Q1.

The circle *C* has equation

$x^2 + y^2 - 6x + 4y = 12$

(a) Find the centre and the radius of *C*.

(5)

The point *P*(−1, 1) and the point *Q*(7, −5) both lie on *C*.

(b) Show that *PQ* is a diameter of *C*.

(2)

The point *R* lies on the positive *y*-axis and the angle *PRQ* = 90°.

(c) Find the coordinates of *R*.

(4)
(Total 11 marks)

Q2.

The circle *C* has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of *C* is at the point *M*.

(a) Find

(i) the coordinates of the point *M*,
(ii) the radius of the circle *C*.

(5)

N is the point with coordinates (25, 32).

(b) Find the length of the line *MN*.

(2)

The tangent to *C* at a point *P* on the circle passes through point *N*.

(c) Find the length of the line *NP*.

(2)
(Total 9 marks)

Q3.

$$f(x) = 2x^3 - 7x^2 - 10x + 24$$

(a) Use the factor theorem to show that $(x + 2)$ is a factor of $f(x)$.

(2)

(b) Factorise $f(x)$ completely.

(4)
(Total 6 marks)

Q4.

$$f(x) = -6x^3 - 7x^2 + 40x + 21$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$

(2)

(b) Factorise $f(x)$ completely.

(4)

(c) Hence solve the equation

$$6(2^{3y}) + 7(2^{2y}) = 40(2^y) + 21$$

giving your answer to 2 decimal places.

(3)

(Total for question = 9 marks)

Q5.

(a) Given that $\binom{40}{4} = \frac{40!}{4!b!}$, write down the value of b.

(1)

In the binomial expansion of $(1 + x)^{40}$, the coefficients of x^4 and x^5 are p and q respectively.

(b) Find the value of $\frac{q}{p}$.

(3)

(Total 4 marks)

Q6.

(a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$(2 + kx)^7$$

where k is a non-zero constant. Give each term in its simplest form.

(4)

Given that the coefficient of x^3 in this expansion is 1890

(b) find the value of k.

(3)

(Total for question = 7 marks)

Q7.

(a) Find the first 4 terms, in ascending powers of x, in the binomial expansion of

$$(1 + kx)^{10}$$

where k is a positive constant. Give each term in its simplest form.

(3)

Given that, in this expansion, the coefficients of x and x^3 are equal,

(b) find the exact value of k,

(3)

(c) find the coefficient of x^2

(1)

(Total for question = 7 marks)

Q1.

Question Number		Scheme	Marks
Q	(a)	$(x-3)^2-9+(y+2)^2-4=12$ Centre is $(3,-2)$	M1 A1, A1
		$(x-3)^2+(y+2)^2=12+"9"+"4"$ $r=\sqrt{12+"9"+"4"}=5$ (or $\sqrt{25}$)	M1 A1 (5)
	(b)	$PQ=\sqrt{(7--1)^2+(-5-1)^2}$ or $\sqrt{8^2+6^2}$	M1
		$=10=2\times\text{radius}$, $\therefore$ diam. (N.B. For A1, need a comment or conclusion)	A1 (2)
		[ALT: midpt. of PQ $\left(\frac{7+(-1)}{2},\frac{1+(-5)}{2}\right)$: M1, $=(3,-2)=$ centre: A1]	
		[ALT: eqn. of PQ $3x+4y-1=0$: M1, verify $(3,-2)$ lies on this: A1]	
		[ALT: find two grads, e.g. PQ and P to centre: M1, equal $\therefore$ diameter: A1]	
		[ALT: show that point $S(-1,-5)$ or $(7,1)$ lies on circle: M1 because $\angle PSQ=90°$, semicircle $\therefore$ diameter: A1]	
	(c)	R must lie on the circle (angle in a semicircle theorem)… often implied by a diagram with R on the circle or by subsequent working)	B1
		$x=0\Rightarrow\quad y^2+4y-12=0$	M1
		$(y-2)(y+6)=0\quad y=.....$ (M is dependent on previous M)	dM1
		$y=-6$ or 2 (Ignore $y=-6$ if seen, and 'coordinates' are not required))	A1 (4)
			[11]
	(a)	1st M1 for attempt to complete square. Allow $(x\pm3)^2\pm k$, or $(y\pm2)^2\pm k$, $k\neq0$. 1st A1 x-coordinate 3, 2nd A1 y-coordinate -2 2nd M1 for a full method leading to $r=\ldots$, with their 9 and their 4, 3rd A1 5 or $\sqrt{25}$ The 1st M can be implied by $(\pm3,\pm2)$ but a full method must be seen for the 2nd M. Where the 'diameter' in part (b) has clearly been used to answer part (a), no marks in (a), but in this case the M1 (not the A1) for part (b) can be given for work seen in (a). Alternative 1st M1 for comparing with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g,-f)$ directly. Condone sign errors for this M mark. 2nd M1 for using $r=\sqrt{g^2+f^2-c}$. Condone sign errors for this M mark.	
	(c)	1st M1 for setting $x=0$ and getting a 3TQ in y by using eqn. of circle. 2nd M1 (dep.) for attempt to solve a 3TQ leading to at least one solution for y. Alternative 1: (Requires the B mark as in the main scheme) 1st M for using (3, 4, 5) triangle with vertices $(3,-2),(0,-2),(0,y)$ to get a linear or quadratic equation in y (e.g. $3^2+(y+2)^2=25$). 2nd M (dep.) as in main scheme, but may be scored by simply solving a linear equation. Alternative 2: (Not requiring realisation that R is on the circle) B1 for attempt at $m_{PR}\times m_{QR}=-1$, (NOT m_{PQ}) or for attempt at Pythag. in triangle PQR. 1st M1 for setting $x=0$, i.e. $(0,y)$, and proceeding to get a 3TQ in y. Then main scheme. Alternative 2 by 'verification': B1 for attempt at $m_{PR}\times m_{QR}=-1$, (NOT m_{PQ}) or for attempt at Pythag. in triangle PQR. 1st M1 for trying (0, 2). 2nd M1 (dep.) for performing all required calculations. A1 for fully correct working and conclusion.	

