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Centre Number | Candidate Number

# Pearson Edexcel Level 3 GCE

**Time** 1 hour 30 minutes

**Paper reference** **9FM0/3C**

# Further Mathematics

**Advanced**

**PAPER 3C: Further Mechanics 1**

A. Chan
23. June, 2022

**You must have:**
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need*.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a numerical value of $g$ is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 8 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

*Turn over* ▶

**1.** A particle $A$ of mass $3m$ and a particle $B$ of mass $m$ are moving along the same straight line on a smooth horizontal surface. The particles are moving in opposite directions towards each other when they collide directly.

Immediately before the collision, the speed of $A$ is $ku$ and the speed of $B$ is $u$.
Immediately after the collision, the speed of $A$ is $v$ and the speed of $B$ is $2v$.

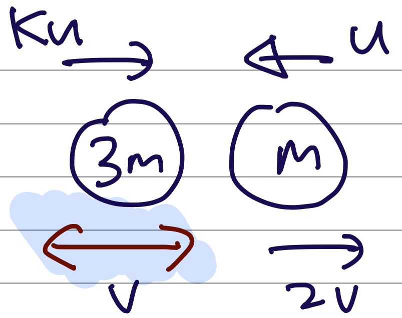
The magnitude of the impulse received by $B$ in the collision is $\frac{3}{2}mu$.

(a) Find $v$ in terms of $u$ only.

**(3)**

(b) Find the two possible values of $k$.

**(5)**




$2v > v$ therefore $\rightarrow$

or $v \leftarrow$

$I = mv - mu$

$= m(2v - -u)$

$\frac{3}{2}mu = m(2v+u)$

$v = \frac{u}{4}$ //

$3m(ku) - mu = 3mv + 2mv$ $\rightarrow$

$u(3k-1) = 5v$

$v = \frac{3k-1}{5}$

$k = \frac{3}{4}$

//

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**Question 1 continued**

$$3mku - mu = -3mv + 2mv$$

$$3k = (1 - \tfrac{1}{4})$$

$$k = \tfrac{1}{4}$$ //

**2.**

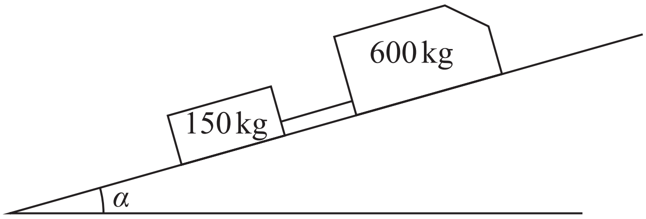



**Figure 1**

A van of mass 600 kg is moving up a straight road which is inclined at an angle $\alpha$ to the horizontal, where $\sin\alpha = \frac{1}{15}$. The van is towing a trailer of mass 150 kg. The van is attached to the trailer by a towbar which is parallel to the direction of motion of the van and the trailer, as shown in Figure 1.

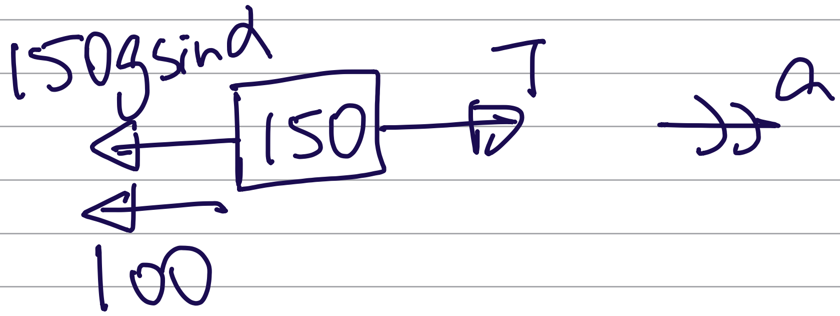
The resistance to the motion of the van from non-gravitational forces is modelled as a constant force of magnitude 200 N.
The resistance to the motion of the trailer from non-gravitational forces is modelled as a constant force of magnitude 100 N.

