# Year 13 Further Mathematics Mock Set#02b
# Further Pure FP1

- Advised to print in "A3-booklets", this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

This exam paper has 7 questions, for a total of 75 marks.

| Question | Marks | Score |
|---|---|---|
| 1 | 3 | |
| 2 | 6 | |
| 3 | 10 | |
| 4 | 7 | |
| 5 | 19 | |
| 6 | 14 | |
| 7 | 16 | |
| Total: | 75 | |

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**1.**

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y(xy - 1)$$

Given that when $x = 0$, $y = 1$

Use the approximation $\dfrac{y_1 - y_0}{h} \approx \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)_0$ to estimate the value of $y$ at $x = 0.1$.

**(3)**

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**Question 1 continued**

**(Total for Question 1 is 3 marks)**

**2.** Evaluate

$$\lim_{x \to 0} \left[ \frac{\cos 7x - 1}{x \sin x} \right]$$

**(6)**

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**Question 2 continued**

**(Total for Question 2 is 6 marks)**

**3.**

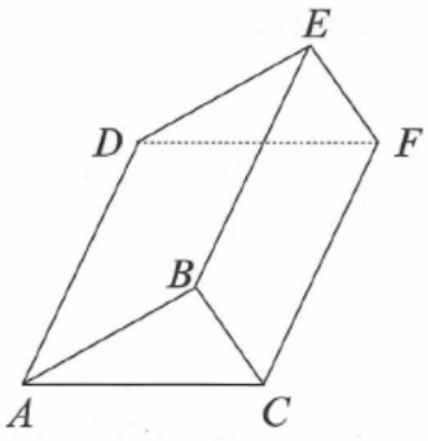


**Figure 1**

Figure 1 shows a right prism with triangular ends $ABC$ and $DEF$, parallel edges $AD$, $BE$, $CF$, and vertices $A(2, 7, -1), B(5, 8, 2), C(6, 7, 4)$ and $D(12, 1, -9)$.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

**(5)**

(b) Find $\overrightarrow{AD} \cdot (\overrightarrow{AB} \times \overrightarrow{AC})$.

**(3)**

(c) Find the volume of the prism.

**(2)**

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**Question 3 continued**

**Question 3 continued**

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**Question 3 continued**

**(Total for Question 3 is 10 marks)**

**4.** (a) On the same diagram, sketch the graphs of

$$y = |x - 5| \text{ and } y = |3x - 2|$$

distinguishing between them clearly.

**(2)**

(b) Find the set of values of $x$ for which

$$|x - 5| < |3x - 2|$$

**(5)**

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**Question 4 continued**

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**Question 4 continued**

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**Question 4 continued**

**(Total for Question 4 is 7 marks)**

**5.** Given that $y$ is a function of $t$ and that $t = \ln x \qquad \{x > 0\}$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}t} = x\frac{\mathrm{d}y}{\mathrm{d}x}$$

**(3)**

(b) Hence show that

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = x\frac{\mathrm{d}y}{\mathrm{d}x} + x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$$

**(4)**

(c) Using the results in part (a) and (b), show that the differential equation

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 3x\frac{\mathrm{d}y}{\mathrm{d}x} + 4y = 4\ln x \qquad \text{(I)}$$

transforms into the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}t^2} - 4\frac{\mathrm{d}y}{\mathrm{d}t} + 4y = 4t \qquad \text{(II)}$$

**(2)**

(d) Find the general solution of the differential equation (II).

**(5)**

(e) Given that $y = 0$ at $x = 1$ and that $y = 2$ at $x = \mathrm{e}$, find $y$ in terms of $x$.

**(5)**

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**Question 5 continued**

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**Question 5 continued**

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**Question 5 continued**

**(Total for Question 5 is 19 marks)**

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**6.** The rectangular hyperbola $H$ has equation

$$xy = c^2 \qquad \{c \neq 0\}$$

(a) Show that an equation of the tangent to $H$ at the point $P\left(ct, \frac{c}{t}\right)$ is

$$x + t^2y - 2ct = 0$$

**(5)**

The tangent at $P$ meets the $x$-axis at the point $Q$.
The mid-point of $PQ$ is $R$.

(b) Show that, as $P$ varies, the locus of $R$ is another rectangular hyperbola and find its equation.

**(9)**

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**Question 6 continued**

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**Question 6 continued**

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**Question 6 continued**

**(Total for Question 6 is 14 marks)**

**7.**

$$\mathrm{f}(x) = \frac{3}{13 + 6\sin x - 5\cos x}$$

Using the substitution $t = \tan\left(\frac{x}{2}\right)$.

(a) Show that $\mathrm{f}(x)$ can be written in the form

$$\frac{3(1+t^2)}{2(3t+1)^2+6}$$

**(3)**

(b) Hence solve, for $0 < x < 2\pi$, the equation

$$\mathrm{f}(x) = \frac{3}{7}$$

giving your answers to 2 decimal places where appropriate.

**(5)**

(c) Use the result of part (a) to show that

$$\int_{\frac{\pi}{3}}^{\frac{4\pi}{3}} \mathrm{f}(x)\,\mathrm{d}x = K\left(\arctan\left(\frac{\sqrt{3}-9}{3}\right) - \arctan\left(\frac{\sqrt{3}+3}{3}\right) + \pi\right)$$

where $K$ is a constant to be determined.

**(8)**

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**Question 7 continued**

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**Question 7 continued**

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**Question 7 continued**

**(Total for Question 7 is 16 marks)**

**Total for paper is 75 marks**

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