Year 13 Mathematics Mock Set#03c
Pure Paper 2

- Advised to print in "A3-booklets", this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

This exam paper has 15 questions, for a total of 100 marks.

Question	Marks	Score
1	5	
2	7	
3	6	
4	7	
5	5	
6	6	
7	8	
8	10	
9	3	
10	7	
11	6	
12	7	
13	9	
14	6	
15	8	
Total:	100	

Andrew Chan
Last updated: 4th March 2023

1. A sequence is defined by

$$u_1 = 6$$
$$u_{n+1} = ku_n + 3$$

where k is a positive constant.

(a) Find, in terms of k, an expression for u_3

(2)

Given that $\sum_{n=1}^{3} u_n = 117$

(b) find the value of k

(3)

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Question 1 continued

Question 1 continued

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Question 1 continued

(Total for Question 1 is 5 marks)

2. Given that a is a positive constant and

$$\mathrm{f}(x) = |3x - a| \qquad \{x \in \mathbb{R}\}$$

(a) sketch the graph with equation $y = \mathrm{f}(x)$, showing the coordinates of the points where the graph cuts or meets the coordinate axes.

(2)

Given that $x = 4$ is a solution to the equation

$$|3x - a| = \frac{1}{2}x + 2$$

(b) find the two possible values of a.

(3)

For one of the values of a, $x = 4$ is the smaller of the two equations.
For this value of a,

(c) find the value of the larger solution.

(2)

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Question 2 continued

Question 2 continued

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Question 2 continued

(Total for Question 2 is 7 marks)

3. Let $f(x) = 2x^3 + 3x$.

Use differentiation from first principles to show that $f'(x) = 6x^2 + 3$

(6)

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Question 3 continued

Question 3 continued

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Question 3 continued

(Total for Question 3 is 6 marks)

4.

$$\mathrm{f}(x) = \frac{10}{\sqrt{4-3x}}$$

(a) Show that the first 4 terms of the binomial expansion of $\mathrm{f}(x)$, in ascending powers of x, are

$$A + Bx + Cx^2 + \frac{675}{1024}x^3$$

where A, B and C are constants to be found.

Give each constant in simplest form.

(4)

Given that this expansion is valid for $|x| < k$

(b) state the largest value of k.

(1)

By substituting $x = \frac{1}{3}$ into $\mathrm{f}(x)$ and into the answer for part (a),

(c) find an approximation for $\sqrt{3}$.

Give your answer in the form $\frac{a}{b}$, where a and b are integers to be found.

(2)

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Question 4 continued

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 7 marks)

5. Prove that the sum of a rational number and an irrational number is always irrational.

(5)

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Question 5 continued

(Total for Question 5 is 5 marks)

6.

$$\mathrm{f}(x) = 4\cos x + e^{-x}$$

(a) Show that the equation $\mathrm{f}(x) = 0$ has a root α between 1.6 and 1.7

(2)

(b) Taking 1.6 as your first approximation to α, apply the Newton-Raphson procedure once to $\mathrm{f}(x)$ to obtain a second approximation to α.
Give your answer to 3 significant figures.

(4)

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Question 6 continued

(Total for Question 6 is 6 marks)

7.

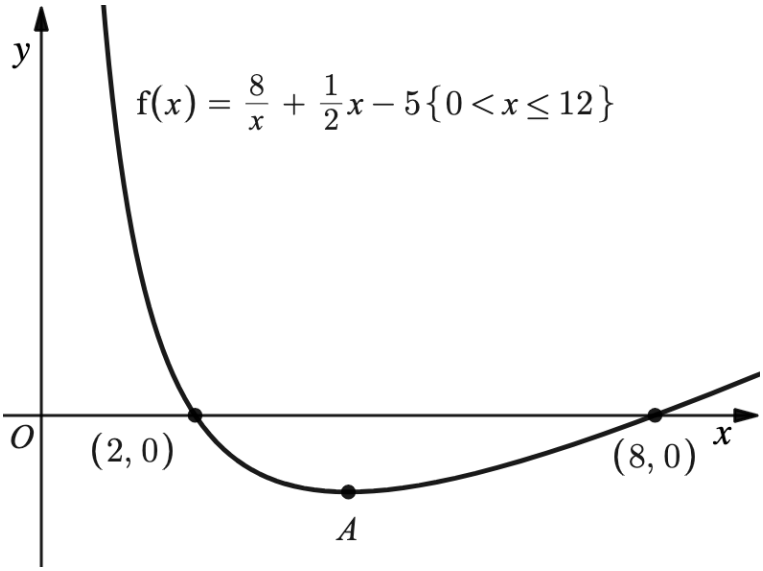

Figure 1: https://www.desmos.com/calculator/0vavyopdns

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{8}{x} + \frac{1}{2}x - 5 \qquad \{0 < x \leq 12\}$$

The curve crosses the x-axis at $(2, 0)$ and $(8, 0)$ and has a minimum point at A.

