Name: ____

IAL C34 exponentials, year 12

Date:

Time:

Total marks available:

Total marks achieved: ______

Questions

Q1.

A hot piece of metal is dropped into a cool liquid. As the metal cools, its temperature T degrees Celsius, t minutes after it enters the liquid, is modelled by

$$T = 300e^{-0.04t} + 20, \quad t \geq 0$$

(a) Find the temperature of the piece of metal as it enters the liquid.

(1)

(b) Find the value of t for which $T = 180$, giving your answer to 3 significant figures.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(c) Show, by differentiation, that the rate, in degrees Celsius per minute, at which the temperature of the metal is changing, is given by the expression.

$$\frac{20 - T}{25}$$

(3)

(Total for question = 8 marks)

Q2.

A population of insects is being studied. The number of insects, N, in the population, is modelled by the equation

$$N = \frac{300}{3 + 17e^{-0.2t}} \quad t \in \mathbb{R},\ t \geqslant 0$$

where t is the time, in weeks, from the start of the study.

Using the model,

(a) find the number of insects at the start of the study,

(1)

(b) find the number of insects when $t = 10$,

(2)

(c) find the time from the start of the study when there are 82 insects.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

(d) Find, by differentiating, the rate, measured in insects per week, at which the number of insects is increasing when $t = 5$. Give your answer to the nearest whole number.

(3)

(Total for question = 10 marks)

Q3.

A bath is filled with hot water. The temperature, θ °C, of the water in the bath, t minutes after the bath has been filled, is given by

$$\theta = 20 + Ae^{-kt}$$

where A and k are positive constants.

Given that the temperature of the water in the bath is initially 38°C,

(a) find the value of A.

(2)

The temperature of the water in the bath 16 minutes after the bath has been filled is 24.5°C.

(b) Show that $k = \frac{1}{8}\ln 2$

(4)

Using the values for k and A,

(c) find the temperature of the water 40 minutes after the bath has been filled, giving your answer to 3 significant figures.

(2)

(d) Explain why the temperature of the water in the bath cannot fall to 19°C.

(1)

(Total for question = 9 marks)

Q4.

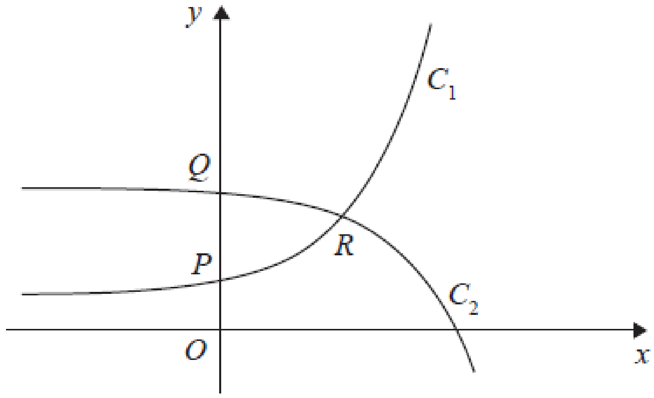

Figure 5

Figure 5 shows a sketch of the curves C_1 and C_2

C_1 has equation $y = 3 + e^{x+1} \quad x \in \mathbb{R}$

C_2 has equation $y = 10 - e^{x} \quad x \in \mathbb{R}$

Given that C_1 and C_2 cut the y-axis at the points P and Q respectively,

(a) find the exact distance PQ.

(2)

C_1 and C_2 intersect at the point R.

(b) Find the exact coordinates of R.

(5)

(Total for question = 7 marks)

Q5.