Q2.

Question Number	Scheme		Marks
(a)			
(i)	The centre is at (10, 12)	B1: $x = 10$ B1: $y = 12$	B1 B1
(ii)	Uses $(x-10)^2+(y-12)^2=-195+100+144 \Rightarrow r = ...$		M1
	Completes the square for both x and y in an attempt to find r. $(x \pm "10")^2 \pm a$ and $(y \pm "12")^2 \pm b$ and $+195=0, (a, b \neq 0)$ Allow errors in obtaining their r^2 but must find square root		
	$r = \sqrt{10^2+12^2-195}$	A correct numerical expression for r including the square root and can implied by a correct value for r	A1
	$r = 7$	Not $r = \pm 7$ unless -7 is rejected	A1
			(5)
(a) Way 2	Compares the given equation with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g, -f)$ i.e. (10, 12)	B1: $x = 10$ B1: $y = 12$	B1B1
	Uses $r = \sqrt{(\pm "10")^2+(\pm "12")^2 - c}$		M1
	$r = \sqrt{10^2+12^2-195}$	A correct numerical expression for r	A1
	$r = 7$		A1
			(5)
(b)	$MN = \sqrt{(25-"10")^2+(32-"12")^2}$	Correct use of Pythagoras	M1
	$MN\left(=\sqrt{625}\right)=25$		A1
			(2)
(c)	$NP = \sqrt{("25"^2 - "7"^2)}$	$NP = \sqrt{(MN^2 - r^2)}$	M1
	$NP\left(=\sqrt{576}\right)=24$		A1
			(2)
(c) Way 2	$\cos(NMP) = \frac{7}{"25"} \Rightarrow NP = "25"\sin(NMP)$	Correct strategy for finding NP	M1
	$NP = 24$		A1
			(2)
			[9]

Q3.

