadratic equation with roots $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$. [7]

Let me go home

8	$\alpha+\beta=-k$	**B1**	State or use correct value
	$\alpha\beta=2k$	**B1**	State or use correct value
		M1	Attempt to express sum of new roots in terms of $\alpha+\beta$, $\alpha\beta$
	$\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$	**A1**	Obtain correct expression
	$\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}=\dfrac{1}{2}(k-4)$	**A1**	Obtain correct answer a.e.f.
	$\alpha'\beta'=1$	**B1**	Correct product of new roots seen
	$x^2-\dfrac{1}{2}(k-4)x+1=0$	**B1ft**	Obtain correct answer, must be an eqn.
		7	
			Alternative for last 5 marks
		M1	Obtain expression for $u=\dfrac{\alpha}{\beta}$ in terms of k and α or k and β
		A1	Obtain a correct expression
		A1	rearrange to get α in terms of u
		M1	Substitute into given equation
		A1	Obtain correct answer

Let me go home

OCR Jan 2009

8 **(i)** Show that $(\alpha - \beta)^2 \equiv (\alpha + \beta)^2 - 4\alpha\beta$. **[2]**

The quadratic equation $x^2 - 6kx + k^2 = 0$, where k is a positive constant, has roots α and β, with $\alpha > \beta$.

(ii) Show that $\alpha - \beta = 4\sqrt{2}k$. **[4]**

(iii) Hence find a quadratic equation with roots $\alpha + 1$ and $\beta - 1$. **[4]**

Let me go home

8	**(i)**	M1 A1	2	Expand at least 1 of the brackets Derive given answer correctly
	(ii) $\alpha + \beta = 6k, \alpha\beta = k^2$ $\alpha - \beta = (4\sqrt{2})k$	B1 B1 M1 A1	4	State or use correct values Find value of $\alpha - \beta$ using (i) Obtain given value correctly (allow if $-6k$ used)
	(iii) $\sum \alpha' = 6k$ $\alpha' \beta' = \alpha\beta - (\alpha - \beta) - 1$ $\alpha' \beta' = k^2 - (4\sqrt{2})k - 1$ $x^2 - 6kx + k^2 - (4\sqrt{2})k - 1 = 0$	B1ft M1 A1ft B1ft	4 **10**	Sum of new roots stated or used Express new product in terms of old roots Obtain correct value for new product Write down correct quadratic **equation**

Let me go home

OCR June 2009

4 The roots of the quadratic equation $x^2 + x - 8 = 0$ are p and q. Find the value of $p + q + \frac{1}{p} + \frac{1}{q}$. **[4]**

5 The cubic equation $x^3 + 5x^2 + 7 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \sqrt{u}$ to find a cubic equation in u with integer coefficients. **[3]**

(ii) Hence find the value of $\alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2$. **[2]**

Let me go home

4.	Either $p+q=-1, pq=-8$	B1		Both values stated or used
	$\frac{p+q}{pq}$	B1		Correct expression seen
		M1		Use their values in their expression
	$-\frac{7}{8}$	A1	4	Obtain correct answer
			4	
	Or $\frac{1}{p}+\frac{1}{q}=8$	B1		Substitute $x=\frac{1}{u}$ and use new quadratic
	$p+q=1$	B1		Correct value stated
		M1		Use their values in given expression
	$-\frac{7}{8}$	A1		Obtain correct answer
	Or $\frac{-1\pm\sqrt{33}}{2}$	M1		Find roots of given quadratic equation
		A1		Correct values seen
		M1		Use their values in given expression
	$-\frac{7}{8}$	A1		Obtain correct answer
5.	(i) $u^3=\{(-)(5u+7)\}^2$	M1		Use given substitution and rearrange
		A1		Obtain correct expression, or equivalent
	$u^3-25u^2-70u-49=0$	A1	3	Obtain correct final answer
	(ii)	M1		Use coefficient of u of their cubic or identity connecting the symmetric functions and substitute values from given equation
	-70	A1 ft	2	Obtain correct answer
			5	