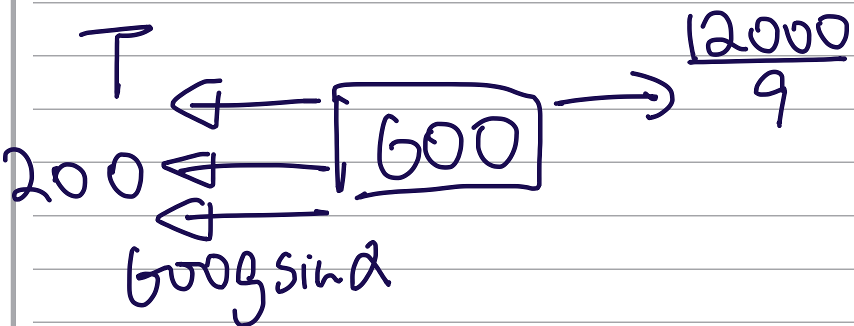
The towbar is modelled as a light rod.

The engine of the van is working at a constant rate of 12 kW.

Find the tension in the towbar at the instant when the speed of the van is $9\,\text{m}\,\text{s}^{-1}$

**(8)**




$\frac{12000}{9} - 200 - 100 - 150g\sin\alpha - 600g\sin\alpha = 750a$

$a = 0.724\ldots$

$a = \frac{163}{225}$ AO

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**Question 2 continued**

$T - 150g\sin\alpha - 100 = 150a$

oR

$\frac{12000}{9} - T - 200 - 600g\sin\alpha = 600a$

$T = 307N\ (3sf)$

$T = \frac{920}{3}$



P 7 2 0 9 2 R A 0 7 3 2

**3.**

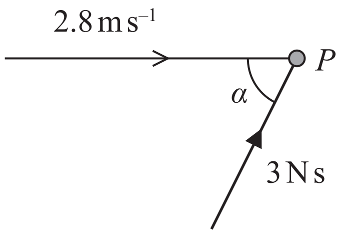
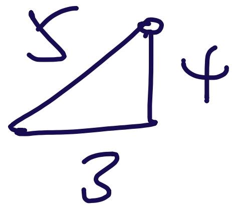



**Figure 2**




A particle $P$ of mass 0.5 kg is moving in a straight line with speed 2.8 m s$^{-1}$ when it receives an impulse of magnitude 3 N s.
The angle between the direction of motion of $P$ immediately before receiving the impulse and the line of action of the impulse is $\alpha$, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 2.

Find the speed of $P$ immediately after receiving the impulse.

**(5)**

$u = \begin{pmatrix} 2.8 \\ 0 \end{pmatrix}$ $I = \begin{pmatrix} 3\cos\alpha \\ 3\sin\alpha \end{pmatrix}$

$\cos\alpha = \frac{3}{5}$

$\sin\alpha = \frac{4}{5}$

$I = mv - mu$

$3\cos\alpha = 0.5(x - 2.8)$

$x = 32/5$

$3\sin\alpha = 0.5(y)$

$y = 24/5$

$\text{speed} = \sqrt{\left(\frac{32}{5}\right)^2 + \left(\frac{24}{5}\right)^2}$

$= 8 \text{ ms}^{-1}$

**4.**

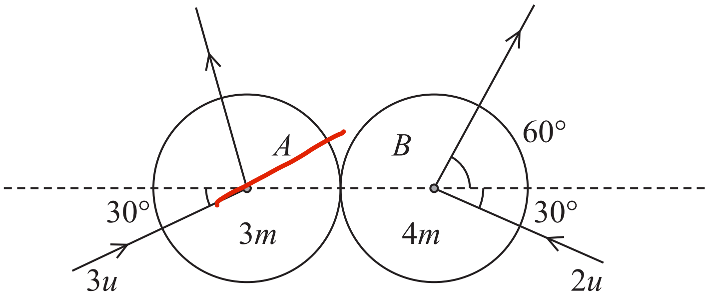



**Figure 3**

Two smooth uniform spheres, $A$ and $B$, have equal radii. The mass of $A$ is $3m$ and the mass of $B$ is $4m$. The spheres are moving on a smooth horizontal plane when they collide obliquely. Immediately before they collide, $A$ is moving with speed $3u$ at 30° to the line of centres of the spheres and $B$ is moving with speed $2u$ at 30° to the line of centres of the spheres. The direction of motion of $B$ is turned through an angle of 90° by the collision, as shown in Figure 3.