Question number	Scheme	Marks
(a)	$f(-2) = 2.(-2)^3 - 7.(-2)^2 - 10.(-2) + 24$	M1
	$= 0$ so $(x+2)$ is a factor	A1
		(2)
(b)	$\text{f}(x) = (x+2)(2x^2 - 11x + 12)$	M1 A1
	$\text{f}(x) = (x+2)(2x-3)(x-4)$	dM1 A1
		(4)
		6 marks
Notes (a)	**M1** : Attempts f(± 2) (Long division is M0) **A1** : is for =0 **and conclusion** **Note**: Stating "hence factor" or "it is a factor" or a "√" (tick) or "QED" is fine for the conclusion. **Note** also that a conclusion can be implied from a <u>preamble</u>, eg: "If $\text{f}(-2) = 0$, $(x+2)$ is a factor…." (Not just f(-2)=0)	
(b)	**1st M1**: Attempts long division by correct factor or other method leading to obtaining $(2x^2 \pm ax \pm b)$, $a \neq 0$, $b \neq 0$, even with a remainder. Working need not be seen as could be done "by inspection." Or *Alternative Method* : **1st M1**: Use $(x+2)(ax^2+bx+c) = 2x^3 - 7x^2 - 10x + 24$ with expansion and comparison of coefficients to obtain $a = 2$ and to obtain values for b and c **1st A1**: For seeing $(2x^2 - 11x + 12)$. [Can be seen here in (b) after work done in (a)] **2nd M1**: Factorises quadratic. (see rule for factorising a quadratic). This is dependent on the previous method mark being awarded and **needs factors** **2nd A1**: is cao and needs all three factors together. Ignore subsequent work (such as a solution to a quadratic equation.) Note: Some candidates will go from $\{(x+2)\}(2x^2 - 11x + 12)$ to $\{x = -2\}$, $x = \frac{3}{2}, 4$, and not list all three factors. Award these responses M1A1M0A0. Finds $x = 4$ and $x = 1.5$ by factor theorem, formula or calculator and produces factors M1 $\text{f}(x) = (x+2)(2x-3)(x-4)$ **or** $\text{f}(x) = 2(x+2)(x-1.5)(x-4)$ **o.e. is full marks** $\text{f}(x) = (x+2)(x-1.5)(x-4)$ **loses last A1**	

Q4.

Question Number	Scheme	Marks
(a)	Attempt $f(3)$ or $f(-3)$ Use of long division is M0A0 as factor theorem was required. $f(-3)=162-63-120+21=0$ so $(x+3)$ is a factor	M1 A1 **(2)**
(b)	Either (Way 1): $f(x)=(x+3)(-6x^2+11x+7)$ $=(x+3)(-3x+7)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$	M1A1 M1A1 **(4)**
	Or (Way 2) Uses trial or factor theorem to obtain $x=-1/2$ **or** $x=7/3$ Uses trial or factor theorem to obtain both $x=-1/2$ **and** $x=7/3$ Puts three factors together (see notes below) Correct factorisation : $(x+3)(7-3x)(2x+1)$ or $-(x+3)(3x-7)(2x+1)$ oe	M1 A1 M1 A1 **(4)**
	Or (Way 3) No working three factors $(x+3)(-3x+7)(2x+1)$ otherwise need working	M1A1M1A1 (4)
(c)	$2^y=\frac{7}{3}, \rightarrow \log(2^y)=\log\left(\frac{7}{3}\right)$ or $y=\log_2\left(\frac{7}{3}\right)$ or $\frac{\log(7/3)}{\log 2}$ $\{y=1.222392421...\} \Rightarrow y=$ awrt 1.22	B1, M1 A1 **(3)** **[9]**

	Notes
(a)	M1 for attempting either $f(3)$ or $f(-3)$ – with **numbers substituted into expression** A1 for calculating $f(-3)$ **correctly to 0**, and they must state $(x+3)$ **is a factor** for A1 (or equivalent ie. QED, □ or a tick). A conclusion may be implied by a preamble, "if $f(-3)=0$, $(x+3)$ is a factor". $-6(-3)^3-7(-3)^2+40(-3)+21=0$ so $(x+3)$ is a factor of $f(x)$ is M1A1 providing bracketing is correct.
(b)	1st M1: attempting to divide by $(x+3)$ leading to a 3TQ beginning with the correct term, usually $-6x^2$. This may be done by a variety of methods including long division, comparison of coefficients, inspection etc. Allow for work in part (a) if the result is used in (b). 1st A1: usually for $(-6x^2+11x+7)$ … Credit when seen and use isw if miscopied 2nd M1: for a ***valid**** attempt to factorise their quadratic (* see notes on page 6 - General Principles for Core Mathematics Marking section 1) 2nd A1 is cao and needs all three factors together fully factorised. Accept e.g. $-3(x+3)(x-\frac{7}{3})(2x+1)$ but $(x+3)(x-\frac{7}{3})(-6x-3)$ and $(x+3)(3x-7)(-2x-1)$ are A0 as not fully factorised. Ignore subsequent work (such as a solution to a quadratic equation.) Way 2: The second M mark needs three roots together so $\pm 6(x-\alpha)(x-\beta)(x+3)$ or equivalent where they obtained α and β by trial, so if correct roots identified, then $(x+3)(3x-7)(2x+1)$ can gain M1A1M1A0. N.B. Replacing $(-6x^2+11x+7)$ (already awarded M1A1) by $(6x^2-11x-7)$ giving $(x+3)(3x-7)(2x+1)$ can have M1A0 for factorization so M1A1M1A0
(c)	B1: $2^y=\frac{7}{3}$ M1: Attempt to take logs to solve $2^y=\alpha$ or $2^y=1/\alpha$, where $\alpha>0$ and α was a root of their factorization. A1: for an answer that rounds to 1.22. If other answers are included (and not "rejected") such as $\ln(-3)$ or -1 lose final A mark **Special case:** Those who deal **throughout** with $f(x)=6x^3+7x^2-40x-21$ They may have full credit in part (a). In part (b) they can achieve a maximum of M1A0M1A0 unless they return the negative sign to give the correct answer. This is then full marks. Part (c) is fine. So they could lose 2 marks on the factorisation. (Like a misread)

