MR. CHAN'S 12FM
MATRICES AND LINEAR TRANSFORMATIONS
QUESTIONS BY TOPIC PACK

https://www.youtube.com/c/SoHokMathsByAChan

Version 1 - Spring 2021

MATRICES AND LINEAR TRANSFORMATION

- I have tried to exclude questions that have orthogonal/eigenvalues/eigenvectors (they are now not in ”Core Pure” of all the exam boards, but in “Further Pure 2” in most exam boards.
- Replacing orthogonal/eigenvalues/eigenvectors is a new topic that used to be Further Pure 4 - FP4 (Invariant Lines/Points)
- For this pack, I have only included/selected some, but not all questions from old spec FP1, FP3, FP4
- Instead, I have included new specifications questions from different exam boards.
- 2018, 2019 OCR/MEI/AQA/Edexcel Questions are included.
- 2020 have not been added to allow them to remain “unseen”
- Feel free to check out other questions by topic pack, located in my Google Drive.
- Also included newer type of questions such AS system of linear equations -> that means knowledge of further vectors, in particular planes and cross product are assumed.
- I have not included proof by induction matrices questions, this will be in a different “proof” pack.

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EDEXCEL FP1

- Mainly transformation/ up to 2x2 matrices

EDEXCEL JUNE 2017 FP1

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$$

(a) Find $\mathbf{A}^{-1}$

(2)

The transformation represented by the matrix **B** followed by the transformation represented by the matrix **A** is equivalent to the transformation represented by the matrix **P**.

(b) Find **B**, giving your answer in its simplest form.

(3)

(Total for question = 5 marks)

Question Number	Scheme		Marks
	$\mathbf{A}=\begin{pmatrix}2 & -1\\ 4 & 3\end{pmatrix}$, $\mathbf{P}=\begin{pmatrix}3 & 6\\ 11 & -8\end{pmatrix}$		
(a)	$\mathbf{A}^{-1}=\frac{1}{10}\begin{pmatrix}3 & 1\\ -4 & 2\end{pmatrix}$	Either $\frac{1}{10}$ or $\begin{pmatrix}3 & 1\\ -4 & 2\end{pmatrix}$	M1
		Correct matrix seen.	A1
			[2]
(b) **Way 1**	$\mathbf{P}=\mathbf{AB}$ $\Rightarrow\ \mathbf{A}^{-1}\mathbf{P}=\mathbf{A}^{-1}\mathbf{AB}\ \Rightarrow\ \mathbf{B}=\mathbf{A}^{-1}\mathbf{P}$		
	$\mathbf{B}\quad=\frac{1}{10}\begin{pmatrix}3 & 1\\ -4 & 2\end{pmatrix}\begin{pmatrix}3 & 6\\ 11 & -8\end{pmatrix}$	Multiplies **their** $\mathbf{A}^{-1}$ by $\mathbf{P}$ in correct order. This substituted statement is sufficient.	M1
	$=\begin{pmatrix}2 & 1\\ 1 & -4\end{pmatrix}$	At least 2 elements correct or $k\begin{pmatrix}20 & 10\\ 10 & -40\end{pmatrix}$ oe. May be unsimplified	A1
		Correct simplified matrix.	A1
			[3]
(b) **Way 2**	$\{\mathbf{P}=\mathbf{AB}\Rightarrow\}$ $\begin{pmatrix}3 & 6\\ 11 & -8\end{pmatrix}=\begin{pmatrix}2 & -1\\ 4 & 3\end{pmatrix}\begin{pmatrix}a & b\\ c & d\end{pmatrix}$		
	$\begin{pmatrix}3 & 6\\ 11 & -8\end{pmatrix}=\begin{pmatrix}2a-c & 2b-d\\ 4a+3c & 4b+3d\end{pmatrix}$	Attempt to multiply **A** by **B** in the correct order and puts equal to **P**	M1
	$\Rightarrow a=2,\ c=1,\ b=1,\ d=-4$		
		At least 2 elements are correct.	A1
	So, $\mathbf{B}=\begin{pmatrix}2 & 1\\ 1 & -4\end{pmatrix}$	Correct matrix.	A1
			[3] **5**

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EDEXCEL JUNE 2017 FP1

(i)

$$\mathbf{A} = \begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -5 & 4 \\ 6 & -5 \end{pmatrix}$$

where p is a constant.

(a) Find, in terms of p, the matrix **AB**

(2)

Given that

$$\mathbf{AB} + 2\mathbf{A} = k\mathbf{I}$$

where k is a constant and $\mathbf{I}$ is the 2×2 identity matrix,

(b) find the value of p and the value of k.

(4)

(ii)

$$\mathbf{M} = \begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix}, \text{ where } a \text{ is a real constant}$$

Triangle T has an area of 15 square units.