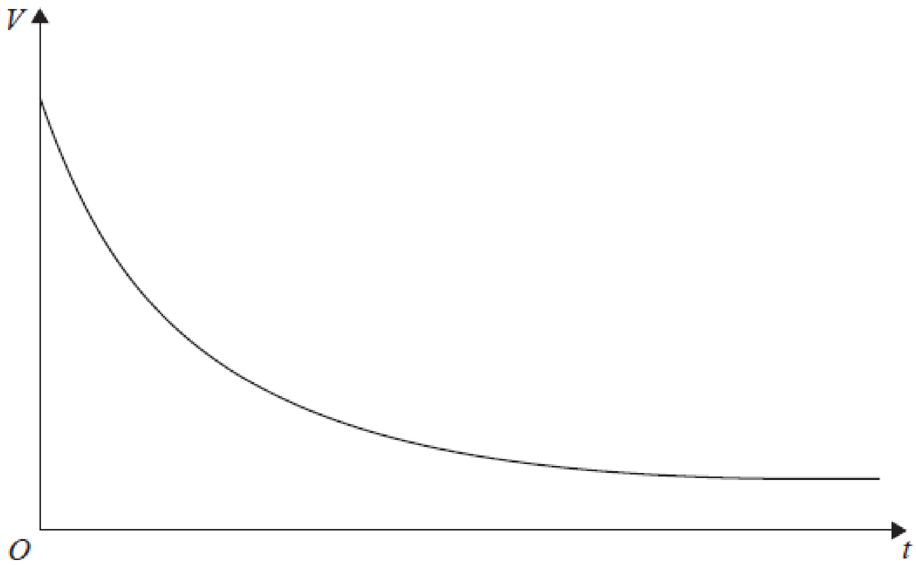

Figure 1

The value of Lin's car is modelled by the formula

$$V = 18\,000e^{-0.2t} + 4000e^{-0.1t} + 1000, \quad t \geqslant 0$$

where the value of the car is V pounds when the age of the car is t years.

A sketch of t against V is shown in Figure 1.

(a) State the range of V.

(2)

According to this model,

(b) find the rate at which the value of the car is decreasing when $t = 10$
Give your answer in pounds per year.

(3)

(c) Calculate the exact value of t when $V = 15000$.

(4)

(Total for question = 9 marks)

Q6.

The growth of pond weed on the surface of a pond is being investigated.

The surface area of the pond covered by the weed, A m^2, is modelled by the equation

$$A = \frac{1200e^{0.04t}}{4e^{0.04t} + 1} \qquad t \in \mathbb{R}, t \geqslant 0$$

where t is the number of weeks after the start of the investigation.

Using the model,

(a) calculate the surface area of the pond covered by the weed at the start of the investigation,

(1)

(b) calculate the value of t when **A** = 260, giving your answer to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

The pond weed continues to grow until it completely covers the surface of the pond.

Using the model,

(c) deduce the maximum possible surface area of the pond.

(1)

(Total for question = 6 marks)

Q7.

The mass, m grams, of a radioactive substance t years after first being observed, is modelled by the equation

$$m = 25e^{1-kt}$$

where k is a positive constant.

(a) State the value of m when the radioactive substance was first observed.

(1)

Given that the mass is 50 grams, 10 years after first being observed,

(b) show that $k = \frac{1}{10}\ln\left(\frac{1}{2}e\right)$

(4)

(c) Find the value of t when m = 20, giving your answer to the nearest year.

(3)

(Total for question = 8 marks)

Q8.

A scientist is studying a population of lizards on an island.

The number of lizards, N, in the population, t years after the start of the study, is modelled by the equation

$$N = \frac{1800}{2 + 3e^{-0.2t}} \qquad t \in \mathbb{R}, t \geqslant 0$$

Use the model to answer parts (a), (b), (c) and (d).

(a) Find the number of lizards in the population at the start of the study.

(1)

The model predicts an upper limit to the number of lizards on the island.

(b) State the value of this limit.

(1)

(c) Find the value of t when N = 780. Give your answer to one decimal place.

(4)

(d) (i) Show that the rate of growth, $\frac{dN}{dt}$, is given by

$$\frac{dN}{dt} = \frac{N(900 - N)}{A}$$

where A is a constant to be found.

(ii) Hence state the value of N at which the rate of growth is a maximum.

(5)

(Total for question = 11 marks)

Mark Scheme

Q1.

Question Number	Scheme	Marks
(a)	$320\ (°C)$	B1 [1]
(b)	$T=180 \Rightarrow 300e^{-0.04t}=160, \Rightarrow e^{-0.04t}=\frac{160}{300}(\text{awrt } 0.53)$	M1, A1
	$t=\frac{1}{-0.04}\ln\left(\frac{160}{300}\right)$ or $\frac{1}{0.04}\ln\left(\frac{300}{160}\right)$	dM1
	15.7 (minutes) cao	A1cso [4]
(c)	$\frac{dT}{dt}=(-0.04)\times 300e^{-0.04t}=(-0.04)\times(T-20)$	M1 A1
	$=\frac{20-T}{25}$ *	A1* [3]
		(8 marks)
Alt (b)	Puts $T=180$ so $180=300e^{-0.04t}+20$ and $300e^{-0.04t}=160$	M1
	$\ln 300-0.04t=\ln 160 \Rightarrow t=...,\quad \frac{\ln 300-\ln 160}{0.04}$	dM1, A1
	15.7 (minutes) cao	A1cso [4]

(a)

B1 320 cao - do not need ° C

(b)

M1 Substitutes T= 180 and proceeds to a form $Ae^{-0.04t}=B$ or $Ce^{0.04t}=D$

Condone slips on the power for this mark. For example condone $Ae^{-0.4t}=B$

A1 For $e^{-0.04t}=\frac{160}{300}$ or $e^{0.04t}=\frac{300}{160}$ or exact equivalent such as $e^{-0.04t}=\frac{8}{15}$

