# 13Fm Further Mechanics (FM1) Mock Paper 2021.05.28

- Advised to print in "A3-booklets", this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

This exam paper has 7 questions, for a total of 75 marks.

| Question | Marks | Score |
|---|---|---|
| 1 | 7 | |
| 2 | 8 | |
| 3 | 8 | |
| 4 | 14 | |
| 5 | 12 | |
| 6 | 11 | |
| 7 | 15 | |
| Total: | 75 | |

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**1.** A particle $P$ of mass 1.5 kg is moving with velocity $(4\mathbf{i} + 6\mathbf{j})$ ms$^{-1}$ when it receives an impulse of 15 Ns. Immediately after $P$ receives the impulse, the velocity of $P$ is $v\mathbf{i}$ ms$^{-1}$

Find the two possible values of $v$.

**(7)**

**Question 1 continued**

**(Total for Question 1 is 7 marks)**

**2.** A small box is projected with speed 7 $\text{ms}^{-1}$ from a point $O$ on a fixed rough inclined plane. The plane is inclined to the horizontal at an angle $\alpha$, where $\tan\alpha = \frac{3}{4}$.

The box moves up a line of greatest slope of the plane and comes to instantaneous rest at the point $A$. The coefficient of friction between the box and the plane is $\frac{1}{4}$.

In a model of the motion, the box is modelled as a particle.

(a) Show that, after coming to rest at the point $A$, the box immediately slides back down the plane.

**(2)**

The speed of the box at the instant when it returns to $O$ is $V$ $\text{ms}^{-1}$.

Given that $OA = \frac{25}{8}$ m,

(b) use the work-energy principle to find the value of $V$.

**(4)**

(c) Suggest two ways in which the model can be refined to make it more realistic.

**(2)**

**Question 2 continued**

**Question 2 continued**

**Question 2 continued**

**(Total for Question 2 is 8 marks)**

**3.** A car of mass 600 kg travels along a straight horizontal road with the engine of the car working at a constant rate of $P$ watts. The resistance to the motion of the car is modelled as a constant force of magnitude $R$ newtons. At the instant when the speed of the car is 15 $\text{ms}^{-1}$, the magnitude of the acceleration of the car is 0.2 $\text{ms}^{-2}$.

Later, the same car travels up a straight road inclined at angle $\theta$ to the horizontal, where $\sin\theta = \frac{1}{20}$. The resistance to the motion of the car from non-gravitational forces is again modelled as a constant force of magnitude $R$ newtons. When the engine of the car is working at a constant rate of $P$ watts, the car has a constant speed of 10 $\text{ms}^{-1}$.

Find the value of $P$.

(8)

**Question 3 continued**

**Question 3 continued**

**Question 3 continued**

**(Total for Question 3 is 8 marks)**

**4.**

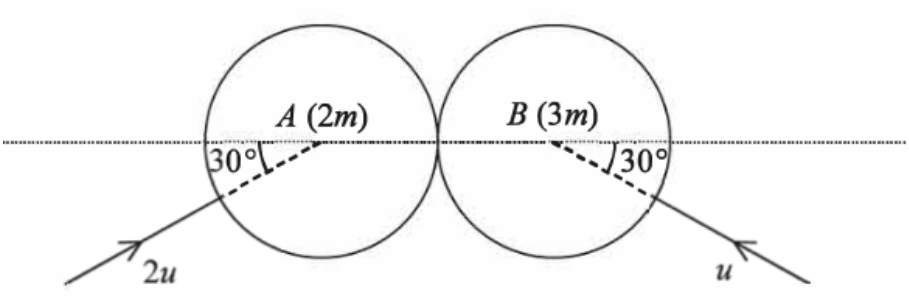


**Figure 1**

Two smooth uniform spheres, $A$ and $B$, with equal radii, have masses $2m$ and $3m$ respectively. The spheres are moving on a smooth horizontal surface when they collide obliquely. Immediately before the collision, $A$ is moving with speed $2u$ at 30° to the line of centres, and $B$ is moving with speed $u$ at 30° to the line of centres, as shown in Figure 1.