Let me go home

OCR Jan 2010

2 The cubic equation $2x^3 + 3x - 3 = 0$ has roots α, β and γ.

(i) Use the substitution $x = u - 1$ to find a cubic equation in u with integer coefficients. **[3]**

(ii) Hence find the value of $(\alpha + 1)(\beta + 1)(\gamma + 1)$. **[2]**

Let me go home

2	**(i)**	$u^3 - 3u^2 + 3u - 1$	B1		Correct unsimplified expansion of $(u-1)^3$
			M1		Substitute for x
		$2u^3 - 6u^2 + 9u - 8 = 0$	A1	**3**	Obtain correct **equation**
	(ii)		M1		Use $(\pm)\frac{\mathrm{d}}{\mathrm{a}}$ of new equation
		4	A1ft	**2**	Obtain correct answer from their equation
			5		

Let me go home

OCR Jan 2010

6 One root of the cubic equation $x^3 + px^2 + 6x + q = 0$, where p and q are real, is the complex number $5 - \mathrm{i}$.

(i) Find the real root of the cubic equation. **[3]**

(ii) Find the values of p and q. **[4]**

Let me go home

6 (i)		B1		State or use 5 + i as a root
		M1		Use $\sum \alpha\beta = 6$
	$x = -2$	A1	**3**	Obtain correct answer
(ii)	***Either***	M1		Use $p = -\sum \alpha$
	$p = -8$	A1ft		Obtain correct answer, from their root
		M1		Use $q = -\alpha\beta\gamma$
	$q = 52$	A1ft	**4**	Obtain correct answer, from their root
	Or	M1		Attempt to find quadratic factor
		M1		Attempt to expand quadratic and linear
		A1A1		Obtain correct answers
	Or	M1		Substitute (5 – i)into equation
		M1		Equate real and imaginary parts
		A1		Obtain correct answer for p
		A1ft		Obtain correct answer for q, ft their p
		7		

OCR June 2010

7 The quadratic equation $x^2 + 2kx + k = 0$, where k is a non-zero constant, has roots α and β. Find a quadratic equation with roots $\frac{\alpha+\beta}{\alpha}$ and $\frac{\alpha+\beta}{\beta}$. **[7]**

Let me go home

7	*Either*	B1B1	State or use correct results
	$\alpha+\beta=-2k \quad \alpha\beta=k$	M1	Attempt to find sum of new roots
		A1	Obtain $4k$
		M1	Attempt to find product of new roots
		A1	Obtain $4k$
		B1ft **7**	Correct quadratic equation a.e.f.
	$y^2-4ky+4k=0$		
	Or		
	$\alpha+\beta=-2k$	B1	State or use correct result
	$\frac{-2k}{\alpha}$	B1	State or imply form of new roots
	$y=\frac{-2k}{x}$	B1	State correct substitution
		M1	Rearrange and substitute for x
		A1	Correct unsimplified equation
		M1	Attempt to clear fractions
	$y^2-4ky+4k=0$	A1	Correct quadratic equation a.e.f.
	Or		
	$-k\pm\sqrt{k^2-k}$	B1	Find roots of original equation
	$\frac{\alpha+\beta}{\alpha}=\frac{2k}{k+\sqrt{k^2-k}},\ \frac{\alpha+\beta}{\beta}=\frac{2k}{k-\sqrt{k^2-k}}$	B1	Express both new roots in terms of k
		M1	Attempt to find sum of new roots
		A1	Obtain $4k$
		M1	Attempt to find product of new roots
	$y^2-4ky+4k=0$	A1	Obtain $4k$
		B1ft	Correct quadratic equation a.e.f.

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OCR Jan 2011

8 The quadratic equation $2x^2 - x + 3 = 0$ has roots α and β, and the quadratic equation $x^2 - px + q = 0$ has roots $\alpha + \dfrac{1}{\alpha}$ and $\beta + \dfrac{1}{\beta}$.