Accept decimals here $e^{-0.04t}=0.53..$ or $e^{0.04t}=1.875$

dM1 Dependent upon having scored the first M1, it is for moving from $e^{kt}=c, c>0 \Rightarrow t=\frac{\ln c}{k}$

A1 15.7 correct answer and correct solution only. Do not accept awrt

(c)

M1 Differentiates to give $\frac{dT}{dt}=ke^{-0.04t}$. Condone $\frac{dT}{dt}=ke^{-0.4t}$ following $T=300e^{-0.4t}+20$

This can be achieved from $T=300e^{-0.04t}+20 \Rightarrow t=\frac{1}{-0.04}\ln\left(\frac{T-20}{300}\right) \Rightarrow \frac{dt}{dT}=\frac{k}{(T-20)}$ for M1

A1 Correct derivative and correctly eliminates t to achieve $\frac{dT}{dt}=(-0.04)\times(T-20)$ oe

If candidate changes the subject it is for $\frac{dt}{dT}=\frac{-25}{(T-20)}$ oe

Alternatively obtains the correct derivative, substitutes T in $\frac{dT}{dt}=\frac{20-T}{25} \rightarrow \frac{dT}{dt}=-12e^{-0.04t}$ and compares. To score the A1* under this method there must be a statement.

A1* Obtains printed answer correctly – no errors

Q2.

Qu	Scheme	Marks
(a)	When $t=0$ $N=15$	B1 (1)
(b)	Puts $t=10$ so $N=$ 56.6 (accept 56 or 57)	M1A1 (2)
(c)	$82=\dfrac{300}{3+17e^{-0.2t}} \Rightarrow e^{-0.2t}=\dfrac{54}{1394}=\text{awrt } 0.039$	M1 A1
	$-0.2t=\ln\left(\dfrac{54}{1394}\right) \Rightarrow t=$	dM1
	$t=\text{awrt } 16.3$	A1 (4)
(d)	$\dfrac{dN}{dt}=(-0.2)\times300\times(-1)\times17e^{-0.2t}(3+17e^{-0.2t})^{-2}$	M1 A1
	=4.38 so 4 insects per week	A1 cso (3)
		(10 marks)

(a)
B1: 15 cao
(b)
M1: Substitutes $t=10$ into the correct formula. Sight of $N=\dfrac{300}{3+17e^{-0.2\times10}}$ is fine
A1: Accept 56 or 57 or awrt 56.6. These values would imply the M.
(c)
M1: Substitutes 82 and proceeds to obtain $e^{\pm0.2t}=C$ Condone slips on the power
A1: For $e^{-0.2t}=\dfrac{27}{697}$ oe $e^{0.2t}=\dfrac{697}{27}$ oe Accept decimals Eg $e^{-0.2t}=\text{awrt } 0.039$ or $e^{0.2t}=\text{awrt } 25.8$
dM1: Dependent upon previous M, scored for taking ln's (of a positive value) and proceeding to $t=$
A1: awrt 16.3 Accept 16 (weeks), 16.25 (weeks), 16 weeks 2 days or 17 weeks following correct log work and acceptable accuracy. Accept $t=5\ln\left(\dfrac{1394}{54}\right)$ *oe* for this mark

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It is possible to answer this by taking ln's at the point $1394e^{-0.2t}=54$
M1A1 $\ln(1394)-0.2t=\ln(54)$ dM1 A1 As scheme

(d)
M1: Differentiates to give a form equivalent to $\dfrac{dN}{dt}=ke^{-0.2t}(3+17e^{-0.2t})^{-2}$ (may use quotient rule)
A1: Correct derivative which may be unsimplified $\dfrac{dN}{dt}=1020e^{-0.2t}(3+17e^{-0.2t})^{-2}$
A1: Obtains awrt 4 **following a correct derivative**. This is cso

Q3.