(a) Use calculus to find the coordinates of A.

(5)

(b) State

(i) the roots of the equation $2\mathrm{f}(x) = 0$

(ii) the coordinates of the turning point on the curve $y = \mathrm{f}(x) + 2$

(iii) the roots of the equation $\mathrm{f}(4x) = 0$

(3)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 8 marks)

8. (a) Express $\dfrac{3x^2 - 4}{x^2(3x - 2)}$ in partial fractions.

(4)

(b) Given that $x > \frac{2}{3}$, find the general solution of the differential equation

$$x^2(3x - 2)\frac{\mathrm{d}y}{\mathrm{d}x} = y\,(3x^2 - 4)$$

Give your answer in the form $y = \mathrm{f}(x)$

(6)

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Question 8 continued

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

9.

$$\sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n \cos(180n)^\circ = \frac{9}{28}$$

(3)

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Question 9 continued

(Total for Question 9 is 3 marks)

10. The growth of duckweed on a pond is being studied.

The surface area of the pond covered by duckweed, A m^2, at a time t days after the start of the study is modelled by the equation

$$A = pq^t$$

where p and q are positive constants.

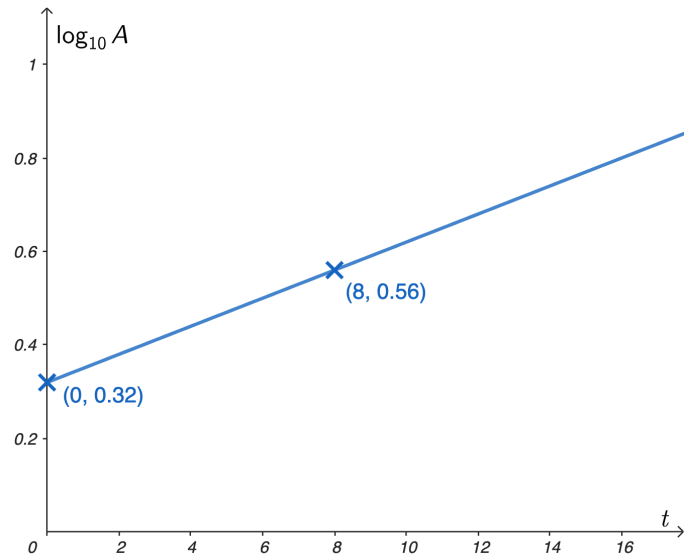

Figure 2: https://www.geogebra.org/calculator/xvhnge2d

Figure 2 shows the linear relationship between $\log_{10} A$ and t.

The points $(0, 0.32)$ and (8,0.56) lie on the line as shown.

(a) Find the value of p and the value of q.
Give your answers to 3 decimal places.

(4)

Using the model with the values of p and the value of q found in part (a),

(b) find the rate of increase of the surface area of the pond covered by duckweed, in m^2/day, exactly 6 days after the start of the study.
Give your answer to 2 decimal places.

(3)

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Question 10 continued

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Question 10 continued

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Question 10 continued

(Total for Question 10 is 7 marks)

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11.

x	0.5	0.6	0.7	0.8	0.9
y	1.632	1.711	1.786	1.869	1.930

The table above shows corresponding values of x and y for the curve with equation $y = \mathrm{f}(x)$ between $x = 0.5$ and $x = 0.9$

The values of y are given to 4 significant figures.

(a) Using the trapezium rule, with all the values of y in the table, to find an estimate for

$$\int_{0.5}^{0.9} \mathrm{f}(x)\,\mathrm{d}x$$

Give your answer to 3 significant figures.

(3)

(b) Using your answer to part (a), deduce an estimate for

$$\int_{0.5}^{0.9} \Big(3\mathrm{f}(x) + 2\Big)\,\mathrm{d}x$$

(3)

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Question 11 continued

(Total for Question 11 is 6 marks)

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12. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

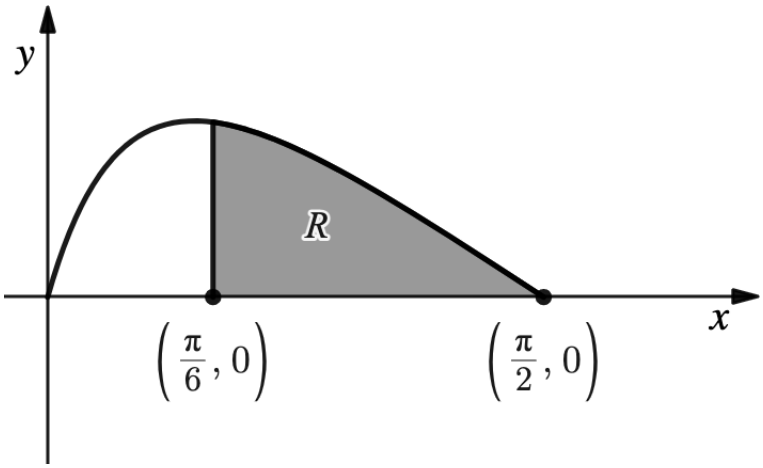

Figure 3

Figure 3 shows a sketch of the curve with equation

$$y = \frac{16 \sin 2x}{(3 + 4 \sin x)^2} \qquad \{0 \leq x \leq \frac{\pi}{2}\}$$