Triangle T is transformed to the triangle T' by the transformation represented by the matrix **M**.

Given that the area of triangle T' is 270 square units, find the possible values of a.

(5)

(Total for question = 11 marks)

Question Number	Scheme		Marks
(i)	$\mathbf{A}=\begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix}$, $\mathbf{B}=\begin{pmatrix} -5 & 4 \\ 6 & -5 \end{pmatrix}$, $\mathbf{M}=\begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix}$	p, a are constants.	
(a)	$\{\mathbf{AB}\}=\begin{pmatrix} -5p+12 & 4p-10 \\ -15+6p & 12-5p \end{pmatrix}$	At least 2 elements are correct.	M1
		Correct matrix.	A1
			[2]
(b)	$\{\mathbf{AB}+2\mathbf{A}=k\mathbf{I}\}$		
	$\begin{pmatrix} -5p+12 & 4p-10 \\ -15+6p & 12-5p \end{pmatrix}+2\begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix}=k\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	If 'simultaneous equations' used, allocate marks as below.	
	$\begin{pmatrix} -3p+12 & 4p-6 \\ -9+6p & 12-3p \end{pmatrix}=\begin{pmatrix} k & 0 \\ 0 & k \end{pmatrix}$		
	$"4p-10"+4=0$ or $"-15+6p"+6=0$ or $"-9+6p"="4p-6"$	Forms an equation in p	M1
	$\Rightarrow p=\dfrac{3}{2}$	$p=\dfrac{3}{2}$ o.e.	A1
	$k=-5\left(\dfrac{3}{2}\right)+12+2\left(\dfrac{3}{2}\right)\Rightarrow k=\ldots$	Substitutes their $p=\dfrac{3}{2}$ into "their $(-5p+12)$"$+2p$ to find a value for k or eliminates p to find k.	M1
	$k=\dfrac{15}{2}$	$k=\dfrac{15}{2}$ oe	A1
			[4]

(ii) **Way 1**	$\pm\frac{270}{15}$ $\{=\pm 18\}$	Can be implied from calculations.	B1	
	$\det \mathbf{M} = (a)(2) - (-9)(1)$	Applies $ad - bc$ to $\mathbf{M}$. Require clear evidence of correct formula being used for M1 if errors seen.	M1	
	$\Rightarrow 2a + 9 = 18$ or $2a + 9 = -18$	Equates their $\det \mathbf{A}$ to either 18 or -18	M1	
	$\Rightarrow a = 4.5$ or $a = -13.5$	At least one of either $a = 4.5$ or $a = -13.5$	A1	
		Both $a = 4.5$ and $a = -13.5$	A1	
				[5]
(ii) **Way 2**	Consider vertices of triangle with area 15 units e.g. (0,0),(15,0) and (0,2) and attempting 2 values of a.		B1	
	e.g. $\begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 15 & 0 \\ 0 & 0 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 15a & -18 \\ 0 & 15 & 4 \end{pmatrix}$	Pre-multiplies their matrix by $\mathbf{M}$ and obtains single matrix	M1	
	e.g. $\frac{1}{2}\begin{vmatrix} 0 & 15a & -18 & 0 \\ 0 & 15 & 4 & 0 \end{vmatrix} = 270$	Equates their determinant to 270 and attempts to solve.	M1	
	$\Rightarrow a = 4.5$ or $a = -13.5$	At least one of either $a = 4.5$ or $a = -13.5$	A1	
		Both $a = 4.5$ and $a = -13.5$	A1	
				[5]
				11

EDEXCEL JUNE 2018 FP1

(i) Given that

$$\mathbf{A} = \begin{pmatrix} -2 & 3 \\ 1 & 1 \end{pmatrix}, \quad \mathbf{AB} = \begin{pmatrix} -1 & 5 & 12 \\ 3 & -5 & -1 \end{pmatrix}$$

(a) find $\mathbf{A}^{-1}$

(2)

(b) Hence, or otherwise, find the matrix **B**, giving your answer in its simplest form.

(3)

(ii) Given that

$$\mathbf{C} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(a) describe fully the single geometrical transformation represented by the matrix **C**.

