Year 13 Further Mathematics Mock Set#03c
Further Mechanics FM1

- Advised to print in “A3-booklets”, this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

This exam paper has 7 questions, for a total of 75 marks.

Question	Marks	Score
1	8	
2	8	
3	9	
4	12	
5	10	
6	13	
7	15	
Total:	75	

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1.

Figure 1

A tree bark is attached to a rope and released from rest, as shown in Figure 1.

The tree bark is modelled as a particle P of mass 0.4 kg, the rope is modelled as a light elastic string of natural length 0.5 m and modulus of elasticity 6 N.

The other end of the string is attached to a fixed origin O.

The acceleration due to gravity is modelled as having magnitude 10 ms^{-2}.

The particle P is released from rest at the point $(0.5 + x)$ m vertically below O. The particle P comes to instantaneous rest at O.

(a) Find x.

(3)

(b) Find the greatest speed of P.

(5)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 8 marks)

2.

Figure 2

A firework is shown in Figure 2.

The firework is instantaneously at rest in the air when it explodes into two parts.

One part is the body B of mass 0.06 kg and the other a cap C of mass 0.004 kg.

The total kinetic energy given to B and C is 0.8 J. B moves off horizontally in the $\mathbf{i}$ direction.

By considering both kinetic energy and conservation of linear momentum, calculate the velocities of B and C immediately after the explosion.

(8)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3.

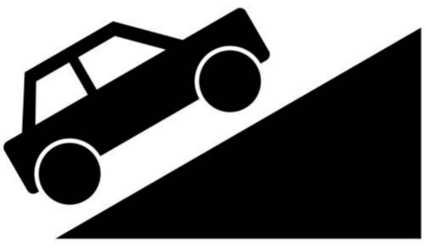

Figure 3

A car of mass 800 kg is travelling up some hills, as shown in Figure 3.
In one situation, the car climbs a vertical height of 20 m while its speed decreases from 30 ms^{-1} to 12 ms^{-1}.
The car is subject to a resistance to its motion but there is no driving force and the brakes are not being applied.

(a) Using an energy method, calculate the work done by the car against the resistance to its motion.

(4)

On another situation, the car is travelling at a constant speed of 18 ms^{-1} and climbs a vertical height of 20 m in 25 s up a uniform slope.
The resistance to its motion is now 750 N.

(b) Calculate the power of the driving force required.

(5)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4.

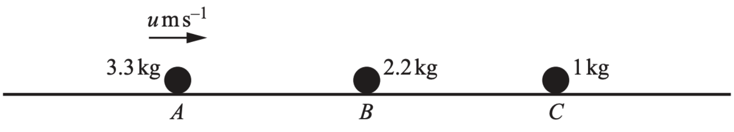

Figure 4

Three particles A, B and C are free to move in the same straight line on a large smooth horizontal surface. Their masses are 3.3 kg, 2.2 kg and 1 kg respectively.

Initially, B and C are at rest and A is moving towards B with speed u ms^{-1}, as shown in Figure 4. A collides directly with B and subsequently, B goes on to collide directly with C.

The coefficient of restitution in collisions between any two particles is e.

The velocities of A and B immediately after the first collision are denoted by v_A ms^{-1} and v_B ms^{-1} respectively.

(a) (i) Show that $v_A = \frac{u}{5}(3 - 2e)$

(ii) Find an similar expression for v_B in terms of u and e.

(4)

(b) Find an expression in terms of u and e for the velocity of B immediately after its collision with C.

(4)

After the collision between B and C there is a further collision between A and B.

(c) Determine the range of possible values of e.

(4)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 12 marks)

5. A light elastic string of natural length 1.5 m and modulus of elasticity 490 N has one end attached to a particle at P of mass 30 kg. Initially, P is held at rest vertically below A such that the distance AP is 0.6 m. It is then allowed to fall vertically.

(a) Calculate the distance AP when P is instantaneously at rest for the first time. Give your answer correct to 2 decimal places.

(8)

(b) Estimate the distance AP when P is instantaneously at rest for the second time, clearly state one assumption that you have made in making your estimate.

(2)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6. Two uniform small smooth spheres A and B have equal radii and equal masses. The spheres are on a smooth horizontal surface. Sphere A is moving at an acute angle α to the line of centres, when it collides with B, which is stationary.

After the impact A is moving at an acute angle β to the line of centres. The coefficient of restitution between A and B is $\frac{1}{3}$.

(a) Show that $\tan\beta = 3\tan\alpha$

(5)

(b) Explain why the assumption that the contact between the spheres is smooth is needed in answering part (a)

(1)

It is given that A is deflected through an angle γ.

(c) Determine, in terms of α, an expression for $\tan\gamma$

(2)

(d) Hence determine the maximum value of γ. You do not need to justify that this value is a maximum.

(5)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 13 marks)

7.

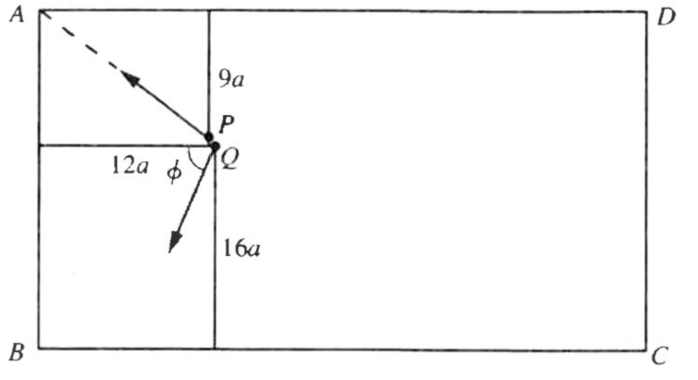

Figure 5

Figure 5 shows a smooth horizontal rectangular pool table $ABCD$ with pockets at A, B, C and D. A small uniform smooth pool ball P is stationary at a point on the table whose distances from AD, BC, and AB are $9a$, $16a$ and $12a$ respectively, where a is a constant.

Mr Wong strikes a second identical cue ball Q, so that it travels with speed u on the table in a direction parallel to DA when it strikes ball P obliquely.

As a result of the collision, ball P falls into the pocket at A, and the direction of motion of ball Q is deflected through an angle ϕ.

Given that P and Q are of equal mass and that the coefficient of restitution between the balls is e.

(a) Show that

$$\tan\phi = \frac{6(e+1)}{17-8e}$$

(11)

Situations where the cue ball falls into one of the pockets on the table is considered a *foul*. Given that Mr Wong conceded a foul, and that ball Q falls into the pocket at B, find,

(b) the coefficient of restitution between the balls.

(2)

(c) the angle between the directions of motion of P and Q immediately after the impact.

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 15 marks)

Total for paper is 75 marks