# 13Ma Pre January Exams Mini Test <br> Trigonometry (Harmonic Forms) <br> Implicit Differentiation <br> Sector, Radians, Numerical Methods 

Surname $\qquad$

B Mr Chan/Mr Phillips

Other names $\qquad$

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

## Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill at the top of this page with your name, and tick the box with the class you belong to.
- Answer all questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
- there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.


## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is

| Question | Marks | Score |
| :---: | :---: | :---: |
| 1 | 11 |  |
| 2 | 8 |  |
| 3 | 11 |  |
| Total: | 30 |  | provided.

- There are 3 questions in this question paper.

The total mark for this paper is 30 .

- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. (a) Express $7 \cos \theta+24 \sin \theta$ in the form $R \cos (\theta-\alpha)$, where $R>0$ and $0^{\circ}<\alpha<90^{\circ}$.

Give the value of $\alpha$ correct to 2 decimal places.
(b) Solve the equation $7 \cos \theta+24 \sin \theta=18$ for $0^{\circ}<\theta<360^{\circ}$.

As $\beta$ varies, the greatest possible value of

$$
\frac{150}{7 \cos \frac{1}{2} \beta+24 \sin \frac{1}{2} \beta+50}
$$

is denoted by $V$,
(c) (i) find the value of $V$,
(ii) determine the smallest positive value of $\beta$ (in degrees) for which the value of $V$ occurs.
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Question 1 continued
2. The equation of a curve is $x^{2} y-a y^{2}=4 a^{3}$, where $a$ is a non-zero constant.
(a) Show that $\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{2 x y}{2 a y-x^{2}}$.
(b) Hence find the coordinates of the points where the tangent to the curve is parallel to the $y$-axis.
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Question 2 continued
3.


Figure 1

Figure 1 shows a sector $O A B$ of a circle with centre $O$ and radius $O A$.
The angle $A O B$ is $\theta$ radians.
$M$ is the mid-point of $O A$.
The ratio of areas $O M B: M A B$ is $2: 3$.
(a) Show that $\theta=1.25 \sin \theta$.

The equation $x=1.25 \sin x$ has only one root for $x>0$.
This root can be found by using the iterative formula $x_{n+1}=1.25 \sin x$.
(b) Using a starting value of $x_{1}=0.5$,
(i) write down the values of $x_{2}, x_{3}$ and $x_{4}$.
(ii) find the value of this root correct to 3 significant figures.
(iii) show, using a change in sign method, that this root is accurate to 3 significant figures.

Diagram 1 in the next page shows the graphs of $y=1.25 \sin x$ and $y=x$ sketched on a graphical calculator.
(c) Use this diagram to show how the iterative process used in (b) converges to this root, you should state the type of convergence, and the values of $x_{2}, x_{3}$ and $x_{4}$ in the diagram.

Question 3 continued



## Diagram 1

A copy of Diagram 1, in case you need to redraw your sketch:


Copy of Diagram 1
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Question 3 continued

Question 3 continued
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(Total for Question 3 is 11 marks)

