Year 12 Full AS Mock Set#02b
AS Pure Pre-mock Nov 2022

This exam has 15 questions, for a total of 100 marks.

- Print in "booklets" will allow all questions to be on the left hand side.

Question	Marks	Score
1	5	
2	6	
3	6	
4	7	
5	6	
6	6	
7	5	
8	5	
9	6	
10	7	
11	9	
12	10	
13	7	
14	4	
15	11	
Total:	100	

Andrew Chan
7th May 2023

In this question you should show all stages of your working.
Solutions relying on calculator technology are not acceptable.

1. A curve has equation

$$y = x^3 - 9x^2 + 26x - 18$$

Find the equation of the normal to the curve at the point $P(4, 6)$. Write your answer in the form $ax + by = c$, where a, b, and c are integers to be found.

(5)

Question 1 continued

(Total for Question 1 is 5 marks)

2. [In this question the unit vectors **i** and **j** are due east and due north respectively]

A boat B is moving with constant velocity. At noon, B is at the point with position vector $(3\mathbf{i} - 4\mathbf{j})$ km with respect to a fixed origin O. At 1430 on the same day, B is at the point with position vector $(8\mathbf{i} + 11\mathbf{j})$ km.

(a) Calculate the bearing on which the boat is moving.

(3)

(b) Calculate the speed of the boat, giving your answer in km h^{-1}

(3)

Question 2 continued

(Total for Question 2 is 6 marks)

In this question you should show all stages of your working.
Solutions relying on calculator technology are not acceptable.

3. (i) Solve the equation

$$4^{2x+1} = 8^{4x}$$

(3)

(ii) Solve

$$3\sqrt{18} - \sqrt{32} = \sqrt{n}$$

(3)

Question 3 continued

(Total for Question 3 is 6 marks)

4. A bakery makes pies.

On any day, the total cost to the bakery, £y, of making x pies is modelled to be the sum of two separate elements:

- a fixed cost, C,
- a cost that is proportional to the number of pies that are made that day, K.

(a) Write down a general equation linking y with x, for this model.

(1)

The pies are sold for £5 each.

On a day when 650 pies are made and sold, the bakery makes a profit of £200.

On a day when 230 pies are made and sold, the bakery makes a loss of £80.

Using the above information,

(b) Rewrite your answer to part (a), giving the values of the constants C and K as simplified fractions.

(3)

(c) With reference to the model, interpret the significance of the value for the gradient in the equation derived above.

(1)

Assuming that each pie is sold on the day it is made,

(d) find the least number of pies that must be made on any given day for the bakery to make a profit that day.

(2)

Question 4 continued

(Total for Question 4 is 7 marks)

5.

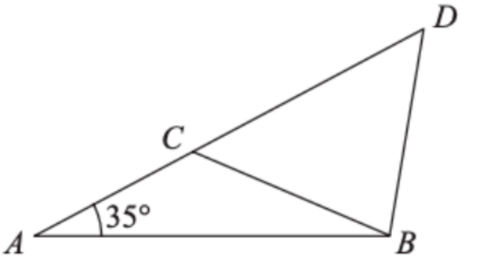

Figure 1

Figure 1 shows the design for a structure used to support a roof.

The structure consists of four wooden beams, AB, BD, BC, AD.

- $AB = 6.5$ m
- $BC = BD = 4.7$ m
- $\angle BAC = 35°$

(a) Find, to one decimal place, the size of $\angle ACB$.

(3)

(b) Find, to the nearest metre, the total length of wood required to make this structure.

(3)

Question 5 continued

(Total for Question 5 is 6 marks)

6. (a) Find the first 3 terms in ascending powers of x of the binomial expansion of

$$(2 + ax)^6$$

where a is a non-zero constant. Give each term in simplest form.

(4)

Given that, in the expansion, the coefficient of x is equal to the coefficient of x^2.

(b) Find the value of a.