(i) Find the size of the angle through which the direction of motion of $A$ is turned as a result of the collision.

(ii) Find, in terms of $m$ and $u$, the magnitude of the impulse received by $B$ in the collision.

**(9)**

$\begin{pmatrix} -2u\cos 30 \\ 2u\sin 30 \end{pmatrix}$

$3u\cos 30$ $\quad$ $2u\cos 30$

3m $\rightarrow$ $\leftarrow$ 4m

$\leftarrow x$ $\quad$ $\rightarrow y$

$\begin{pmatrix} y \\ 2u\sin 30 \end{pmatrix}$

$3m(3u\cos 30) - 4m(2u\cos 30) = -3mx + 4my$

$-3x + 4y = \frac{\sqrt{3}}{2}u$

$V_{B\,before} \cdot V_{B\,After} = 0$

$(-2u\cos 30)(y) + (2u\sin 30)^2 = 0$

$y = \frac{\sqrt{3}}{3}u$ $\quad$ $x = \frac{5\sqrt{3}}{18}u$ //



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**Question 4 continued**

$$\begin{pmatrix} 3u\cos 30 \\ 3u\sin 30 \end{pmatrix} \text{ vs } \begin{pmatrix} -\frac{5\sqrt{3}}{18}u \\ 3u\sin 30 \end{pmatrix}$$

$$\tan\alpha = \tan 30 \qquad \tan\beta = -\frac{9\sqrt{3}}{5}$$

$$= \frac{\sqrt{3}}{3}$$

$$\tan(\alpha-\beta) = \frac{\frac{9\sqrt{3}}{5} + \frac{\sqrt{3}}{3}}{1 - \frac{9\sqrt{3}}{5}\cdot\frac{\sqrt{3}}{3}}$$

$$= \frac{-8\sqrt{3}}{3}$$

Angle of deflection = 77.8

anticlockwise

$$\begin{pmatrix} -2u\cos 30 \\ 2u\sin 30 \end{pmatrix}$$

$$I = mv - mu$$

$$\begin{pmatrix} y \\ 2u\sin 30 \end{pmatrix} \qquad I = 4m\begin{pmatrix} \frac{\sqrt{3}}{3}u + 2u\cos 30 \\ 2u\sin 30 - 2u\sin 30 \end{pmatrix}$$

$$I = \frac{16\sqrt{3}}{3}mu$$

**5.** Two particles, $P$ and $Q$, are moving in opposite directions along the same straight line on a smooth horizontal surface when they collide directly.
The mass of $P$ is $3m$ and the mass of $Q$ is $4m$.
Immediately before the collision the speed of $P$ is $2u$ and the speed of $Q$ is $u$.
The coefficient of restitution between $P$ and $Q$ is $e$.

(a) Show that the speed of $Q$ immediately after the collision is $\frac{u}{7}(9e + 2)$

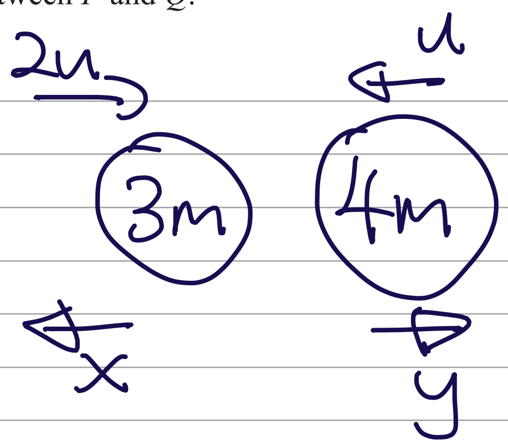
**(6)**

After the collision with $P$, particle $Q$ collides directly with a fixed vertical wall and rebounds. The wall is perpendicular to the direction of motion of $Q$.