Q5.

Question Number	Scheme		Marks
(a)	$\binom{40}{4}=\frac{40!}{4!b!}$; $(1+x)^n$ coefficients of x^4 and x^5 are p and q respectively. $b=36$ Candidates should usually "identify" two terms as their p and q respectively.		B1 (1)
(b)	Term 1: $\binom{40}{4}$ or ${}^{40}C_4$ or $\frac{40!}{4!36!}$ or $\frac{40(39)(38)(37)}{4!}$ or 91390	Any one of Term 1 or Term 2 correct. (Ignore the label of p and/or q.)	M1
	Term 2: $\binom{40}{5}$ or ${}^{40}C_5$ or $\frac{40!}{5!35!}$ or $\frac{40(39)(38)(37)(36)}{5!}$ or 658008	Both of them correct. (Ignore the label of p and/or q.)	A1
	Hence, $\frac{q}{p}=\frac{658008}{91390}\left\{=\frac{36}{5}=7.2\right\}$	for $\frac{658008}{91390}$ oe	A1 oe cso (3) **[4]**
	Notes		
(a)	B1: for only $b=36$.		
(b)	The candidate may expand out their binomial series. At this stage no marks should be awarded until they start to identify either one or both of the terms that they want to focus on. Once they identify their terms then if one out of two of them (ignoring which one is p and which one is q) is correct then award M1. If both of the terms are identified correctly (ignoring which one is p and which one is q) then award the first A1. Term 1 $=\binom{40}{4}x^4$ or ${}^{40}C_4(x^4)$ or $\frac{40!}{4!36!}x^4$ or $\frac{40(39)(38)(37)}{4!}x^4$ or $91390x^4$, Term 2 $=\binom{40}{5}x^5$ or ${}^{40}C_5(x^5)$ or $\frac{40!}{5!35!}x^5$ or $\frac{40(39)(38)(37)(36)}{5!}x^5$ or $658008x^5$ are fine for any (or both) of the first two marks in part (b). 2^{nd} A1 for stating $\frac{q}{p}$ as $\frac{658008}{91390}$ or equivalent. Note that $\frac{q}{p}$ must be independent of x. Also note that $\frac{36}{5}$ or 7.2 or any equivalent fraction is fine for the 2^{nd} A1 mark. SC: If candidate states $\frac{p}{q}=\frac{5}{36}$, then award M1A1A0. Note that either $\frac{4!36!}{5!35!}$ or $\frac{5!35!}{4!36!}$ would be awarded M1A1.		

Q6.

Question Number	Scheme	Marks
	$(2+kx)^7$	
(a)	$2^7 + {}^7C_1 2^6(kx) + {}^7C_2 2^5(kx)^2 + {}^7C_3 2^4(kx)^3 ...$	
	First term of 128	B1
	$({}^7C_1 \times ... \times x) + ({}^7C_2 \times ... \times x^2) + ({}^7C_3 \times ... \times x^3)...$	M1
	$=(128 ..) + 448kx + 672k^2x^2 + 560k^3x^3 ..$	A1, A1
		(4)
(b)	$560k^3 = 1890$	M1
	$k^3 = \frac{1890}{560}$ so $k =$	dM1
	$k = 1.5$ o.e.	A1
		(3)
		(7marks)
Alternative method For (a)	$(2+kx)^7 = 2^7(1+\frac{kx}{2})^7$	
	$2^7(1 + {}^7C_1(\frac{k}{2}x) + {}^7C_2(\frac{k}{2}x)^2 + {}^7C_3(\frac{k}{2}x)^3 \ ...)$	
	Scheme is applied exactly as before	