(2)

(b) Hence find the matrix $\mathbf{C}^{39}$

(2)

(Total for question = 9 marks)

Question Number	Scheme	Notes	Marks
(i) (a)	$\mathbf{A}^{-1} = \frac{1}{-2-3}\begin{pmatrix} 1 & -3 \\ -1 & -2 \end{pmatrix}$	Either $\frac{1}{-2-3}$ or $-\frac{1}{5}$ or $\begin{pmatrix} 1 & -3 \\ -1 & -2 \end{pmatrix}$	M1
		Correct expression for $\mathbf{A}^{-1}$	A1
			[2]
(b)	$\{\mathbf{B} = \mathbf{A}^{-1}(\mathbf{AB})\}$		
	$\mathbf{B} = -\frac{1}{5}\begin{pmatrix} 1 & -3 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} -1 & 5 & 12 \\ 3 & -5 & -1 \end{pmatrix}$	Writing down their $\mathbf{A}^{-1}$ multiplied by $\mathbf{AB}$	M1
	$= \left\{-\frac{1}{5}\right\}\begin{pmatrix} -10 & 20 & 15 \\ -5 & 5 & -10 \end{pmatrix}$	At least one correct row or at least two correct columns of $\begin{pmatrix} \dots \\ \dots \end{pmatrix}$. (Ignore $-\frac{1}{5}$).	A1
	$= \begin{pmatrix} 2 & -4 & -3 \\ 1 & -1 & 2 \end{pmatrix}$	Correct simplified matrix for $\mathbf{B}$	A1
			[3]
ALT (b)	Let $\mathbf{B} = \begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$		
	$-2a + 3d = -1$ $\quad -2b + 3e = 5$ $a + d = 3$ $\quad b + e = -5$ $-2c + 3f = 12$ $c + f = -1$	Writes down at least 2 correct sets of simultaneous equations	M1
	$\{a = 2, d = 1, b = -4, e = -1, c = -3, f = 2\}$		
	$\mathbf{B} = \begin{pmatrix} 2 & -4 & -3 \\ 1 & -1 & 2 \end{pmatrix}$	At least one correct row or at least two correct columns for the matrix $\mathbf{B}$	A1
		Correct matrix for $\mathbf{B}$	A1
			[3]

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(ii) (a)	Rotation	Rotation only.	M1
	90° clockwise about the origin	**90°** $\left(\text{or } \frac{\pi}{2}\right)$ **clockwise** about the **origin** or **270°** $\left(\text{or } \frac{3\pi}{2}\right)$ (anti-clockwise) about the **origin.** **-90°** $\left(\text{or } -\frac{\pi}{2}\right)$ (anticlockwise) about the **origin.** Origin can be written as $(0, 0)$ or O.	A1
			[2]
(b)	$\{C^{39}\} = C^{-1}$ or $C^3 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	For stating C^{-1} or C^3 or 'rotation of **270°** clockwise o.e. about the **origin.** Can be implied by correct matrix.	M1
		$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ Correct answer with no working award M1A1	A1
			[2]
			Total 9

EDEXCEL JUNE 2018 FP1

$$\mathbf{M} = \begin{pmatrix} 8 & -1 \\ -4 & 2 \end{pmatrix}$$

(a) Find the value of det **M**

(1)

The triangle T has vertices at the points (4, 1), (6, k) and (12, 1), where k is a constant.

The triangle T is transformed onto the triangle T' by the transformation represented by the matrix **M**.

Given that the area of triangle T' is 216 square units,

(b) find the possible values of k.

(5)

(Total for question = 6 marks)

Question Number	Scheme		Marks
(a)	$\{\det \mathbf{M} = (8)(2) - (-1)(-4)\} \Rightarrow \det \mathbf{M} = 12$	12	B1
			[1]
(b)	Area $T = \dfrac{216}{12}\ \{=18\}$	Area $T = \dfrac{216}{\text{their "}\det \mathbf{M}\text{"}}$	M1
	$h = \pm(1-k)$	Uses $(k-1)$ or $(1-k)$ in their solution.	M1
	$\frac{1}{2}8(k-1) = 18$ or $\frac{1}{2}8(1-k) = 18$ or $(k-1) = \dfrac{18}{4}$ or $(1-k) = \dfrac{18}{4}$ or $\{\frac{1}{2}8h = 18\} \Rightarrow h = \dfrac{9}{2}, k = 1 \pm \dfrac{9}{2}$	**dependent on the two previous M marks** $\frac{1}{2}8(k-1)$ or $\frac{1}{2}8(1-k) = \dfrac{216}{\text{their "}\det \mathbf{M}\text{"}}$ or $(k-1)$ or $(1-k) = \dfrac{216}{4(\text{their "}\det \mathbf{M}\text{"})}$ or $h = \dfrac{216}{4(\text{their "}\det \mathbf{M}\text{"})}, k = 1 \pm \dfrac{216}{4(\text{their "}\det \mathbf{M}\text{"})}$	ddM1
	$\Rightarrow k = 5.5$ or $k = -3.5$	At least one of either $k = 5.5$ or $k = -3.5$	A1
		Both $k = 5.5$ and $k = -3.5$	A1
			[5]

