# Year 13 Mathematics Mock Set#03c
# Pure Paper 1

- Advised to print in “A3-booklets”, this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

This exam paper has 14 questions, for a total of 100 marks.

| Question | Marks | Score |
|---|---|---|
| 1 | 3 | |
| 2 | 7 | |
| 3 | 9 | |
| 4 | 8 | |
| 5 | 9 | |
| 6 | 7 | |
| 7 | 15 | |
| 8 | 7 | |
| 9 | 7 | |
| 10 | 7 | |
| 11 | 9 | |
| 12 | 4 | |
| 13 | 3 | |
| 14 | 5 | |
| Total: | 100 | |

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Last updated: 28th February 2023

**1.** Prove that the sum of the squares of 2 consecutive odd integers is always 2 more than a multiple of 8.

(3)

Question 1 continued

(Total for Question 1 is 3 marks)

**2.**

$$f(x) = 3x^3 \quad 7x^2 + 7x \quad 10$$

(a) Use the factor theorem to show that $(x \quad 2)$ is a factor of $f(x)$.

**(2)**

(b) Find the values of the constants $a$, $b$ and $c$ such that

$$f(x) = (x \quad 2)(ax^2 + bx + c)$$

**(3)**

(c) Using your answer to part (b) to show that the equation $f(x) = 0$ has only one real root.

**(2)**

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 7 marks)

3. **In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.**

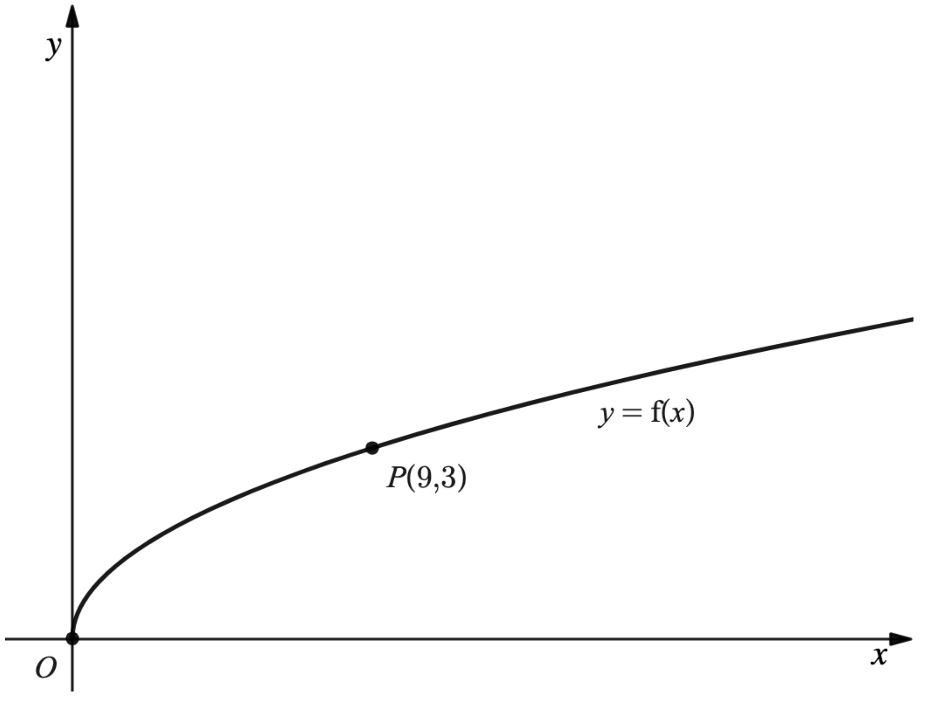


Figure 1: https://www.desmos.com/calculator/mhlj9dp6c7

Figure 1 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = \sqrt{x} \qquad \{x \geq 0\}$$

The point $P(9, 3)$ lies on the curve and is shown in Figure 1.
A copy of Figure 1, labelled Diagram 1 is shown on the next page.

(a) On Diagram 1, sketch and clearly label the graphs of

$$y = \mathrm{f}(2x) \qquad \text{(I)}$$
$$\text{and } y = \mathrm{f}(x) + 3 \qquad \text{(II)}$$

Show on each graph the coordinates of the point to which $P$ is transformed.

(3)

The graph of $y = \mathrm{f}(2x)$ meets the graph of $y = \mathrm{f}(x) + 3$ at the point $Q$.