The coefficient of restitution between $Q$ and the wall is $\frac{1}{2}$

(b) Find the complete range of possible values of $e$ for which there is a second collision between $P$ and $Q$.

**(4)**




$\frac{x+y}{3u} = e$ (CLM)

$6mu - 4mu = -3mx + 4my$

$2u = -3x + 4y$

$x = 3ue - y$

$\Rightarrow 2u = -3(3ue - y) + 4y$

**Question 5 continued**

$$u(2+9e)=7y$$

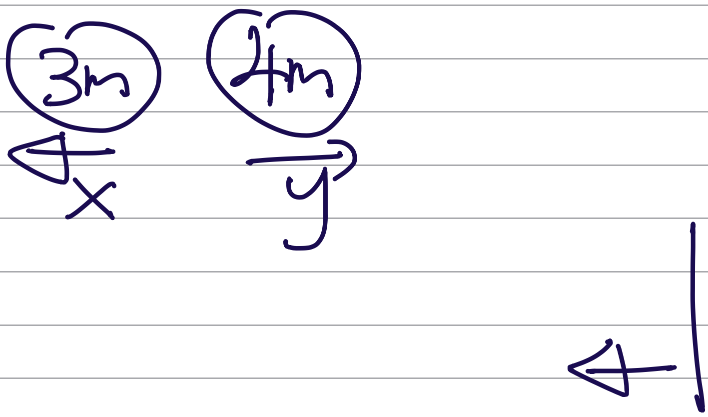
$$y=\frac{u}{7}(2+9e)$$

$$x=3ue-y$$

$$x=3ue-\frac{2u}{7}-\frac{9ue}{7}$$

$$x=u\left(\frac{12e}{7}-\frac{2}{7}\right)$$

$$x=\frac{2u}{7}(6e-1)$$




$$\frac{1}{2}y>x$$

$$\frac{u}{14}(2+9e)>\frac{2u}{7}(6e-1)$$

$$2+9e>24e-4$$

$$6>15e \qquad e<\frac{2}{5}$$

**Question 5 continued**

$0 < e < \frac{2}{5}$ //

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**6.**

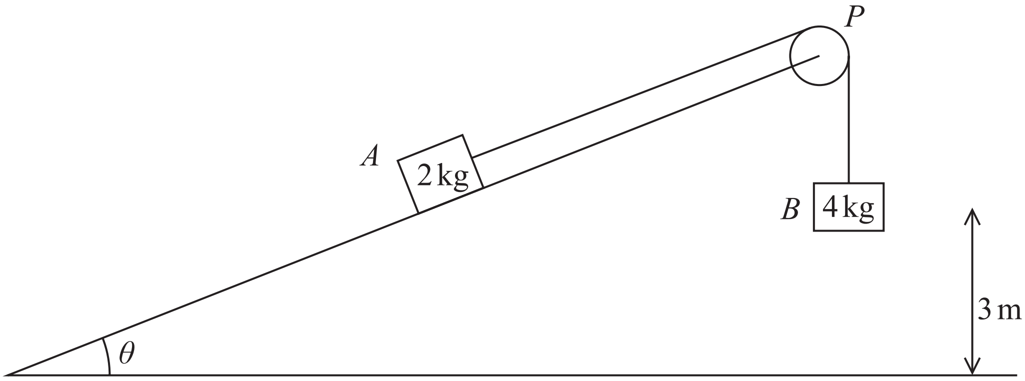



**Figure 4**

Two blocks, $A$ and $B$, of masses 2 kg and 4 kg respectively are attached to the ends of a light inextensible string.

Initially $A$ is held on a fixed rough plane. The plane is inclined to horizontal ground at an angle $\theta$, where $\tan\theta = \frac{3}{4}$

The string passes over a small smooth light pulley $P$ that is fixed at the top of the plane. The part of the string from $A$ to $P$ is parallel to a line of greatest slope of the plane.