(2)

Question 6 continued

(Total for Question 6 is 6 marks)

7. Given $k > 3$ and

$$\int_3^k \left(2x + \frac{6}{x^2}\right) \mathrm{d}x = 10k$$

Show that $k^3 - 10k^2 - 7k - 6 = 0$

(5)

Question 7 continued

(Total for Question 7 is 5 marks)

8. Solve, using algebra, the equation

$$x - 6\sqrt{x} + 4 = 0$$

Fully simplify your answers, writing them in the form $a + b\sqrt{c}$, where a, b and c are integers to be found.

(5)

Question 8 continued

(Total for Question 8 is 5 marks)

9. A population of a rare species of toad is being studied.

The number of toads, N, in the population, t years after the start of the study, is modelled by the equation.

$$N = \frac{900e^{0.12t}}{2e^{0.12t} + 1} \qquad t \geq 0, t \in \mathbb{R}$$

According to this model,

(a) Calculate the number of toads in the population at the start of the study.

(1)

(b) Find the value of t when there are 420 toads in the population, giving your answer to 2 decimal places.

(4)

(c) Explain why, according to this model, the number of toads can never reach 500.

(1)

Question 9 continued

(Total for Question 9 is 6 marks)

10.

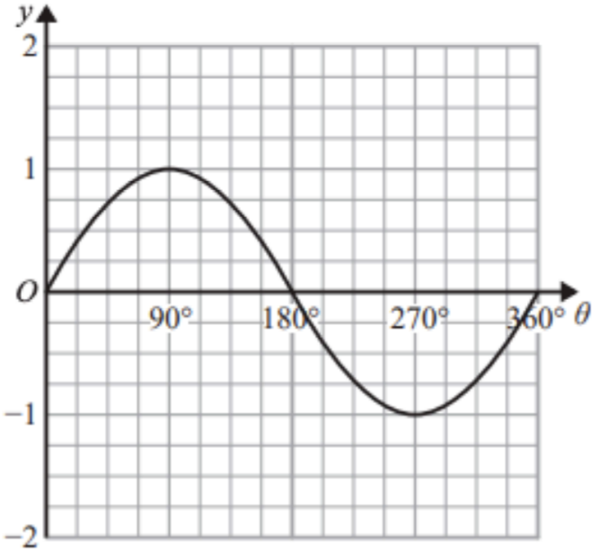

Figure 2

Figure 2 shows a plot of the curve with equation $y = \sin\theta, \quad 0 \leq \theta \leq 360°$

(a) State the coordinates of the minimum point on the curve with equation

$$y = 4\sin\theta, \quad 0 \leq \theta \leq 360°$$

(2)

A copy of Figure 2, called Diagram 1, is shown on the next page.

(b) On Diagram 1, sketch and label the curves

(i) $y = 1 + \sin\theta, \quad 0 \leq \theta \leq 360°$

(ii) $y = \tan\theta, \quad 0 \leq \theta \leq 360°$

(2)

(c) Hence find the number of solutions of the equation

(i) $\tan\theta = 1 + \sin\theta, \quad 0 \leq \theta \leq 2160°$

(ii) $\tan\theta = 1 + \sin\theta, \quad 0 \leq \theta \leq 1980°$

(3)

Question 10 continued

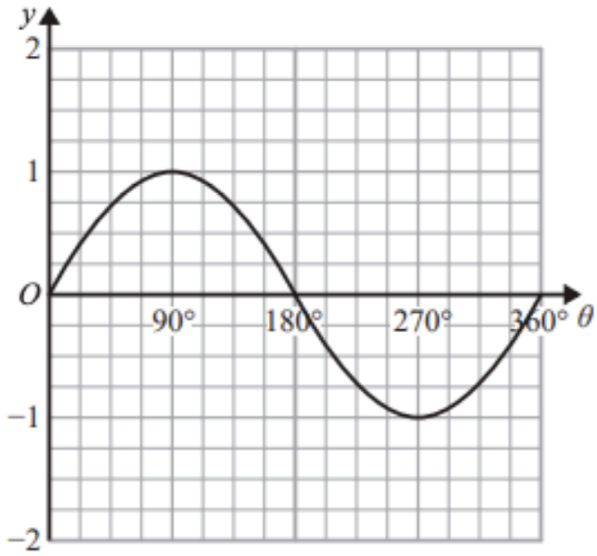

Diagram 1

(Total for Question 10 is 7 marks)

11. A curve has equation $y = (x+2)^2(4-x)$

The curve touches the x-axis at the point P and crosses the x-axis at the point Q.