Question Number	Scheme		Marks
(a)	$t=0, \theta=38 \Rightarrow 38=20+A\mathrm{e}^{-k\times 0}$	For substituting $t=0$ and $\theta=38$ into $\theta=20+A\mathrm{e}^{-kt}$	M1
	$\Rightarrow A=18$	Correct value for A	A1
	A = 18 with no working scores both marks		
			(2)
(b)	$t=16, \theta=24.5 \Rightarrow 24.5=20+\text{"}18\text{"}\mathrm{e}^{-k\times 16}$	For substituting $t=16$ and $\theta=24.5$ into $\theta=20+\text{their "}A\text{"}\mathrm{e}^{-kt}$	M1
	$\Rightarrow 18\mathrm{e}^{-k\times 16}=4.5$ or $\mathrm{e}^{-k\times 16}=\frac{1}{4}$	This mark is for a correct equation with the constants **combined.** Allow equivalent correct equations e.g. $\mathrm{e}^{16k}=4$	A1
	$\Rightarrow \mathrm{e}^{16k}=4 \Rightarrow 16k=\ln 4$ or $\Rightarrow \ln 18\mathrm{e}^{-k\times 16}=\ln 4.5 \Rightarrow \ln 18+\ln \mathrm{e}^{-k\times 16}=\ln 4.5 \Rightarrow \ln \mathrm{e}^{-k\times 16}=\ln\frac{1}{4}$ $\Rightarrow -16k=\ln\frac{1}{4}$ Uses correct log or exponential work to move from: $\mathrm{e}^{\pm nk}=C$ to $\pm nk=\alpha\ln C$ or $p\mathrm{e}^{\pm nk}=q$ to $\pm nk=\alpha\ln\beta$		M1
	$-16k=\ln\frac{1}{4} \Rightarrow k=-\frac{1}{16}\ln\frac{1}{4}=\frac{1}{8}\ln 2^*$ Shows that $k=\frac{1}{8}\ln 2$ There must be **at least one intermediate line** between their $\pm nk=\alpha\ln C$ or their $\pm nk=\alpha\ln\beta$ and the printed answer. So for example $-16k=\ln\frac{1}{4} \Rightarrow k=\frac{1}{8}\ln 2^*$ scores A0 as there is no intermediate line.		A1*
	Note: The marks in part (b) can be scored by using $\theta=20+A\mathrm{e}^{-kt}$ and substituting 2 out of: $A=18$, $\theta=24.5$, $k=\frac{1}{8}\ln 2$ to show that the 3rd variable is correct followed by a conclusion e.g. so $k=\frac{1}{8}\ln 2$		
			(4)

(c)	$t=40 \Rightarrow \theta=20+\text{"}18\text{"}\mathrm{e}^{-\frac{1}{8}\ln 2\times 40}$	Substitutes $t=40$ into the given equation with their A and the given value of k to obtain a value for θ	M1
	$\Rightarrow \theta=\text{awrt } 20.6\,(^\circ C)$	Awrt 20.6	A1
	Correct answer only scores both marks		
			(2)

(d)	**Examples:** • The lower limit is 20 • $\theta>20$ • As t tends to infinity temperature tends to 20 • The temperature cannot go below 20 • e^{-kt} tends towards zero so the temperature tends to 20 • e^{-kt} is always positive so the temperature is always bigger than 20 • Substitutes $\theta=19$ in $\theta=20+\text{"}18\text{"}\mathrm{e}^{-kt}$ (may be implied by e.g. $\mathrm{e}^{-kt}=-\frac{1}{18}$) and states e.g. that you cannot find the log of a negative number or "which is not possible" Do not accept e^{-kt} cannot be negative without reference to the "20"	B1
		(1)
		[9 marks]

Q4.

Question Number	Scheme		Marks
(a)	**Allow e^1 for e throughout.**		
	9 or $3+e$	For sight of either intercept 9 (not $10-e^0$) or $3+e$ or $3+e^1$ or $3+e^{0+1}$	M1
	Distance $PQ = 6-e$	$6-e$ (Not $9-(3+e)$)	A1
			(2)
(b)	Sets $3+e^{x+1}=10-e^x$	Equates the 2 curves	M1
	$e^x(e+1)=7$	Collects exponential terms and takes out a factor of e^x with correct index work.	M1
	$x=\ln\left(\frac{7}{1+e}\right)$ or $\ln\frac{7}{1+e}$	Correct x-coordinate	A1
	Substitutes their $x=\ln\left(\frac{7}{1+e}\right)$ in $y=10-e^x \Rightarrow y=..$ **Dependent upon both M's.** It is for substituting their value of x into either equation to find y		ddM1
	$R=\left(\ln\left(\frac{7}{1+e}\right),\frac{3+10e}{1+e}\right)$	$R=\left(\ln\left(\frac{7}{1+e}\right),\frac{3+10e}{1+e}\right)$ or equivalent such as $x=\ln\left(\frac{7}{1+e}\right)$, $y=10-\frac{7}{1+e}$ or $y=3+\frac{7e}{1+e}$ **but not** $y=10-e^{\ln\frac{7}{1+e}}$	A1
			(5)