Block $A$ is held on the plane with the distance $AP$ greater than 3 m.
Block $B$ hangs freely below $P$ at a distance of 3 m above the ground, as shown in Figure 4.

The coefficient of friction between $A$ and the plane is $\mu$

Block $A$ is released from rest with the string taut.

By modelling the blocks as particles,

(a) find the potential energy lost by the whole system as a result of $B$ falling 3 m.

**(3)**

Given that the speed of $B$ at the instant it hits the ground is $4.5\,\text{m}\,\text{s}^{-1}$ and ignoring air resistance,

(b) use the work-energy principle to find the value of $\mu$

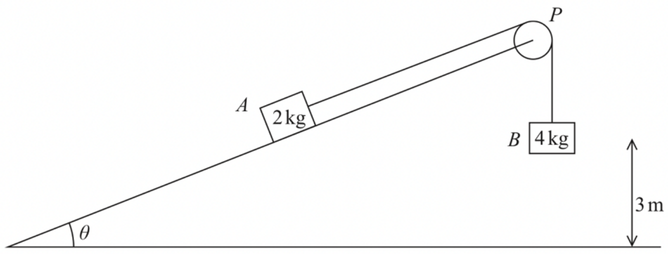
**(6)**

After $B$ hits the ground, $A$ continues to move up the plane but does not reach the pulley in the subsequent motion.
Block $A$ comes to instantaneous rest after moving a total distance of $(3 + d)$ m from its point of release.

Ignoring air resistance,

(c) use the work-energy principle to find the value of $d$

**(4)**

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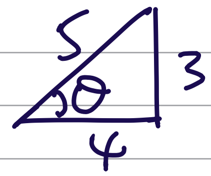


$$\Delta PE = 2g(3\sin\theta) - 4g(3)$$

$$= -6g\left(\tfrac{3}{5}\right) - 12g$$

$$= -\frac{42}{5}g = -82.32\text{ J} \quad (82.3 \text{ to } 3\text{sf})$$

$$82.32 = \tfrac{1}{2}(2)(v^2) + \tfrac{1}{2}(4)(v^2) + WD$$

$$R = 2g\cos\theta \qquad Fr = \mu 2g\left(\tfrac{4}{5}\right)$$

$$\mu = 0.4585459\ldots = \frac{719}{1568}$$

$$\mu \approx 0.459 \ (3\text{sf})$$

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$$\tfrac{1}{2}(2)(4.5)^2 = mg(d\sin\theta) + 2\mu g\left(\tfrac{4}{5}\right)d$$

$$d = \frac{405}{379} = 1.0686\ldots = 1.07 \ (3\text{sf})$$

**7.** A spring of natural length $a$ has one end attached to a fixed point $A$. The other end of the spring is attached to a package $P$ of mass $m$.
The package $P$ is held at rest at the point $B$, which is vertically below $A$ such that $AB = 3a$.
After being released from rest at $B$, the package $P$ first comes to instantaneous rest at $A$.
Air resistance is modelled as being negligible.

By modelling the spring as being light and modelling $P$ as a particle,

(a) show that the modulus of elasticity of the spring is $2mg$

**(5)**

(b) (i) Show that $P$ attains its maximum speed when the extension of the spring is $\frac{1}{2}a$

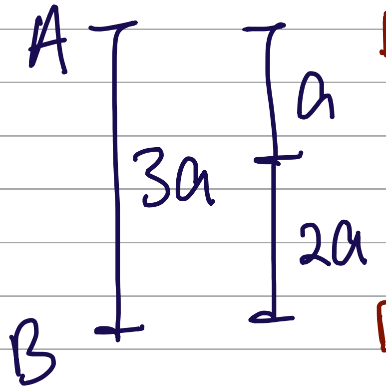
(ii) Use the principle of conservation of mechanical energy to find the maximum speed, giving your answer in terms of $a$ and $g$.

**(6)**

In reality, the spring is not light.

(c) State one way in which this would affect your energy equation in part (b).