(a) State the coordinates of the point Q.

(1)

The finite region R is bounded by the curve and the x-axis.

(b) Using calculus and showing each step of your working, find the exact area of R.

(6)

(c) Using the answer to part (b) and explaining your reasoning, find the area of the finite region bounded by the curve with equation $y = (3x+6)^2\left(2 - \frac{1}{2}x\right)$ and the x-axis.

(2)

Question 11 continued

(Total for Question 11 is 9 marks)

12. (i) A circle C_1 has centre $A(2, 1)$ and passes through the point $B(10, 7)$.

The line l is the tangent to C_1 at the point B.

Find an equation of l in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

(ii) A different circle C_2 has equation

$$x^2 + y^2 - 6x - 14y + k = 0$$

where k is a constant.

Given that C_2 lies entirely in the first quadrant, find the range of possible values for k.

(5)

Question 12 continued

(Total for Question 12 is 10 marks)

13. A doctors' surgery starts a campaiign to reduce missed appointments. The number of missed appointments for each of the first five weeks after the start of the campaign is shown below.

Number of weeks after the start *(x)*	1	2	3	4	5
Number of missed appointments *(y)*	235	149	99	59	38

Figure 3

This data could be modelled by an equation of the form $y = pq^x$ where p and q are constants.

(a) Show that this relationship may be expressed in the form $\log_{10} y = mx + c$, expressing m and c in terms of p and/or q.

(2)

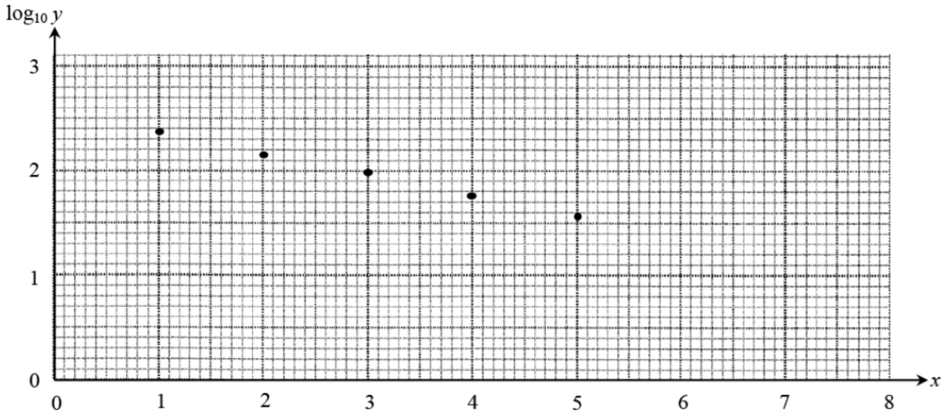

Figure 4

Figure 4 shows $\log_{10} y$ plotted against x, for the given data.

(b) Estimate the values of p and q.

(3)

(c) Use the model to predict when the number of missed appointments will fall below 20. Explain why this answer may not be reliable.

(2)

Question 13 continued

(Total for Question 13 is 7 marks)

14. Prove that for all $n \in \mathbb{N}$, $n^2 - 2n + 2$ is not divisible by 4.

(4)

Question 14 continued

(Total for Question 14 is 4 marks)

15. A curve has equation $y = f(x)$, where

$$f''(x) = \frac{6}{\sqrt{x^3}} + x \qquad x > 0$$

The point $P(4, -50)$ lies on the curve.
Given that $f'(x) = -4$ at P,

(a) Find the equation of the normal at P, write your answer in the form $y = mx + c$, where m and c are constants.

(3)

(b) Find $f(x)$

(8)

Question 15 continued

(Total for Question 15 is 11 marks)

–Total for paper is 100 marks–