ALT (b)	$\mathbf{T}' = \begin{pmatrix} 8 & -1 \\ -4 & 2 \end{pmatrix}\begin{pmatrix} 4 & 6 & 12 \\ 1 & k & 1 \end{pmatrix}$		
	$\mathbf{T}' = \begin{pmatrix} 31 & 48-k & 95 \\ -14 & -24+2k & -46 \end{pmatrix}$ or 18 seen	At least 5 out of 6 elements are correct or 18 seen..	M1
	$\frac{1}{2}\begin{vmatrix} 31 & 48-k & 95 & 31 \\ -14 & -24+2k & -46 & -14 \end{vmatrix} = 216$ or $\frac{1}{2}\begin{vmatrix} 4 & 6 & 12 & 4 \\ 1 & k & 1 & 1 \end{vmatrix} = 18$	$\frac{1}{2}\lvert\text{their } \mathbf{T}'\rvert = 216$ or $\frac{1}{2}\begin{vmatrix} 4 & 6 & 12 & 4 \\ 1 & k & 1 & 1 \end{vmatrix} = 18$	M1
	$\frac{1}{2}\begin{vmatrix} -744 + 62k + 672 - 14k - 2208 + 46k \\ +2280 - 190k - 1330 + 1426 \end{vmatrix} = 216$ $\frac{1}{2}\lvert 4k - 6 + 6 - 12k + 12 - 4\rvert = 18$	**Dependent on the two previous M marks.** Full method of evaluating a determinant.	ddM1
	$\frac{1}{2}\lvert 96 - 96k\rvert = 216$ or $\frac{1}{2}\lvert 8 - 8k\rvert = 18$		
	So, $1-k=4.5$ or $k-1=4.5$		
	$\Rightarrow k=-3.5$ or $k=5.5$	At least one of either $k=-3.5$ or $k=5.5$	A1
		Both $k=-3.5$ and $k=5.5$	A1
			[5]
			Total 6

EDEXCEL JUNE 2019 FP1

$$\mathbf{M} = \begin{pmatrix} 9+k & -3 \\ 4-k & 2 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

The triangle T has vertices at the points (2, 1), (7, 1) and (7, 12).

Triangle T is transformed onto triangle T' by the transformation represented by the matrix **M**.

(a) Find, in terms of k, the coordinates of the vertices of the triangle T'

(3)

Given that the area of triangle T' is 770 square units,

(b) find the two possible values of k.

(4)

(Total for question = 7 marks)

Question Number	Scheme	Notes	Marks
(a)	$\begin{pmatrix} 9+k & -3 \\ 4-k & 2 \end{pmatrix}\begin{pmatrix} 2 & 7 & 7 \\ 1 & 1 & 12 \end{pmatrix} = \ldots$	Attempt to multiply all the coordinates appropriately. This statement oe is sufficient. Coordinate pairs could be multiplied separately	M1
	$\begin{pmatrix} 2(9+k)-3 & 7(9+k)-3 & 7(9+k)-36 \\ 2(4-k)+2 & 7(4-k)+2 & 7(4-k)+24 \end{pmatrix}$ or $\begin{pmatrix} 18+2k-3 & 63+7k-3 & 63+7k-36 \\ 8-2k+2 & 28-7k+2 & 28-7k+24 \end{pmatrix}$ or $\begin{pmatrix} 15+2k & 60+7k & 27+7k \\ 10-2k & 30-7k & 52-7k \end{pmatrix}$	Deduct 1 mark for each incorrect element up to a maximum of 2 marks. Accept all unsimplified and isw. No requirement to extract coordinates from the matrix and may be given individually as column vectors	A1A1
			(3)
(b)	Area of triangle $T = \frac{1}{2}(7-2)(12-1)$ or $\frac{1}{2}\lvert(2+84+7)-(7+7+24)\rvert$	Correct unsimplified expression for area for triangle T Could come from shoelace method	B1
	$\lvert\mathbf{M}\rvert = (9+k)\times 2-(4-k)\times -3$	Correct unsimplified expression for det$\mathbf{M}$. Seen in part (b)	B1
	$"(9+k)\times 2-(4-k)\times -3"\times"\frac{55}{2}" = \pm 770$ $\Rightarrow k = \ldots$	Uses 770 correctly with their numerical area of T and their det$\mathbf{M}$ to give an equation in terms of k and attempts to solve. The $\pm$ sign is not required. $\lvert\det\mathbf{M}\rvert$ replaced with $(\det\mathbf{M})^2$ is M0 unless $(\det\mathbf{M})^2 = (\text{their } 28)^2$	M1
	$30-k = \pm 28 \Rightarrow k = 2,\ 58$	Both values and no extra	A1
			(4)
			Total 7

EDEXCEL JUNE 2019 FP1

$$\mathbf{P} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation U represented by the matrix **P**.

(2)

The transformation V, represented by the matrix **Q**, is a reflection in the line $y = -x$

(b) Write down the matrix **Q**.

(1)

The transformation U followed by the transformation V is the transformation T.

Transformation T is represented by the matrix **R**.

(c) Find the matrix **R**.