(b) Way 2	$y=10-e^x \Rightarrow e^x=10-y \Rightarrow x=\ln(10-y) \Rightarrow y=3+e^{1+\ln(10-y)}$ Makes x the subject of equation 2 and substitutes into equation 1		M1
	$y=3+(10-y)e$	Uses correct index work to eliminate the "ln"	M1
	$\Rightarrow y=\frac{10e+3}{1+e}\left(\text{or } 10-\frac{7}{1+e}\right)$ or $y=3+\frac{7e}{1+e}$	Correct y-coordinate. **Not** $y=10-e^{\ln\frac{7}{1+e}}$	A1
	$x=\ln(10-y)=\ln\left(10-\frac{10e+3}{1+e}\right)$	Dependent upon both M's. It is for substituting their value of y into either equation to find x. **Dependent upon both M's.**	ddM1
	$R=\left(\ln\left(\frac{7}{1+e}\right),\frac{3+10e}{1+e}\right)$	$R=\left(\ln\left(\frac{7}{1+e}\right),\frac{3+10e}{1+e}\right)$ or equivalent such as $x=\ln\left(\frac{7}{1+e}\right)$, $y=10-\frac{7}{1+e}$ Allow $x=\ln\left(10-\frac{10e+3}{1+e}\right)$	A1

(b) Way 3	$y=3+e^{x+1} \Rightarrow y-3=e^{x+1} \Rightarrow e^x=\frac{y-3}{e} \Rightarrow 10-y=\frac{y-3}{e}$ Makes e^x the subject of equation 1 and substitutes into equation 2		M1
	$10-y=\frac{y-3}{e} \Rightarrow 10e-ye=y-3 \Rightarrow y(1+e)=10e+3$ Uses correct algebra and factorises y		M1
	$\Rightarrow y=\frac{10e+3}{1+e}\left(\text{or } 10-\frac{7}{1+e}\right)$ or $y=3+\frac{7e}{1+e}$	Correct y-coordinate	A1
	Then as above.		
(b) Way 4	$y=10-e^x \Rightarrow e^x=10-y \Rightarrow x=\ln(10-y)$ $y=3+e^{x+1} \Rightarrow e^{x+1}=y-3 \Rightarrow x+1=\ln(y-3)$ $y=3+e^{x+1} \Rightarrow e^{x+1}=y-3 \Rightarrow \ln(10-y)=\ln(y-3)-1$ Makes x and $x+1$ the subject and uses $x=x$		M1
	$\Rightarrow \frac{y-3}{(10-y)}=e$	Uses correct work to eliminate the "ln's"	M1
	$\Rightarrow y=\frac{10e+3}{1+e}\left(\text{or } 10-\frac{7}{1+e}\right)$ or $y=3+\frac{7e}{1+e}$	Correct y-coordinate	A1
	Then as above.		
			[7 marks]

If the candidate works in decimals throughout then the <u>method</u> marks are still available if no exact values are seen. NB: $3 + e = 5.71\ldots$, $6 - e = 3.28\ldots$, $\ln\left(\frac{7}{1+e}\right) = 0.632\ldots$, $\frac{10e+3}{1+e} = 8.11\ldots$

Q5.

Question Number	Scheme	Marks
(a)	$1000 < V \leqslant 23000$	B1,B1
		(2)
(b)	$\frac{dV}{dt} = 18000 \times -0.2e^{-0.2t} + 4000 \times -0.1e^{-0.1t}$	M1
	$\left.\frac{dV}{dt}\right\|_{t=10} = 18000 \times -0.2e^{-2} + 4000 \times -0.1e^{-1} = \text{awrt}(-)634$	M1A1
		(3)
(c)	$15000 = 18000e^{-0.2t} + 4000e^{-0.1t} + 1000$	
	$0 = 9e^{-0.2t} + 2e^{-0.1t} - 7$	
	$0 = (9e^{-0.1t} - 7)(e^{-0.1t} + 1)$	M1A1
	$9e^{-0.1t} = 7 \Rightarrow t = 10\ln\left(\frac{9}{7}\right)$ oe	dM1A1
		(4)
		(9 marks)

(a)

B1 Accept either boundary: $V < 23000$ or $V \leqslant 23000$ or $V_{\max} 23000$ for the upper boundary and $V > 1000$ or $V \geqslant 1000$ or $V_{\min} 1000$ for the lower boundary. Answers like $V \geqslant 23000$ are B0

B1 Completely correct solution.
Accept $1000 < V \leqslant 23000$, $1000 < Range$ or $y \leqslant 23000$, $(1000, 23000]$, $V > 1000$ *and* $V \leqslant 23000$

(b)

M1 Score for a $\frac{dV}{dt} = Ae^{-0.2t} + Be^{-0.1t}$, where $A \neq 18000$, $B \neq 4000$

M1 Sub $t = 10$ into a $\frac{dV}{dt}$ of the form $Ae^{-0.2t} + Be^{-0.1t}$ where $A \neq 18000$, $B \neq 4000$

Condone substitution of $t = 10$ into a $\frac{dV}{dt}$ of the form $Ae^{-0.2t} + Be^{-0.1t} + 1000$ $A \neq 18000$, $B \neq 4000$

A1 Correct solution and answer only. Accept ± 634 following correct $\frac{dV}{dt} = -3600e^{-0.2t} - 400e^{-0.1t}$

Watch for students who sub $t = 10$ into their V first and then differentiate. This is 0,0,0.