(b) Show that the $x$-coordinate of $Q$ is the solution of

$$\sqrt{x} = 3\sqrt{2} + 1$$

(3)

(c) Hence find, in simplest form, the coordinates of $Q$.

(3)

Question 3 continued

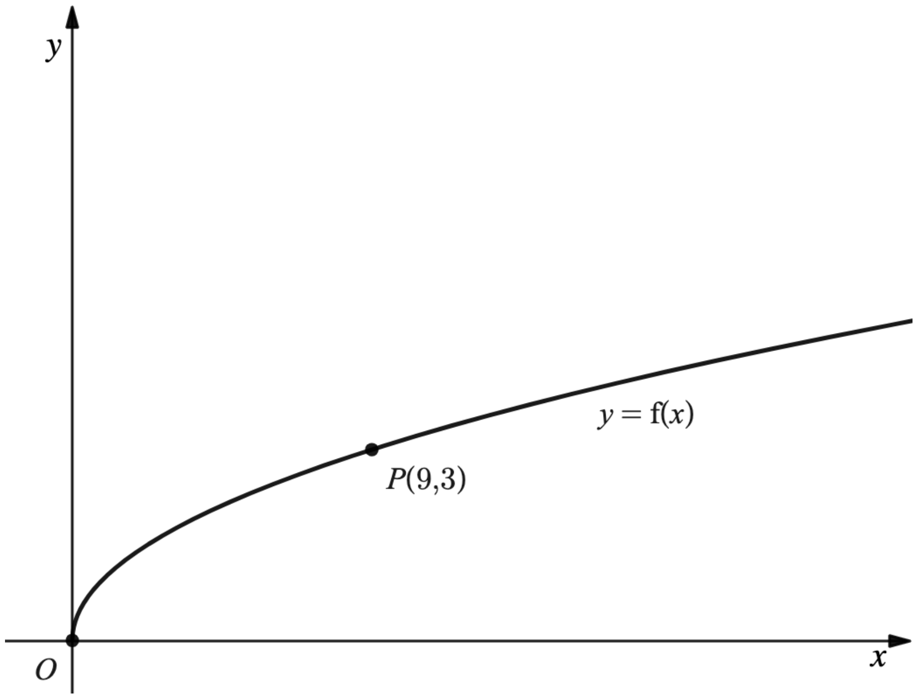


Diagram 1

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

**4.** The fuel consumption of a car, $C$ miles per gallon, varies with the speed, $v$ miles per hour. Mr Chan models the fuel consumption of his car by the formula

$$C = 2.4v - 0.024v^2 \qquad \{0 \leq v \leq 80\}$$

(a) Suggest a reason why Mr Chan has included an upper limit in his model.

(1)

(b) Determine the speed that gives the maximum fuel consumption.
You should justify that the speed you have found gives the maximum fuel consumption.

(4)

Miss Anderson's car does more miles per gallon than Mr Chan's car. She proposes to model the fuel consumption of her car using a formula of the form

$$C = 2.4v - 0.024v^2 + k \qquad \{0 \leq v \leq 80\}$$

(c) Give a reason why this model is **not** suitable.

(1)

(d) Suggest a different change to Mr Chan's formula which would give a more suitable model.

(2)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 8 marks)

5.