**(1)**

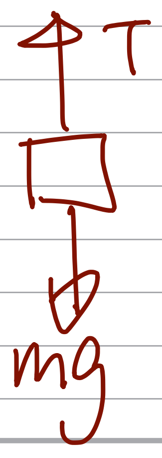



$\square$ $PE + EPE = mg(3a) + \frac{\lambda a^2}{2a}$

$\square$ $EPE = \frac{\lambda(2a)^2}{2a}$

$3mga + \frac{1}{2}\lambda a = 2\lambda a$

$\lambda = 2mg$ //




$\frac{\lambda x}{l} = mg$

$\frac{2mgx}{a} = mg$

Question 7 continued

$x = \frac{a}{2}$ //

$KE + EPE + PE = 2mg(2a)$

$\frac{1}{2}mv^2 + \frac{2mg\left(\frac{a}{2}\right)^2}{2a} + mg\left(2a - \frac{1}{2}a\right) = 4mga$

$m\frac{v^2}{2} = \frac{9}{4}mga$

$v = \frac{3\sqrt{2}}{2}\sqrt{ga}$

$v = \frac{3}{2}\sqrt{2ga}$

(f) different extension gives different equilibrium.

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**8.**

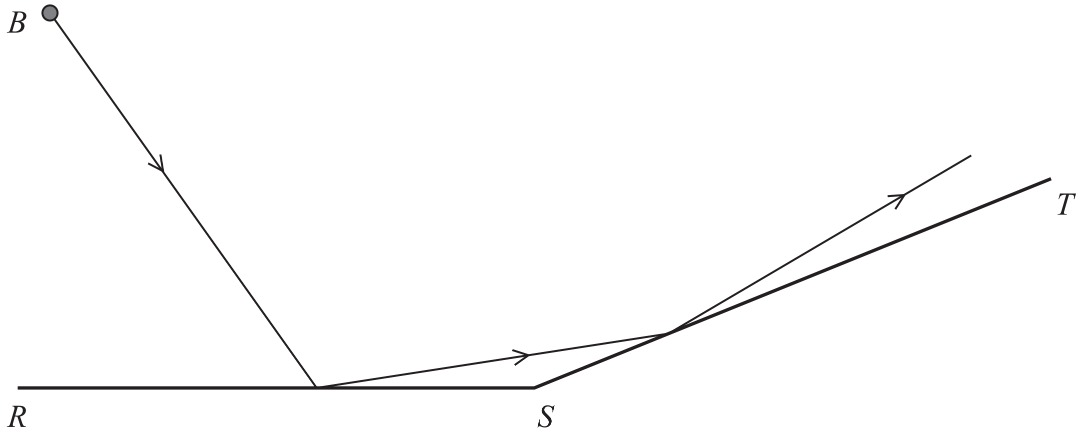



**Figure 5**

Figure 5 represents the plan view of part of a smooth horizontal floor, where $RS$ and $ST$ are smooth fixed vertical walls. The vector $\overrightarrow{RS}$ is in the direction of $\mathbf{i}$ and the vector $\overrightarrow{ST}$ is in the direction of $(2\mathbf{i} + \mathbf{j})$.

A small ball $B$ is projected across the floor towards $RS$. Immediately before the impact with $RS$, the velocity of $B$ is $(6\mathbf{i} - 8\mathbf{j})\,\mathrm{m\,s^{-1}}$. The ball bounces off $RS$ and then hits $ST$.

The ball is modelled as a particle.

Given that the coefficient of restitution between $B$ and $RS$ is $e$,

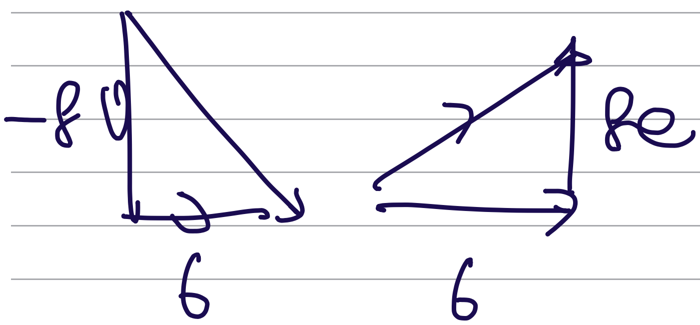
(a) find the full range of possible values of $e$.