Watch for students who achieve +634 following $\frac{dV}{dt} = 3600e^{-0.2t} + 400e^{-0.1t}$. This is 1,1,0

A correct answer with no working can score all marks.

(c)

M1 Setting up 3TQ in $e^{\pm 0.1t}$ AND correct attempt to factorise or solve by the formula.

For this to be scored the $e^{\pm 0.2t}$ term must be the x^2 term.

A1 Correct factors $(9e^{-0.1t} - 7)(e^{-0.1t} + 1)$ or $(7e^{0.1t} - 9)(e^{0.1t} + 1)$ or a root $e^{-0.1t} = \frac{7}{9}$

dM1 Dependent upon the previous M1.
This is scored for setting the $ae^{\pm 0.1t} - b = 0$ and proceeding using correct ln work to $t = ...$

A1 $t = 10\ln\left(\frac{9}{7}\right)$. Accept alternatives such as $t = \frac{1}{0.1}\ln\left(\frac{9}{7}\right), \frac{1}{-0.1}\ln\left(\frac{7}{9}\right), -10\ln\left(\frac{7}{9}\right)$

If any extra solutions are given withhold this mark.

Q6.

Question Number	Scheme	Marks
(a)	240 (m^2)	B1
		[1]
(b)	$260 = \dfrac{1200e^{0.04t}}{4e^{0.04t}+1} \Rightarrow 160e^{0.04t} = 260$	M1 A1
	$\Rightarrow 0.04t = \ln\left(\dfrac{260}{160}\right) \Rightarrow t = 12.14$	dM1 A1
		[4]
(c)	300(m^2)	B1
		[1]
		(6 marks)

(a)
B1 240(m^2) Units are not required
(b)

M1 Proceeds from $260 = \dfrac{1200e^{0.04t}}{4e^{0.04t}+1}$ to a form $Pe^{\pm 0.04t} = Q$

Condone P or Q negative. Condone slips

A1 $160e^{0.04t} = 260$ or exact equivalent for example $e^{0.04t} = \dfrac{13}{8}$.

dM1 Dependent upon the first M. It is for correctly taking lns and proceeding to a value of t.
Eg proceeding from $e^{0.04t} = \ldots$ to $0.04t = \ln(\ldots)$ to $t = \ldots$
$160e^{0.04t} = 260 \Rightarrow \ln 160 + 0.04t = \ln 260 \Rightarrow t = \ldots$ is also fine.
It can only be scored from a solvable equation E.g $Pe^{\pm 0.04t} = Q,\quad P, Q > 0$

A1 Awrt 12.14. Allow for the exact answer of $25\ln\left(\dfrac{13}{8}\right)$ or $25\ln\left(\dfrac{a}{b}\right)$ where $\dfrac{a}{b} \equiv \dfrac{13}{8}$

(c)
B1 300(m^2) Units are not required. Condone 299.99

Part (b) No working

Going from $260 = \dfrac{1200e^{0.04t}}{4e^{0.04t}+1}$ to 12.14 scores SC 1000

Q7.

Question Number	Scheme	Marks
(a)	$25e$ or equivalent decimal - Accept awrt 68	B1
		(1)
(b)	$50 = 25e^{1-10k}$	M1
	Way 1 $e^{1-10k} = 2$; Way 2 $e^{-10k} = 2/e$; Way 3 $e^{10k} = e/2$	A1
	Way 1 $\Rightarrow 1-10k = \ln 2$; Way 2 $-10k = \ln(2/e)$; Way 3 $10k = \ln(e/2)$	M1
	Way 1 $\Rightarrow k = \dfrac{\ln e - \ln 2}{10}$; Way 2 $k = \dfrac{-\ln\left(\frac{2}{e}\right)}{10}$; Way 3 No intermediate step needed	
	$\Rightarrow k = \dfrac{\ln\left(\frac{1}{2}e\right)}{10}$ *	A1*
		(4)
(c)	Uses $m = 20$ and their numerical k so $20 = 25e^{1-'k't} \Rightarrow e^{1-'k't} = 0.8$ o.e.	M1
	$\Rightarrow t = \dfrac{1-\ln 0.8}{'k'}$	dM1
	$\Rightarrow t =$ awrt 40 (years)	A1
		(3)
		(8 marks)