(2)

(d) Find the exact value of the real constant k for which the transformation T maps the point $(1, k)$ onto itself.

(4)

(Total for question = 9 marks)

Question Number	Scheme	Notes	Marks
(a)	Rotation 45° clockwise about the origin	B1: Rotation B1: 45° clockwise (or 315°/– 45° anticlockwise) about/around/at/from/with centre the origin or (0, 0) or O Allow just "315°/– 45° about O" Allow radian equivalents	B1B1
			(2)
(b)	$\mathbf{Q}=\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}$	Correct matrix	B1
	Condone straight lines used as matrix brackets throughout		
			(1)
(c)	$\mathbf{R}=\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix}$	Multiplies in the correct order with their $\mathbf{Q} \neq \mathbf{I}$. This statement is sufficient. May be implied by their $\mathbf{R}$.	M1
	$\mathbf{R}=\begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix}$	Correct matrix. Ignore labelling, such as $\mathbf{T} = \ldots$ Allow exact equivalents for elements and isw. Allow $\begin{pmatrix}\text{awrt } 0.707 & \text{awrt } -0.707\\ \text{awrt } -0.707 & \text{awrt } -0.707\end{pmatrix}$	A1
			(2)

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(d)	$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 1 \\ k \end{pmatrix} = \begin{pmatrix} 1 \\ k \end{pmatrix}$	Forms matrix equation correctly with their **R**. Implied by a correct ft linear equation. $(1, k)\mathbf{R} = (1, k)$ is M0	M1
	$\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}k = 1$ or $-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}k = k$	Either correct equation (or equivalent) from a correct **R**	A1
	$\frac{1}{\sqrt{2}}k = \frac{1}{\sqrt{2}} - 1 \Rightarrow k = \ldots$	Solves one of their equations to obtain a value for k. Allow obtaining and solving an equation from $\begin{pmatrix} 1 \\ k \end{pmatrix} = \mathbf{R}^{-1}\begin{pmatrix} 1 \\ k \end{pmatrix}$ **Dependent on previous method mark.**	dM1
	$k = 1 - \frac{2}{\sqrt{2}}$ or $\frac{\sqrt{2}-2}{\sqrt{2}}$ or $1 - \sqrt{2}$ or $\frac{-1}{\sqrt{2}+1}$	Cao. Accept equivalent exact answers and isw. No extra non-equivalent solutions.	A1
			(4)
			Total 9

EDEXCEL FP3

- 3x3 matrices inverse

EDEXCEL FP3 JUNE 2013

(b) The matrix **P** is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 1 & d \\ -1 & 0 & 1 \end{pmatrix}, \text{ where } d \text{ is constant, } d \neq -1$$

Find

(i) the determinant of **P** in terms of d,

(ii) the matrix $\mathbf{P}^{-1}$ in terms of d.

(5)

(Total 13 marks)

(b)(i)	detP = $-d - 1$	Allow $1 - d - 2$ or $1 - (2 + d)$ A correct (possibly un-simplified) determinant	B1
(ii)	$\mathbf{P}^T = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 1 & 0 \\ 0 & d & 1 \end{pmatrix}$ **or minors** $\begin{pmatrix} 1 & d+2 & 1 \\ 1 & 1 & 1 \\ d & d & -1 \end{pmatrix}$ **or** cofactors $\begin{pmatrix} 1 & -2-d & 1 \\ -1 & 1 & -1 \\ d & -d & -1 \end{pmatrix}$ **a correct first step**		B1
	$\dfrac{1}{-d-1}\begin{pmatrix} 1 & -1 & d \\ -2-d & 1 & -d \\ 1 & -1 & -1 \end{pmatrix}$	M1: Identifiable full attempt at inverse **including reciprocal of determinant**. Could be indicated by at least 6 correct elements. A1: Two rows or two columns correct (ignoring determinant) **BUT M0A1A0 or M0A1A1 is not possible** A1: Fully correct inverse	M1 A1 A1
			(5)
			Total 13

EDEXCEL FP3 JUNE 2016

$$\mathbf{A} = \begin{pmatrix} -2 & 1 & -3 \\ k & 1 & 3 \\ 2 & -1 & k \end{pmatrix}, \text{ where } k \text{ is a constant}$$

Given that the matrix **A** is singular, find the possible values of *k*.