The direction of motion of $A$ immediately after the collision is perpendicular to the direction of motion of $A$ immediately before the collision. The direction of motion of $B$ is deflected through an angle $\theta$ as a result of the collision.

The coefficient of restitution between A and B is $e$.

(a) the speed of $A$ immediately after the collision,

**(2)**

(b) the size of angle $\theta$,

**(8)**

(c) the value of $e$.

**(3)**

(d) Explain how you have used the fact that the two spheres have equal radii in this question.

**(1)**

**Question 4 continued**

**Question 4 continued**

**Question 4 continued**

**(Total for Question 4 is 14 marks)**

**5.** Two particles, $A$ and $B$, have masses $3m$ and $4m$ respectively. The particles are moving towards each other along the same straight line on a smooth horizontal surface. The particles collide directly. Immediately after the collision, $A$ and $B$ are moving in the same direction with speeds $\frac{u}{3}$ and $u$ respectively. In the collision, $A$ receives an impulse of magnitude $8mu$.

(a) Find the coefficient of restitution between $A$ and $B$

**(6)**

When $A$ and $B$ collide they are at a distance $d$ from a smooth vertical wall, which is perpendicular to their direction of motion. After the collision with $A$, particle $B$ collides directly with the wall and rebounds so that there is a second collision between $A$ and $B$. This second collision takes place at a distance $x$ from the wall.

Given that the coefficient of restitution between $B$ and the wall is $\frac{1}{4}$

(b) find $x$ in terms of $d$.

**(6)**

**Question 5 continued**

**Question 5 continued**

**Question 5 continued**

**(Total for Question 5 is 12 marks)**

**6.** Two fixed points, $A$ and $B$, lie on a horizontal ceiling with $AB = 6a$. A light elastic string of modulus of elasticity $\frac{5mg}{3}$ has one end attached to $A$ and the other end attached to $B$.

A particle $P$ of mass $4m$ is attached to the midpoint of the string and $P$ hangs in equilibrium at a distance $4a$ below $AB$.

(a) Show that the natural length of the string is $4a$.

**(5)**

The particle $P$ is now held at the midpoint of $AB$ and released from rest.

(b) Find the maximum speed of P as it falls.

**(6)**

**Question 6 continued**

**Question 6 continued**

**Question 6 continued**

**(Total for Question 6 is 11 marks)**

**7.** A small ball is projected with speed 14 $\text{ms}^{-1}$ from a point $O$ on the ground. The ball is projected at an angle $\alpha$ to the ground, where $\tan\alpha = \frac{3}{4}$. The ball bounces on the ground for the first time at the point $A_1$. The coefficient of restitution between the ball and the ground is $\frac{1}{2}$. The ball is modelled as a particle moving freely under gravity from $O$ to $A_1$ and between bounces. The ground is modelled as a smooth horizontal plane.

(a) Find the size of the angle between the direction of motion of the ball and the ground immediately after the ball bounces on the ground at $A_1$.

**(4)**

(b) Explain how, in your calculation, you have used the fact that the ball is moving freely under gravity from $O$ to $A_1$.

**(1)**

The ball bounces on the ground for the second time at the point $A_2$.

(c) Find the total time taken by the ball to travel from $O$ to $A_2$.

**(4)**

The ball bounces on the ground for the $n^{\text{th}}$ time at the point $A_n$.

Immediately after the ball bounces at $A_n$, the angle between the direction of motion of the ball and the ground is $\phi$.

(d) Find, in terms of $n$ only, an expression for $\tan\phi$.

**(3)**

(e) Describe, according to the model, the subsequent motion of the ball after it has bounced on the ground at $A_2$.

**(1)**

Given instead that the coefficient of restitution between the ball and the ground is 0.

(f) describe fully the motion of the ball from the instant when it is projected from $O$.

**(2)**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**Question 7 continued**

**(Total for Question 7 is 15 marks)**

**Total for paper is 75 marks**