Notes for question

(a) B1 for 25e or for numerical answer, e.g. 67.957 – allow awrt 68

(b) M1 Uses $t=10$, $m=50$, in $m=25e^{1-kt}$ to give $50=25e^{1-10k}$

A1 $e^{1-10k}=2$ (way 1) or $e^{-10k}=2/e$ o.e. (way 2) or $e^{10k}=e/2$ (way 3) or $e^{10k-1}=\frac{1}{2}$ (variant on Way 1) to give a correct equation $e^{f(k)}=B$. Some solutions will move from one of these options to another by sound algebra – this is acceptable.

M1 Taking logs **correctly** to give $f(k)=\ln B$
i.e. $1-10k=\ln 2$ (way 1) or $-10k=\ln(2/e)$ o.e. (way 2) or $10k=\ln(e/2)$ (way 3) or $10k-1=\ln(1/2)$ (way 4*)
(There are a number of correct alternatives but this line should follow directly from the previous one) This must be a correct equation.

A1* cso- Needs both M marks, everything should have been correct and exact. Makes k the subject of the Formula. Needs an intermediate step for ways 1 and 2 but not for way 3.

e.g. $k=\dfrac{\ln e-\ln 2}{10}$ (Way 1 or Way 4*) or $k=\dfrac{-\ln(2/e)}{10}$ (Way2) or straight to

$k=\dfrac{\ln\left(\frac{1}{2}e\right)}{10}$ (Way 3)

Must conclude with the printed answer $k=\dfrac{\ln\left(\frac{1}{2}e\right)}{10}$ or $k=\dfrac{\ln\left(\frac{e}{2}\right)}{10}$ or $k=\dfrac{1}{10}\ln\left(\dfrac{e}{2}\right)$ o.e.

Special Case Taking the mass as 50 + 25e in part (b) should be treated as misread. Can earn M1A0M1A0 and obtains $k=\dfrac{1-\ln(2+e)}{10}$

(c) M1 Uses $m=20$ and their numerical k in $m=25e^{1-kt}$ $\Rightarrow e^{1-'k't}=0.8$ (NB $k=0.030685\ldots$)

NB $\Rightarrow e^{(1-'k')t}=0.8$ is M0 (usually $e^{0.97t}=0.8$)

dM1 Use of correct work to reach $\Rightarrow t=\dfrac{1-\ln 0.8}{'k'}$ or equivalent e.g. $\Rightarrow t=\dfrac{\ln\left(\frac{5e}{4}\right)}{'k'}$

A1 Allow awrt 40 (may see 39.86 or 39.9). (Decimals are acceptable in part (c)) Do not allow -40

Special Case
If the answer 40 appears with no working or after minimal working where no marks have been scored then award M1M0A0 – special case.
If the first M mark in (c) has been awarded and they give the answer 40 with no further working, then award M1M1A1

Q8.

Question Number	Scheme		Marks
(a)	360	Cao. No need for $N =$ … just look for the correct value	B1
			(1)
(b)	900	Cao. No need for $N =$ … just look for the correct value. Allow e.g. $N < 900$	B1
			(1)
(c)	$780 = \dfrac{1800}{2+3\mathrm{e}^{-0.2t}} \Rightarrow 2340\mathrm{e}^{-0.2t} = 240$	Substitutes $N = 780$ and proceeds to $A\mathrm{e}^{\pm 0.2t} = B$, where A and B are both positive or both negative	M1
	$2340\mathrm{e}^{-0.2t} = 240$	Correct equation oe e.g. $\mathrm{e}^{-0.2t} = \dfrac{4}{39}$, $3\mathrm{e}^{-0.2t} = \dfrac{4}{13}$, $\mathrm{e}^{0.2t} = \dfrac{39}{4}$	A1
	$2340\mathrm{e}^{-0.2t} = 240 \Rightarrow \mathrm{e}^{-0.2t} = \dfrac{4}{39} \Rightarrow -0.2t = \ln\left(\dfrac{4}{39}\right) \Rightarrow t = \ldots$ **or** $2340\mathrm{e}^{-0.2t} = 240 \Rightarrow \ln 2340\mathrm{e}^{-0.2t} = \ln 240 \Rightarrow \ln \mathrm{e}^{-0.2t} = \ln 240 - \ln 2340$ $\mathrm{e}^{-0.2t} = \dfrac{4}{39} \Rightarrow -0.2t = \ln\left(\dfrac{4}{39}\right) \Rightarrow t = \ldots$ This mark is for <u>fully correct processing</u> from $A\mathrm{e}^{\pm 0.2t} = B$ to obtain a value for t **Dependent on the first method mark**		dM1
	$(t =)\ 11.4$	For awrt 11.4	A1
			(4)