(4)

(Total for question = 4 marks)

Question Number	Scheme	Notes	Marks
	$\mathbf{A}=\begin{pmatrix}-2 & 1 & -3\\ k & 1 & 3\\ 2 & -1 & k\end{pmatrix}$		
	$\det\mathbf{A}=-2(k+3)-(k^2-6)-3(-k-2)\ row1$ or e.g. $\det\mathbf{A}=-k(k-3)+(-2k+6)-3(2-2)\ row2$ $\det\mathbf{A}=2(3+3)+(-6+3k)+k(-2-k)\ row3$ $\det\mathbf{A}=-2(k+3)-k(k-3)+2(3+3)\ col1$ $\det\mathbf{A}=-(k^2-6)+(-2k+6)+(-6+3k)\ col2$ $\det\mathbf{A}=-3(-2-k)-3(2-2)+k(-2-k)\ col3$	M1: Correct attempt at determinant (3 'elements' (may be implied if one is zero) with at least two elements correct). **Note that there are various alternatives depending on the choice of row or column.** A1: Correct determinant **in any form**	M1A1
	Note that e.g. $\det\mathbf{A}=-2\begin{vmatrix}1 & 3\\ -1 & k\end{vmatrix}-\begin{vmatrix}k & 3\\ 2 & k\end{vmatrix}-3\begin{vmatrix}k & 1\\ 2 & -1\end{vmatrix}$ scores no marks until the determinants are 'extracted'.		
	$-2(k+3)-(k^2-6)-3(-k-2)=0\Rightarrow k=\ldots$	Sets their det**A** = 0 (= 0 may be implied) and attempts to solve a 3 term quadratic (see general guidance) as far as $k=\ldots$ NB Correct quadratic is $k^2-k-6=0$	M1
	$(k+2)(k-3)=0\Rightarrow k=-2,\ 3$	Both values correct	A1
			(4)
			Total 4

EDEXCEL FP3 JUNE 2017

The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & k & 0 \\ 2 & -2 & 1 \\ -4 & 1 & -1 \end{pmatrix}, \; k \in \mathbb{R}, \; k \neq \frac{1}{2}$$

(a) Show that det **M** = $1 - 2k$.

(2)

(b) Find $\mathbf{M}^{-1}$ in terms of k.

(4)

The straight line l_1 is mapped onto the straight line l_2 by the transformation represented by the matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 2 & -2 & 1 \\ -4 & 1 & -1 \end{pmatrix}$$

Given that l_2 has cartesian equation

$$\frac{x-1}{5} = \frac{y+2}{2} = \frac{z-3}{1}$$

(c) find a cartesian equation of the line l_1

(6)

(Total for question = 12 marks)

Question Number	Scheme	Notes	Marks
(a)	$\det \mathbf{M} = 1\times(2-1)-k(-2+4)(+0)=1-2k^*$ or e.g. $\det \mathbf{M} = (0)-1(1+4k)-1(-2-2k)=1-2k^*$	M1: Correct attempt at determinant (at least 2 'elements' correct). May need to check as they might use a different row/column.	M1A1*
	or rule of Sarrus: $\det \mathbf{M} = 2-4k-1+2k=1-2k^*$ Or e.g. $(1)\begin{vmatrix}-2 & 1\\ 1 & -1\end{vmatrix}-k\begin{vmatrix}2 & 1\\ -4 & -1\end{vmatrix}+0\begin{vmatrix}2 & -2\\ -4 & 1\end{vmatrix}$	A1: Obtains printed answer with no errors. If they use determinant notation as in the last example, then you must see at least one intermediate step before the printed answer e.g. minimally 1 - 2k + 0.	
			(2)
(b)	$(\mathbf{M}^{\mathrm{T}})$ $\begin{pmatrix}1 & 2 & -4\\ k & -2 & 1\\ 0 & 1 & -1\end{pmatrix}$ or (minors) $\begin{pmatrix}1 & 2 & -6\\ -k & -1 & 1+4k\\ k & 1 & -2-2k\end{pmatrix}$ or (cofactors) $\begin{pmatrix}1 & -2 & -6\\ k & -1 & -1-4k\\ k & -1 & -2-2k\end{pmatrix}$		B1
	$\mathbf{M}^{-1}=\dfrac{1}{1-2k}\begin{pmatrix}1 & k & k\\ -2 & -1 & -1\\ -6 & -1-4k & -2-2k\end{pmatrix}$	M1: Full attempt at inverse ignoring determinant. Need to see all stages but allow numerical slips. A1: 2 correct rows or 2 correct columns including reciprocal of determinant A1: All correct including reciprocal of determinant	M1A1A1
			(4)