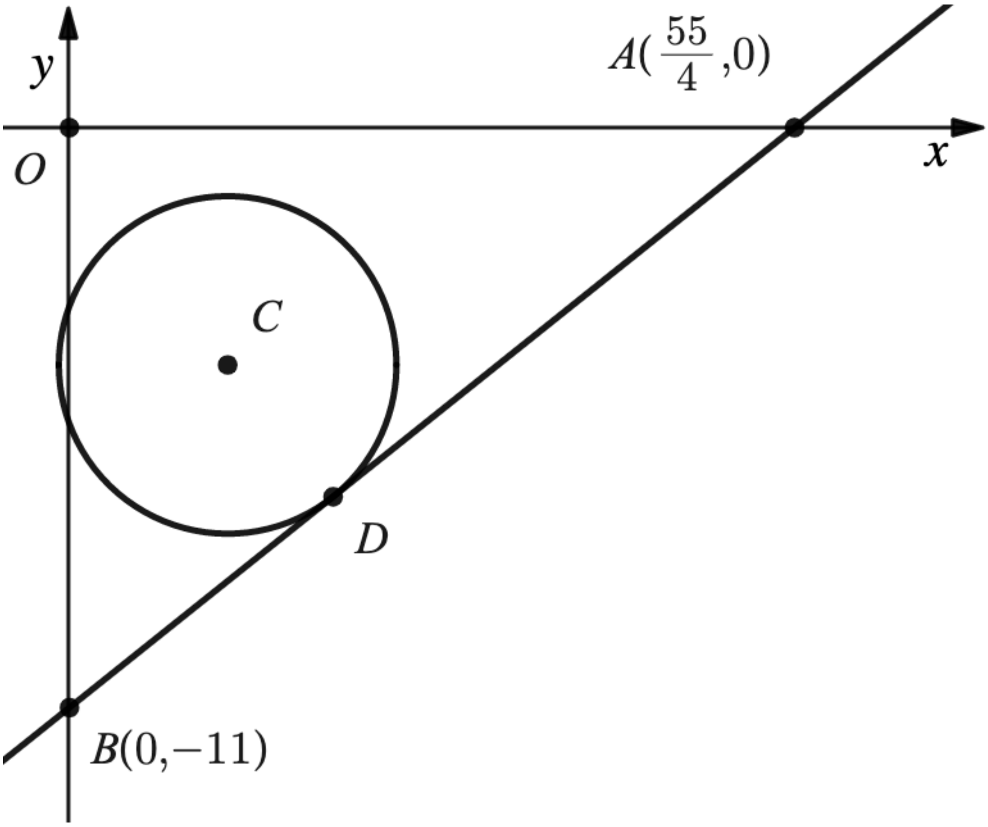


Figure 2: https://www.desmos.com/calculator/4w4ughepqx

Figure 2 shows a circle with equation $x^2 + y^2 \quad 6x + 9y + 19 = 0$ and centre $C$.

(a) Find

(i) the coordinates of the centre of $C$,

(ii) the exact radius of $C$.
Give your answer as a simplified surd.

(3)

The tangent to the circle at $D$ meets the $x$-axis at the point $A \ \frac{55}{4}; 0$ and the $y$-axis at the point $B(0; \quad 11)$.

(b) Determine the area of the triangle $OBD$.

(6)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

**6.** Debbie is saving money to buy a new iPad. She saves £5 in week 1, £5.25 in week 2, £5.50 in week 3 and so on until she has enough money, in total, to buy the iPad.

She decides to model her savings using either an arithmetic series or a geometric series.

Using the information given,

(a) (i) state with a reason whether an arithmetic series or a geometric series should be used,

(ii) write down an expression, in terms of $n$, for the amount, in pounds (£), saved in week $n$.

(3)

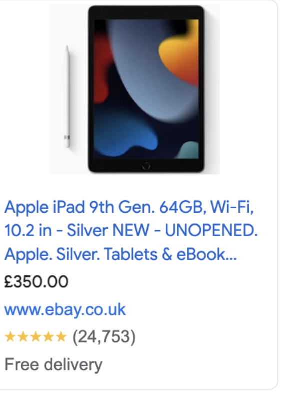


**Figure 3**

Given that the iPad Debbie wants to buy costs £350, as shown in Figure 3.

(b) Find the number of weeks it will take for Debbie to save enough money to buy the iPad.

(4)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 7 marks)

7.