(d)(i) Way 1	Quotient: $\dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{\left(2+3\mathrm{e}^{-0.2t}\right)\times 0 - 1800 \times -0.6 \times \mathrm{e}^{-0.2t}}{\left(2+3\mathrm{e}^{-0.2t}\right)^2}$ Chain: $\dfrac{\mathrm{d}N}{\mathrm{d}t} = -1800 \times -0.6 \times \mathrm{e}^{-0.2t}\left(2+3\mathrm{e}^{-0.2t}\right)^{-2}$ M1: For obtaining a derivative of the form $\dfrac{A\mathrm{e}^{-0.2t}}{\left(2+3\mathrm{e}^{-0.2t}\right)^2}$ A1: Correct derivative in any form which may be unsimplified as above. Often seen as $\dfrac{1080\mathrm{e}^{-0.2t}}{\left(2+3\mathrm{e}^{-0.2t}\right)^2}$		M1 A1
	$\Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{1800 \times 0.6 \times \frac{1}{3}\left(\frac{1800}{N} - 2\right)}{\left(\frac{1800}{N}\right)^2}$ A full attempt to get $\dfrac{\mathrm{d}N}{\mathrm{d}t}$ in terms of N. Both $\mathrm{e}^{-0.2t}$ and $\left(2+3\mathrm{e}^{-0.2t}\right)^2$ must be replaced by a function of N. **Dependent on the first method mark**		dM1
	$\Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{N(900-N)}{4500}$ $\therefore A = 4500$	$\dfrac{\mathrm{d}N}{\mathrm{d}t} = \dfrac{N(900-N)}{4500}$	A1

(d)(i) Way 2	$N=\frac{1800}{2+3e^{-0.2t}}\Rightarrow N\left(2+3e^{-0.2t}\right)=1800\Rightarrow\left(2+3e^{-0.2t}\right)\frac{dN}{dt}+N\left(-0.6e^{-0.2t}\right)=0$ M1: $\left(2+3e^{-0.2t}\right)\frac{dN}{dt}+Ae^{-0.2t}=0$ A1: Correct equation **or** $N=\frac{1800}{2+3e^{-0.2t}}\Rightarrow 2N+3Ne^{-0.2t}=1800\Rightarrow 2\frac{dN}{dt}+3e^{-0.2t}\frac{dN}{dt}+N\left(-0.6e^{-0.2t}\right)=0$ M1: $A\frac{dN}{dt}+Be^{-0.2t}\frac{dN}{dt}+CNe^{-0.2t}=0$ A1: Correct equation	M1A1
	$\frac{dN}{dt}=\frac{0.6Ne^{-0.2t}}{2+3e^{-0.2t}}=\frac{0.6N\left(\frac{1800}{3N}-\frac{2}{3}\right)}{\frac{1800}{N}}$ Makes $\frac{dN}{dt}$ the subject and a full attempt to get $\frac{dN}{dt}$ in terms of N. Both $e^{-0.2t}$ and $2+3e^{-0.2t}$ must be replaced by a function of N. **Dependent on the first method mark**	dM1
	$\Rightarrow\frac{dN}{dt}=\frac{N(900-N)}{4500}\quad\therefore A=4500$ | $\frac{dN}{dt}=\frac{N(900-N)}{4500}$	A1

(d)(i) Way 3	$N=\frac{1800}{2+3e^{-0.2t}}\Rightarrow 2N+3Ne^{-0.2t}=1800\Rightarrow e^{-0.2t}=\frac{1800-2N}{3N}$ $\Rightarrow -0.2t=\ln\left(\frac{1800-2N}{3N}\right)\Rightarrow\frac{dt}{dN}=-5\times\left(\frac{3N}{1800-2N}\right)\times -600N^{-2}$ M1: For an attempt to make t or $-0.2t$ the subject and then applies the chain rule to obtain $\frac{dt}{dN}$ A1: Correct derivative in any form	M1 A1
	$\Rightarrow\frac{dN}{dt}=\frac{(1800-2N)N^2}{9000}$ A full attempt to get $\frac{dN}{dt}$ in terms of N. **Dependent on the first method mark**	dM1
	$\Rightarrow\frac{dN}{dt}=\frac{N(900-N)}{4500}\quad\therefore A=4500$ | $\frac{dN}{dt}=\frac{N(900-N)}{4500}$	A1
(ii)	$N=450$ | Cao	B1
		(5)
		[11marks]