(i) Show that $p = \frac{5}{6}$. **[4]**

(ii) Find the value of q. **[5]**

Let me go home

8	(i) *Either*			
	$\alpha+\beta=\frac{1}{2}, \alpha\beta=\frac{3}{2}$	B1		State or use both correct results in (i) or (ii)
	$\alpha+\beta+\frac{\alpha+\beta}{\alpha\beta}$ or $\alpha+\beta+\frac{2}{3}(\alpha+\beta)$	M1		Express sum of new roots in terms of $\alpha+\beta$ and $\alpha\beta$
		M1		Substitute their values into their expression
	$p=\frac{5}{6}$	A1	**4**	Obtain **given** answer correctly
	Or			
	$3u^2-u+2(=0)$	B1		Substitute $x=\frac{1}{u}$ and obtain correct quadratic (equation)
		M1		Use sum of roots of new equation
		M1		Substitute their values into their expression
	$p=\frac{5}{6}$	A1		Obtain **given** answer correctly
	(ii) $\alpha'\beta'=\alpha\beta+\frac{1}{\alpha\beta}+\frac{\beta}{\alpha}+\frac{\alpha}{\beta}$	B1		Correct expansion
	$\frac{\beta}{\alpha}+\frac{\alpha}{\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$	M1		Show how to deal with $\alpha^2+\beta^2$
		A1		Obtain correct expression
	$q=\frac{1}{3}$	M1		Substitute their values into $\alpha'\beta'$
		A1	**5**	Obtain correct answer a.e.f.
		9		

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Ocr June 2011

9 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is $16 - 30\mathrm{i}$.

(i) Write down the other root of the quadratic equation. **[1]**

(ii) Find the values of a and b. **[4]**

(iii) Use an algebraic method to solve the quartic equation $y^4 + ay^2 + b = 0$. **[7]**

Let me go home

9	(i)	$16 + 30i$	B1	1	State correct value
	(ii)		M1		Use $a = -$ (sum of roots)
		$a = -32$	A1		Obtain correct answer
			M1		Use $b =$ product of roots
		$b = 1156$	A1	4	Obtain correct answer
			M1		Substitute, expand and equate imag. parts
			A1		Obtain **a** = -32
			M1		Equate real parts
			A1		Obtain b = 1156
	(iii)		M1		Attempt to equate real and imaginary parts of $(p+iq)^2$ & $16 - 30i$ or root from (ii)
		$p^2 - q^2 = 16$ and $pq = -15$	A1		Obtain both results cao
			M1		Obtain quadratic in p^2 or q^2
			M1		Solve to obtain $p = (\pm)5$ or $q = (\pm)3$
			A1		Obtain 2 correct answers as complex nos
			M1		Attempt at all 4 roots
		$\pm(5 \pm 3i)$	A1	7	State other two roots as complex nos
			12		

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OCR June 2011

10 The cubic equation $x^3 + 3x^2 + 2 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \dfrac{1}{\sqrt{u}}$ to show that $4u^3 + 12u^2 + 9u - 1 = 0$. **[5]**

(ii) Hence find the values of $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2} + \dfrac{1}{\gamma^2}$ and $\dfrac{1}{\alpha^2\beta^2} + \dfrac{1}{\beta^2\gamma^2} + \dfrac{1}{\gamma^2\alpha^2}$. **[5]**

Let me go home

10 (i)

$\frac{1}{u^{\frac{3}{2}}} + \frac{3}{u} + 2 = 0$	B1		Use substitution correctly
EITHER	M1		Rearrange
	M1		Square
$\frac{9}{u^2} + \frac{12}{u} + 4 = \frac{1}{u^3}$	A1		Obtain correct equation
$4u^3 + 12u^2 + 9u - 1 = 0$	A1	**5**	Obtain **given** answer
OR			
e. g. $(2u^{3/2} + 3u^{1/2} + 1)(2u^{3/2} + 3u^{1/2} - 1) = 0$	M2		Multiply their equation in u by appropriate related expression
	A2		Obtain **given** answer