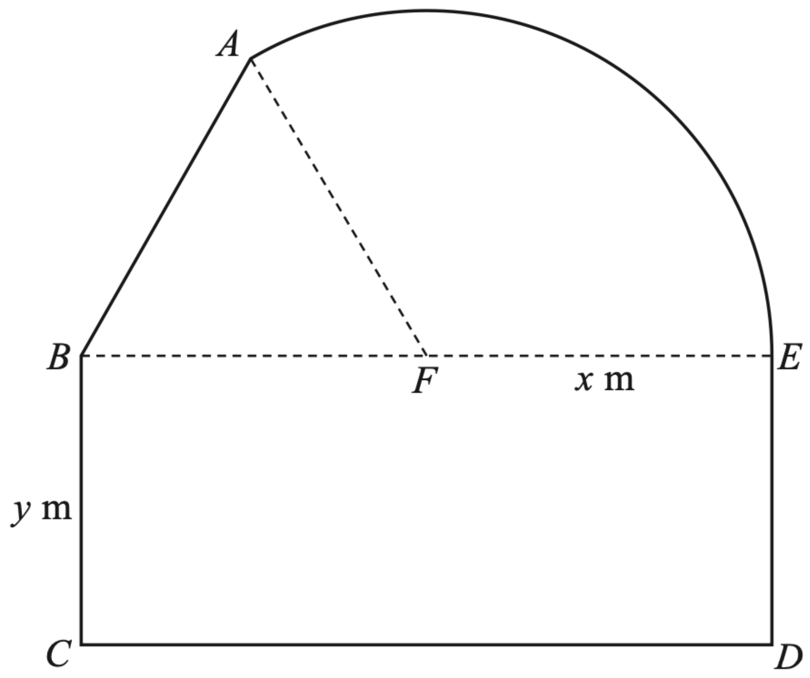


Figure 4

Figure 4 shows a plan view of a sheep enclosure on Clarkson's Farm.

The enclosure $ABCDEA$, as shown in Figure 4, consists of a rectangle $BCDE$ joined to an equilateral triangle $BFA$ and a sector $FEA$ of a circle with radius $x$ metres and centre $F$.

The points $B$, $F$ and $E$ lie on a straight line with $FE = x$ metres and $10 \leqslant x \leqslant 25$

(a) Find, in $\text{m}^2$, the exact area of the sector $FEA$, giving your answer in terms of $x$, in its simplest form.

(2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is $1000\ \text{m}^2$,

(b) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$$

(3)

(c) Hence show that the perimeter $P$ metres of the enclosure is given by

$$y = \frac{1000}{x} + \frac{x}{12}\left(4\pi - 3\sqrt{3} + 36\right)$$

(3)

(d) Use calculus to find the minimum value of $P$, giving your answer to the nearest metre.

(5)

(e) Justify, by further differentiation, that the value of $P$ you have found is a minimum.

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

8. **In this question you must show all stages of your working.**
**Solutions relying on calculator technology are not acceptable.**

(a) Show that the equation

$$2\sin(\quad 30^\circ) = 5\cos$$

can be written in the form

$$\tan \quad = 2^{p}\overline{3}$$

(4)

(b) Hence, or otherwise, solve for $0 \quad x \quad 360^\circ$

$$2\sin(x \quad 10^\circ) = 5\cos(x + 20^\circ)$$

Give your answers to one decimal place.

(3)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 7 marks)

9.

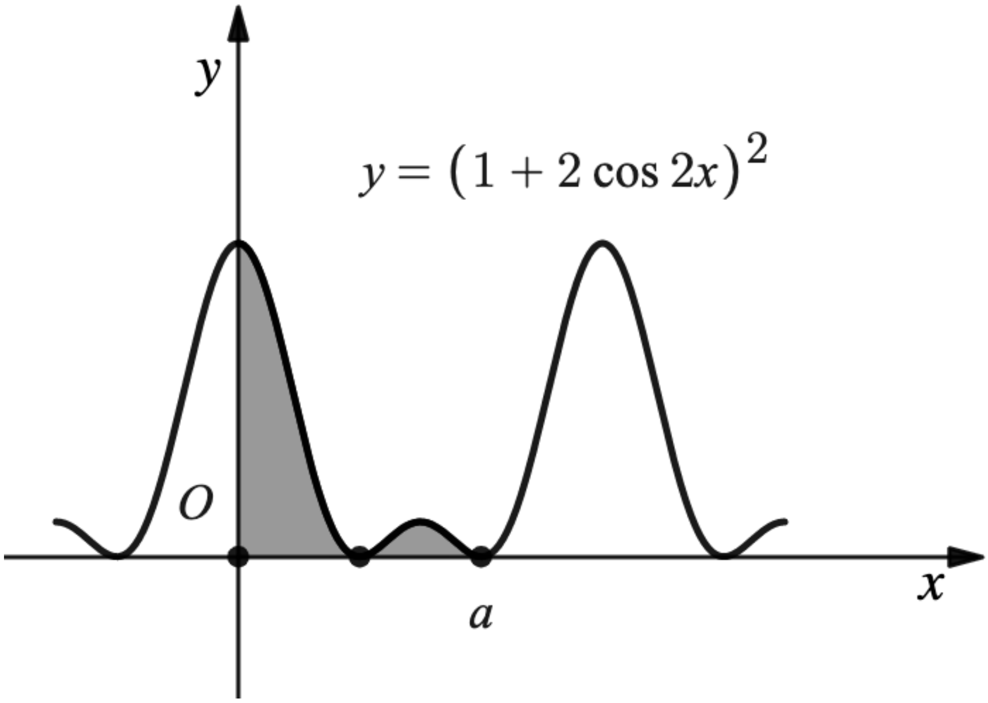


Figure 5: https://www.desmos.com/calculator/xcuvsoj08s

Figure 5 shows a sketch of the equation

$$y = (1 + 2\cos 2x)^2 \qquad \frac{\phantom{\pi}}{2} \quad x \quad \frac{3}{2}$$