Notes

(a)

B1: The constant term should be 128 in their expansion (should not be followed by other constant terms)

M1: Two of the three binomial coefficients must be correct and must be with the correct power of x. Accept ${}^7C_1$ or $\binom{7}{1}$ or 7 **as a coefficient, and** ${}^7C_2$ or $\binom{7}{2}$ or 21 **as another and** ${}^7C_3$ or $\binom{7}{3}$ or 35 **as another……..**

Pascal's triangle may be used to establish coefficients.

A1: Two of the final three terms correct (i.e. two of $448kx + 672k^2x^2 + 560k^3x^3 ..$).

A1: All three final terms correct. (Accept answers without + signs, can be listed with commas or appear on separate lines)

e.g. The common error $=(128 ..) + 448kx + 672kx^2 + 560kx^3 ..$ would earn B1, M1, A0, A0, so 2/4 Then would gain a maximum of 1/3 in part (b)

If extra terms are given then isw

If the **final** answer is given as $=(128 ..) + 448kx + 672(kx)^2 + 560(kx)^3 ..$ with correct brackets and no errors are seen, this may be given full marks. If they continue and remove the brackets wrongly then they lose the accuracy marks.

Special case using Alternative Method: Uses $2\ (1+\frac{kx}{2})^7$ is likely to result in a maximum mark of B0M1A0A0 then M1M1A0

If the correct expansion is seen award the marks and isw

(b)

M1: Sets their **Coefficient** of x^3 equal to 1890. They should have an equation which does not include a power of x. This mark may be recovered if they continue on to get $k = 1.5$

dM1: This mark depends upon the previous M mark. Divides then attempts a cube root of their answer to give k – the intention must be clear. (You may need to check on a calculator) The correct answer implies this mark.

A1: Any equivalent to 1.5 If they give – 1.5 as a second answer this is A0

Q7.

Question Number	Scheme	Marks
	$(1+kx)^{10}$	
(a)	$1+{}^{10}C_1(kx)+{}^{10}C_2(kx)^2+{}^{10}C_3(kx)^3...$	
	$1+\left({}^{10}C_1\times...\times x\right)+\left({}^{10}C_2\times...\times x^2\right)+\left({}^{10}C_3\times...\times x^3\right)...$	M1
	$=1+10kx,+45k^2x^2+120k^3x^3..$	B1, A1
		(3)
(b)	$120k^3=10k$	M1
	$k^2=\frac{1}{12}$ so $k=...$	M1
	$k=\frac{\sqrt{3}}{6}$ o.e	A1
		(3)
(c)	$\frac{15}{4}$ o.e.	B1
		(1)
		(7 marks)

Notes

(a)

M1: All three binomial coefficients must be correct and must be with the correct power of x. (Ignore k) Accept ${}^{10}C_1$ or $\binom{10}{1}$ or 10 **as a coefficient, and** ${}^{10}C_2$ or $\binom{10}{2}$ or 45 **as another and** ${}^{10}C_3$ or $\binom{10}{3}$ or 120 **as another……..**

Pascal's triangle may be used to establish coefficients.

B1: The first two terms correct (i.e. $=1+10kx$)

A1: The third and fourth terms are correct – allow with brackets (kx) (i.e. $45k^2x^2+120k^3x^3$ or $45(kx)^2+120(kx)^3$)

(Accept answers without + signs, can be listed with commas or appear on separate lines)

If extra terms are given then isw

(b)

M1: Sets their **Coefficient** of x equal to their **Coefficient** of x^3 but must have differing powers of k

M1: Divides then takes a square root to give a value for k (May use difference of two squares to find k which is fine)

A1: $k=\frac{1}{\sqrt{12}}$ or $\frac{\sqrt{12}}{12}$ or $\frac{\sqrt{3}}{6}$ o.e. (needs to have just the one positive answer – if negative square root is also given, this is A0)

If there are x terms present e.g. $120k^3x^3=10kx$ then this is M0M0A0

If both powers of k are the same this is also M0M0A0

(c)

B1: $\frac{45}{12}$ or $\frac{15}{4}$ or 3.75 or equivalent Allow $\frac{15}{4}x^2$ (can follow negative value for k)