(c)	$l_2:(1+5\lambda)\mathbf{i}+(-2+2\lambda)\mathbf{j}+(3+\lambda)\mathbf{k}$	M1: Attempt l_2 in parametric form A1: Correct parametric form	M1A1
	$\frac{1}{1}\begin{pmatrix}1&0&0\\-2&-1&-1\\-6&-1&-2\end{pmatrix}\begin{pmatrix}1+5\lambda\\-2+2\lambda\\3+\lambda\end{pmatrix}=\begin{pmatrix}1+5\lambda\\-3-13\lambda\\-10-34\lambda\end{pmatrix}$ or e.g. $\frac{1}{1}\begin{pmatrix}1&0&0\\-2&-1&-1\\-6&-1&-2\end{pmatrix}\begin{pmatrix}1&5\\-2&2\\3&1\end{pmatrix}=\begin{pmatrix}1&5\\-3&-13\\-10&-34\end{pmatrix}$	M1: Puts $k=0$ in their $\mathbf{M}^{-1}$ and multiplies this by their parametric form correctly. Or starts again to find the inverse and multiplies. A1: Correct parametric form for l_1 or correct matrix.	M1A1
	$\frac{x-1}{5}=\frac{y+3}{-13}=\frac{z+10}{-34}$ oe $\begin{matrix}a_1+b_1\lambda\\a_2+b_2\lambda\\a_3+b_3\lambda\end{matrix}\rightarrow\frac{x-a_1}{b_1}=\frac{y-a_2}{b_2}=\frac{z-a_3}{b_3}$	M1: Attempts cartesian form from their parametric l_1 **correctly**. **Dependent on both previous M's.** A1: A complete correct equation	dM1A1
	If their $\mathbf{M}^{-1}$ is incorrect in terms of k but by substituting $k=0$, a correct answer is obtained in (c) allow a full recovery.		
			(6)
			Total 12

	(c) Way 2		
	$\mathbf{i}-2\mathbf{j}+3\mathbf{k}$ and $6\mathbf{i}+4\mathbf{k}$ are on l_2		
	$\mathbf{M}^{-1}(\mathbf{i}-2\mathbf{j}+3\mathbf{k})=\mathbf{i}-3\mathbf{j}-10\mathbf{k}$ $\mathbf{M}^{-1}(6\mathbf{i}+4\mathbf{k})=6\mathbf{i}-16\mathbf{j}-44\mathbf{k}$	M1: Attempt two points on l_1 A1: Two correct points on l_1	M1A1
	$\begin{pmatrix}6+5\lambda\\-16-13\lambda\\-44-34\lambda\end{pmatrix}$	M1: Uses their points to obtain parametric form for l_1 A1: Correct parametric form for l_1 or correct position and direction.	M1A1
	$\frac{x-6}{5}=\frac{y+16}{-13}=\frac{z+44}{-34}$ oe $\begin{matrix}a_1+b_1\lambda\\a_2+b_2\lambda\\a_3+b_3\lambda\end{matrix}\rightarrow\frac{x-a_1}{b_1}=\frac{y-a_2}{b_2}=\frac{z-a_3}{b_3}$	M1: Attempts cartesian form from their parametric l_1 **correctly**. **Dependent on both previous M's.** A1: A complete correct equation	dM1A1

	(c) Way 3		
	$\begin{pmatrix}1&0&0\\2&-2&1\\-4&1&-1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\-2\\3\end{pmatrix}\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\-3\\-10\end{pmatrix}$	M1: Solves $\mathbf{M}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\-2\\3\end{pmatrix}\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\ldots$ A1: $\mathbf{i}-3\mathbf{j}-10\mathbf{k}$. Correct vector or values for x, y and z	M1A1
	$\begin{pmatrix}1&0&0\\2&-2&1\\-4&1&-1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5\\2\\1\end{pmatrix}\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5\\-13\\-34\end{pmatrix}$	M1: Solves $\mathbf{M}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}5\\2\\1\end{pmatrix}\Rightarrow\begin{pmatrix}x\\y\\z\end{pmatrix}=\ldots$ A1: $5\mathbf{i}-13\mathbf{j}-34\mathbf{k}$. Correct vector or values for x, y and z	M1A1
	$\frac{x-1}{5}=\frac{y+3}{-13}=\frac{z+10}{-34}$	M1: Attempts Cartesian form from their values correctly. **Dependent on both previous M's.** A1: A complete correct equation	dM1A1

	(c) Way 4		
	$l_2:(1+5\lambda)\mathbf{i}+(-2+2\lambda)\mathbf{j}+(3+\lambda)\mathbf{k}$	M1: Attempt l_2 in parametric form correctly A1: Correct	M1A1
	$\begin{pmatrix}1&0&0\\2&-2&1\\-4&1&-1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1+5\lambda\\-2+2\lambda\\3+\lambda\end{pmatrix}\Rightarrow\begin{matrix}x=1+5\lambda\\y=-3-13\lambda\\z=-10-34\lambda\end{matrix}$ M1: Uses $\mathbf{Mx}=l_2$ in parametric form A1: Correct expressions for x, y and z		M1A1
	$\frac{x-1}{5}=\frac{y+3}{-13}=\frac{z+10}{-34}$	M1: Attempts Cartesian form from their values correctly. **Dependent on both previous M's.** A1: A complete correct equation	dM1A1

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AQA FP4

- Invariant points/lines

AQA FP4 JUNE 2015

7 The matrix $\mathbf{A} = \begin{bmatrix} 3.4 & 2 \\ 1.2 & 1 \end{bmatrix}$ represents a transformation that is a shear S followed by a transformation T.