(ii)

	B1		Stated or imply that $u = \frac{1}{x^2}$
	M1		Use $-\frac{b}{a}$
-3	A1		Obtain correct answer
	M1		Use $\frac{c}{a}$
$\frac{9}{4}$	A1	**5**	Obtain correct answer
	10		

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OCR JAN 2012

10 The cubic equation $3x^3 - 9x^2 + 6x + 2 = 0$ has roots α, β and γ.

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. **[3]**

The cubic equation $x^3 + ax^2 + bx + c = 0$ has roots α^2, β^2 and γ^2.

(ii) Show that $c = -\frac{4}{9}$ and find the values of a and b. **[9]**

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10	(i)		$\alpha+\beta+\gamma=3$ $\alpha\beta+\beta\gamma+\gamma\alpha=2$ $\alpha\beta\gamma=-\frac{2}{3}$	B1 B1 B1 **[3]**	State correct value State correct value State correct value	
10	(ii)		EITHER			
				M1	$c=(\pm)\alpha^2\beta^2\gamma^2$	
			$c=-\frac{4}{9}$	A1FT	Obtain **given** correct answer	FT for sign error in **(i)**
			$\sum\alpha^2=(\sum\alpha)^2-2\sum\alpha\beta$	M1	Use correct expression	
			5	A1FT	Obtain correct value	FT for sign error in **(i)**
			$a=-5$	A1FT	Obtain answer correctly	Sign change done correctly
				M1*	Attempt to find an identity	
			$\sum\alpha^2\beta^2=(\sum\alpha\beta)^2-2\alpha\beta\gamma\sum\alpha$	A1	Obtain correct identity	
				DM1	Use appropriate values	
			$b=8$	A1 **[9]**	Obtain correct answer cao	
			OR			
				B1	State or use correct substitution	
				M1	Rearrange, fractional indices isolated	
				DM1	Square both sides	
				DM1	Expand and simplify	
			$9y^3-45y^2+72y-4=0$	A1	Obtain correct equation	
				M1	Use coefficients of their cubic	
			$c=-\frac{4}{9}$	A1	Obtain given answer correctly	
			$a=-5$	A1FT	Obtain correct answer	
			$b=8$	A1FT	Obtain correct answer	
				[9]	**SC** mixture of methods only A1FT for a and b	Let me go home

Ocr June 2012

3 One root of the quadratic equation $x^2 + ax + b = 0$, where a and b are real, is the complex number $4 - 3\text{i}$. Find the values of a and b. **[4]**

6 The quadratic equation $2x^2 + x + 5 = 0$ has roots α and β.

(i) Use the substitution $x = \dfrac{1}{u+1}$ to obtain a quadratic equation in u with integer coefficients. **[3]**

(ii) Hence, or otherwise, find the value of $\left(\dfrac{1}{\alpha} - 1\right)\left(\dfrac{1}{\beta} - 1\right)$. **[3]**

Question			Answer	Marks	Guidance	
			EITHER			
3				M1	Use sum of root and conjugate	
			$a = -8$	A1	Obtain correct answer	
				M1	Use product of root and conjugate	
			$b = 25$	A1	Obtain correct answer	
				[4]		
			OR			
				M1	Substitute $4+3i$ or conjugate into equation	
				M1	Equate real and imaginary parts	
			$a = -8$	A1	Obtain correct answer	
			$b = 25$	A1	Obtain correct answer	

Let me go home

6	(i)			M1	Attempt to clear fractions	
				M1	Attempt to expand and simplify to a quadratic	
			$5u^2+11u+8=0$	A1	Obtain correct answer, must be an equation	
				[3]		
6	(ii)		EITHER			
			$u=\frac{1}{x}-1$	B1	State or imply by using roots of new quadratic	
				M1	Use their c/a	
			$\frac{8}{5}$	A1 FT	Obtain correct answer	
				[3]		
			OR			
			$\frac{1}{\alpha\beta}-\frac{\alpha+\beta}{\alpha\beta}+1$	B1	Express in terms of $\alpha+\beta$ and $\alpha\beta$	
				M1	Use values $-\frac{1}{2}$ and $\frac{5}{2}$ correctly	Must be values from original equation
			$\frac{8}{5}$	A1	Obtain correct answer	

