SOHOKMATHS

Year 13 (in class test)
Differentiation (Skills based only)
you CHAN do it

Time: 36 minutes

Surname CHAN Other names Andrew

☐ M2E Mr Chan/Ms Esteban Ruiz ☐ MaB Mr Chan/Mr Phillips

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill at the top of this page with your name, and tick the box with the class you belong to.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 4 questions in this question paper. The total mark for this paper is 30.
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Question	Marks	Score
1	7	
2	6	
3	9	
4	8	
Total:	30	

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2}\right)$, where p is a constant,

(a) find the exact value of p

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

a) $x = \left[4\left(\frac{\pi}{2}\right) - \sin(\pi)\right]^2$

$x = 4\pi^2$

b) $\frac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$

$y = \frac{\pi}{2}$

$\frac{dx}{dy} = 24\pi$

line: $\left(y - \frac{\pi}{2}\right) = \frac{1}{24\pi}(x - 4\pi^2)$

when $x = 0$

$y = \frac{\pi}{3}$ //   A: $\left(0, \frac{\pi}{3}\right)$ //

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2.

$$f(x) = \frac{(2x+5)^2}{x-3} \quad x \neq 3$$

(a) Find $f'(x)$ in the form $\dfrac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are fully factorised quadratic expressions.

(4)

(b) Hence find the range of values of x for which $f(x)$ is increasing.

(2)

a)

$$f'(x) = \frac{(x-3)(2)(2x+5)(2) - (2x+5)^2(1)}{(x-3)^2}$$

$$f'(x) = \frac{(2x+5)(4x-12-2x-5)}{(x-3)^2}$$

$$= \frac{(2x+5)(2x-17)}{(x-3)^2}$$

b)

$$\frac{dy}{dx} > 0$$

$$(2x+5)(2x-17) > 0$$

$$x < -\frac{5}{2}, \quad x > \frac{17}{2}$$

//

3.

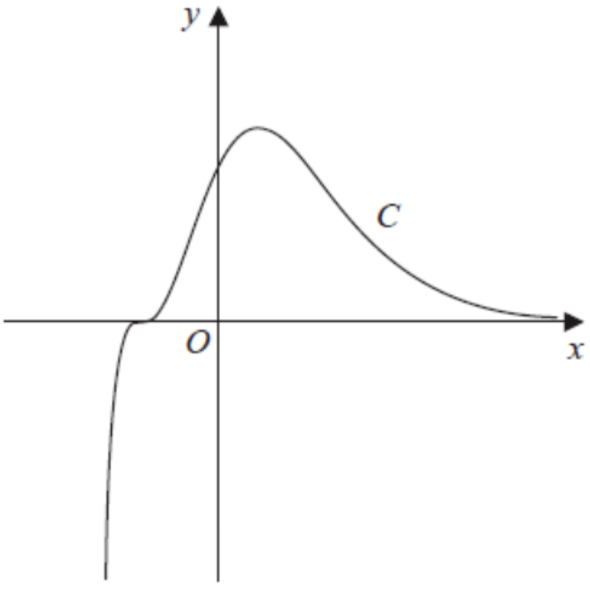

Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$f(x) = (2x+1)^3 e^{-4x}$$

(a) Show that

$$f'(x) = A(2x+1)^2(1-4x)\mathrm{e}^{-4x}$$

where A is a constant to be found.

(4)

(b) Hence find the exact coordinates of the two stationary points on C.

(3)

The function g is defined by

$$g(x) = 8f(x-2)$$

(c) Find the coordinates of the maximum stationary point on the curve with equation $y = g(x)$.

(2)

a) $f'(x) = (2x+1)^3(-4e^{-4x}) + (e^{-4x})(6)(2x+1)^2$

$f'(x) = (e^{-4x})(2x+1)^2\left[(2x+1)(-4)+6\right]$

$= (2x+1)^2(e^{-4x})\left[-8x+2\right]$

$$= (2x+1)^2 (e^{-4x}) [2] [1-4x]$$

$$= 2(2x+1)^2 (1-4x)(e^{-4x})$$ as req

b) $x = \frac{-1}{2}$, $x = \frac{1}{4}$

$y = 0$ $y = \frac{27}{8e}$

$\left(\frac{-1}{2}, 0\right)$, $\left(\frac{1}{4}, \frac{27}{8e}\right)$

c) $\left(\frac{9}{4}, \frac{27}{e}\right)$ //

4.

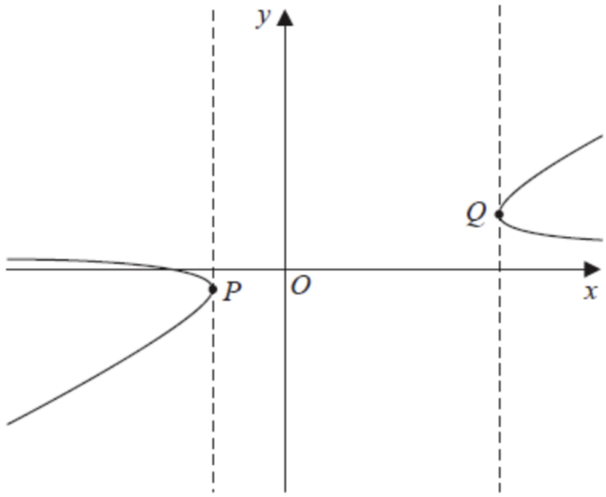

Figure 2

Figure 2 shows a sketch of the curve with equation

$$x = \frac{2y^2 + 6}{3y - 3} = \frac{2(y^2+3)}{3(y-1)}$$

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ giving your answer as a fully simplified fraction.

(4)

The tangents at points P and Q on the curve are parallel to the y-axis, as shown in Figure 2.

(b) Use the answer to part (a) to find the equations of these two tangents.

(4)

Total for paper is 30 marks

a) $$\frac{dx}{dy} = \frac{2}{3}\left[\frac{(y-1)[2y]-[y^2+3][1]}{(y-1)^2}\right]$$

$$\frac{dx}{dy} = \frac{2}{3}\left[\frac{2y^2-2y-y^2-3}{(y-1)^2}\right]$$

$$\frac{dx}{dy} = \frac{2}{3}\left[\frac{y^2-2y-3}{(y-1)^2}\right] = \frac{2}{3}\,\frac{(y+1)(y-3)}{(y-1)^2} \quad //$$

b) $\frac{dx}{dy} = 0$

$y = -1$ or $y = 3$

$x = \frac{-4}{3}$ $\quad$ $x = 4$

$\left(\frac{-4}{3}, -1\right)$ or $(4, 3)$ //