The region R, shown shaded, is bounded by the curve, the x-axis and the line $x = \frac{\pi}{6}$

Using the substitution $u = 3 + 4 \sin x$, show that the area of R can be written in the form $a + \ln b$, where a and b are rational constants to be found.

(7)

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Question 12 continued

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Question 12 continued

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Question 12 continued

(Total for Question 12 is 7 marks)

13.

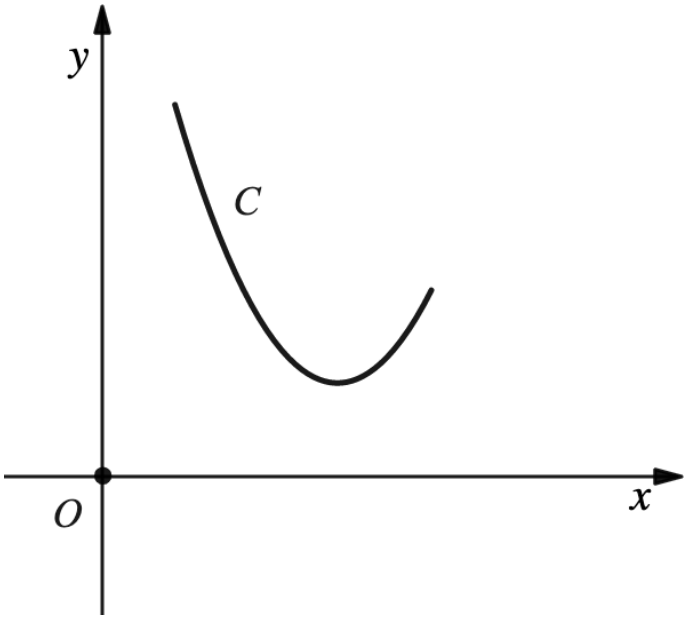

Figure 4: https://www.desmos.com/calculator/6sado6amu7

Figure 4 shows a sketch of the curve C with parametric equations

$$x = 5 + 2\tan t \qquad y = 8\sec^2 t \qquad \{-\frac{\pi}{3} \leq t \leq \frac{\pi}{4}\}$$

(a) Use parametric differentiation to find the gradient of C at $x = 3$

(4)

The curve C has equation $y = \mathrm{f}(x)$, where f is a quadratic function.

(b) Find $\mathrm{f}(x)$ in the form $a(x+b)^2 + c$, where a, b and c are constants to be found.

(3)

(c) Find the range of f.

(2)

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Question 13 continued

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Question 13 continued

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Question 13 continued

(Total for Question 13 is 9 marks)

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14. Relative to a fixed origin O,

- the point A has position vector $5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$
- the point B has position vector $7\mathbf{i} + \mathbf{j} + 2\mathbf{k}$
- the point C has position vector $4\mathbf{i} + 8\mathbf{j} - 3\mathbf{k}$

(a) Find $|\overrightarrow{AB}|$

Give your answer as a simplified surd.

(2)

Given that $ABCD$ is a parallelogram,

(b) find the position vector of the point D.

(2)

The point E is positioned such that

- ACE is a straight line
- $AC : CE = 2 : 1$

(c) Find the coordinates of the point E.

(2)

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Question 14 continued

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Question 14 continued

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Question 14 continued

(Total for Question 14 is 6 marks)

15. (a) Express $12\sin x - 5\cos x$ in the form $R\sin(x - \alpha)$, where R and α are constants, $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Give the exact value of R and give the value of α in radians, to 3 decimal places.

(3)

The function g is defined by

$$\mathrm{g}(\theta) = 10 + 12\sin\left(2\theta - \frac{\pi}{6}\right) - 5\cos\left(2\theta - \frac{\pi}{6}\right) \qquad \{\theta > 0\}$$

(b) (i) the minimum value of $\mathrm{g}(\theta)$

(ii) the smallest value of θ at which this minimum value occurs.

(3)

The function h is defined by

$$\mathrm{h}(\beta) = 10 - 12\left(12\sin\beta - 5\cos\beta\right)^2$$

(c) Find the range of h.

(2)

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Question 15 continued

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Question 15 continued

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Question 15 continued

(Total for Question 15 is 8 marks)

Total for paper is 100 marks