(a) Show that

$$(1 + 2\cos 2x)^2 \quad p + q\cos 2x + r\cos 4x$$

where $p$, $q$ and $r$ are constants to be found.

(2)

The curve touches the positive $x$-axis for the **second** time when $x = a$.

The regions bounded by the curve, the $y$-axis and the $x$-axis up to $x = a$ are shown shaded in Figure 5.

(b) Use your answer to part (a), find, using algebraic integration and making your method clear, the exact total area of the shaded regions.
Write your answer in simplest form.

(5)

Question 9 continued

Question 9 continued

Question 9 continued

(Total for Question 9 is 7 marks)

**10.** **In this question you must show all stages of your working.**
**Solutions relying on calculator technology are not acceptable.**

Figure 6

A scientist, Nemo, is studying a population of fish in a lake.
The number of fish, $N$, in the population, $t$ years after the start of the study, is modelled by the equation

$$N = \frac{600e^{0.3t}}{2 + e^{0.3t}} \qquad \{t \geqslant 0\}$$

(a) Show that

$$\frac{dN}{dt} = \frac{Ae^{0.3t}}{\left(2 + e^{0.3t}\right)^2}$$

where $A$ is a constant to be found.

(3)

Given that when $t = T$, $\frac{dN}{dt} = 8$

(b) find the value of $T$ to one decimal place.

(4)

Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 7 marks)

**11.** The curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = x^2 + \ln(2x^2 \quad 4x + 5)$$

has a single turning point at $x =$

(a) Show that  is a solution for the equation

$$2x^3 \quad 4x^2 + 7x \quad 2 = 0$$

(4)

The iterative formula

$$x_{n+1} = \frac{1}{7}(2 + 4x_n^2 \quad 2x_n^3)$$

is used to find an approximate value for . Starting with $x_1 = 0{:}3$

(b) calculate, giving each answer to 4 decimal places,

(i) the value of $x_2$

(ii) the value of $x_4$

(3)

Using a suitable interval and a suitable function that should be stated,

(c) show that  is 0.341 to 3 decimal places.

(2)

Question 11 continued

Question 11 continued

Question 11 continued

(Total for Question 11 is 9 marks)

**12.** **In this question you must show all stages of your working. Solutions relying on calculator technology are not acceptable.**

(a) Express as an integral

$$\lim_{\delta x \to 0} \sum_{x=4}^{12} (1+2x)^{\frac{1}{2}} \, \delta x$$

(1)

(b) Use your answer to part (a) to show that

$$\lim_{\delta x \to 0} \sum_{x=4}^{12} (1+2x)^{\frac{1}{2}} \, \delta x = \frac{98}{3}$$

(3)

Question 12 continued

(Total for Question 12 is 4 marks)

**13.** Find

$$\int x^2 \ln x \, dx$$

(3)

Question 13 continued

(Total for Question 13 is 3 marks)

**14.**

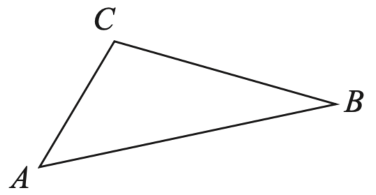


Figure 7

Figure 7 shows a sketch of triangle $ABC$.
Given that

- $\overrightarrow{AB} = \quad 3\mathbf{i} \quad 4\mathbf{j} \quad 5\mathbf{k}$
- $\overrightarrow{BC} = \mathbf{i} + \mathbf{j} + 4\mathbf{k}$

(a) Find $\overrightarrow{AC}$

(2)

(b) Show that $\cos \angle ABC = \dfrac{9}{10}$

(3)

Question 14 continued

Question 14 continued

Question 14 continued

Question 14 continued

(Total for Question 14 is 5 marks)

Total for paper is 100 marks