**(3)**

It is now given that $e = \frac{1}{4}$ and that the coefficient of restitution between $B$ and $ST$ is $\frac{1}{2}$

(b) Find, in terms of $\mathbf{i}$ and $\mathbf{j}$, the velocity of $B$ immediately after its impact with $ST$.

**(7)**




$m = \frac{8e}{6} = \frac{4e}{3}$

$\frac{4e}{3} < \frac{1}{2}$

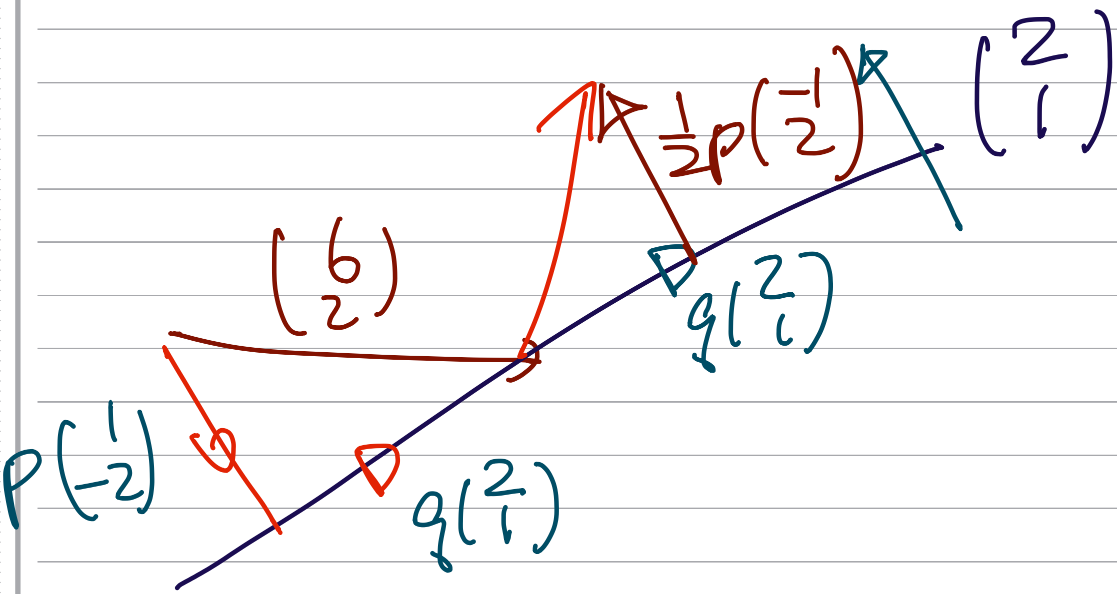
$0 < e < \frac{3}{8}$

**Question 8 continued**

(b) $e = \frac{1}{4}$

$$V_{Before} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$$

$$V_{after} = ?$$




$$\begin{pmatrix} 6 \\ 2 \end{pmatrix} = p\begin{pmatrix} 1 \\ -2 \end{pmatrix} + q\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$p + 2q = 6 \qquad p = \frac{2}{5},\ q = \frac{14}{5}$$
$$-2p + q = 2$$

**Question 8 continued**

$$V_B \text{ After} = \frac{14}{5}\begin{pmatrix}2\\1\end{pmatrix} + \frac{1}{2}\left(\frac{2}{5}\right)\begin{pmatrix}-1\\2\end{pmatrix}$$

$$= \frac{14}{5}(2) + \frac{1}{2}\left(\frac{2}{5}\right)(-1)\ \mathbf{i}$$

$$+$$

$$\frac{14}{5}(1) + \frac{1}{2}\left(\frac{2}{5}\right)(2)\ \mathbf{j}$$

$$= \frac{27}{5}\mathbf{i} + \frac{16}{5}\mathbf{j}$$