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OCR Jan 2013

4 The quadratic equation $x^2 + x + k = 0$ has roots α and β.

(i) Use the substitution $x = 2u + 1$ to obtain a quadratic equation in u. **[2]**

(ii) Hence, or otherwise, find the value of $\left(\frac{\alpha - 1}{2}\right)\left(\frac{\beta - 1}{2}\right)$ in terms of k. **[2]**

9 **(i)** Show that $(\alpha\beta + \beta\gamma + \gamma\alpha)^2 \equiv \alpha^2\beta^2 + \beta^2\gamma^2 + \gamma^2\alpha^2 + 2\alpha\beta\gamma(\alpha + \beta + \gamma)$. **[3]**

(ii) It is given that α, β and γ are the roots of the cubic equation $x^3 + px^2 - 4x + 3 = 0$, where p is a constant. Find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$ in terms of p. **[5]**

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4	**(i)**		$4u^2 + 6u + k + 2 = 0$	M1 A1 **[2]**	Substitute and attempt to simplify Obtain correct answer, must be an **equation**
4	**(ii)**		*Either* $\frac{k+2}{4}$	M1 A1ft **[2]**	Use products of roots of new quadratic i.e. use (±) *c*/*a* Obtain correct answer, from their quadratic
			Or $\frac{k+2}{4}$	M1 A1	Use sum and product of roots of original equation Obtain correct answer

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Question			Answer	Marks	Guidance
9	(i)			M1 A1 A1 **[3]**	Attempt at complete expansion Obtain correct unsimplified answer Obtain **given** answer correctly
9	(ii)		*Either* $\sum\alpha = -p, \sum\alpha\beta = -4, \alpha\beta\gamma = -3$ $\frac{16-6p}{9}$ *Or* $9u^3 + (6p-16)u^2 + (8+p^2)u - 1 = 0$ $\frac{16-6p}{9}$	B1 M1 A1 M1 A1 **[5]** B1 M1 A1 M1 A1	State (anywhere) correct values for $\sum\alpha, \sum\alpha\beta, \sum\alpha\beta\gamma$ Express given expression as a single fraction Obtain correct expression using (i) Use their values for sum of roots etc. in their expression Obtain correct answer Use substitution $1/\sqrt{u}$ Rearrange appropriately and square out Obtain correct co-efficients of u^3 and u^2 Use (+/-)b/a from their cubic Obtain correct answer

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Ocr June 2013

8 The cubic equation $kx^3 + 6x^2 + x - 3 = 0$, where k is a non-zero constant, has roots α, β and γ.

Find the value of $(\alpha + 1)(\beta + 1) + (\beta + 1)(\gamma + 1) + (\gamma + 1)(\alpha + 1)$ in terms of k. **[6]**

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Question			Answer	Marks	Guidance
8			*Either*		
			$\sum\alpha = -\frac{6}{k}, \sum\alpha\beta = \frac{1}{k}$	B1B1	Correct values stated or used
			$\sum\alpha\beta + 2\sum\alpha + 3$	M1 A1	Expand brackets Obtain correct expression aef
			$3 - \frac{11}{k}$	M1 A1	Use their values, in terms of k, for $\sum\alpha$ and $\sum\alpha\beta$ Obtain correct answer aef
				[6]	
			Or		
				B1	State or use substitution $x = u - 1$
				M1	Expand and attempt to simplify coefficients
			$ku^3 + (6-3k)u^2 + (3k-11)u + 2 - k = 0$	A1 A1	Obtain at least correct 1st and 3rd terms
				M1	Use their "$\frac{c}{a}$"
			$3 - \frac{11}{k}$	A1	Obtain correct answer a.e.f.

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