(a) The shear S is such that the image of the point $(1,\ 1)$ is $(5,\ -3)$ and the line $y = -x$ is a line of invariant points. Find the matrix that represents S.

[4 marks]

(b) (i) Hence find the matrix that represents the transformation T.

[4 marks]

(ii) Give a full description of the transformation T.

[2 marks]

Q7	Solution	Mark	Total	Comment
(a)	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -3 \end{bmatrix}$ gives $a+b=5$ $c+d=-3$	B1		both equations correct
	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ gives $a-b=1$ $c-d=-1$	B1		both equations correct
	Hence $a=3, b=2, c=-2, d=-1$	M1		Solving to get at least two correct values
	Matrix is $\begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}$	A1	4	

(b)(i)	$\begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 3.4 & 2 \\ 1.2 & 1 \end{bmatrix}$	**M1**		Multiplication of matrices in correct order to form matrix equation - accept $\mathbf{TS} = \mathbf{A}$
	Hence $\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 3.4 & 2 \\ 1.2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix}^{-1}$	**m1**		Rearranging - correct order on RHS accept $\mathbf{T} = \mathbf{AS}^{-1}$
	$\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 3.4 & 2 \\ 1.2 & 1 \end{bmatrix}\begin{bmatrix} -1 & -2 \\ 2 & 3 \end{bmatrix}$	**B1F**		Correct inverse of their matrix from (a) seen anywhere
	$\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{bmatrix}$	**A1**	**4**	**CAO**
	ALTERNATIVE			
	$\begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 3.4 & 2 \\ 1.2 & 1 \end{bmatrix}$	**(M1)**		Multiplication of matrices in correct order to form matrix equation
	$\begin{bmatrix} 3p-2q & 2p-q \\ 3r-2s & 2r-s \end{bmatrix} = \begin{bmatrix} 3.4 & 2 \\ 1.2 & 1 \end{bmatrix}$	**(A1)**		LHS fully correct
	$3p - 2q = 3.4$ and $2p - q = 2$ Gives $p = 0.6$ and $q = -0.8$			
	$3r - 2s = 1.2$ and $2r - s = 1$ Gives $r = 0.8$ and $s = 0.6$	**(A1)**		Solving to find all correct values
	$\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{bmatrix}$	**(A1)**	**(4)**	**CAO**
(ii)	(Anticlockwise) rotation	**M1**		Matrix must be correct in part **(b)(i)**
	through 53.1^0 (about O)	**A1**	**2**	Correct angle
	Total		**10**	

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AQA FP4 JUNE 2014

6 The plane transformation S is a shear and is represented by the matrix $\begin{bmatrix} a & b \\ c & -2 \end{bmatrix}$, where a, b and c are constants.

(a) Show that $2a + bc = -1$.

[2 marks]

(b) Given further that $(2, 2)$ is an invariant point of S, find the values of a, b and c.

[4 marks]

(c) Show that all lines of the form $y = x + k$, where k is a constant, are invariant lines of S.

[3 marks]

	Total		12	
Q	**Solution**	**Mark**	**Total**	**Comments**
6a)	Determinant = $-2a - bc$ Determinant of **shear** = 1/**shear** leaves area unchanged hence $-2a - bc = 1$ Giving $2a + bc = -1$	M1 A1	 2	Correct evaluation of the determinant Set determinant equal to 1 with **justification** and manipulate correctly to obtain result
b)	Fixed point $\begin{pmatrix} a & b \\ c & -2 \end{pmatrix}\begin{pmatrix} 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ Gives $2a + 2b = 2$ and $2c - 4 = 2$ hence c = 3 $b = -3$ $a = 4$	 M1 A1 A1 A1	 4	 Correct use of fixed point to set up two equations **A1** each correct value

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c)	$\begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ x+k \end{pmatrix} = \begin{pmatrix} 4x - 3x - 3k \\ 3x - 2x - 2k \end{pmatrix}$	M1		Correct substitution and multiplication of their values form part b) – can be unsimplified
	$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} x - 3k \\ x - 2k \end{pmatrix}$			
	$x' + k = x - 3k + k$ $= x - 2k$ $= y'$	m1		Both components correctly simplified and attempt to show that $y' = x' + k$ works
	Hence $y' = x' + k$	A1	3	Fully shown – **CSO**
	ALTERNATIVE for c)			
	$\begin{pmatrix} 4 & -3 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ mx+k \end{pmatrix} = \begin{pmatrix} 4x - 3mx - 3k \\ 3x - 2mx - 2k \end{pmatrix}$	(M1)		Correct substitution and multiplication of their values form part b) – can be unsimplified
	Using $y' = mx' + k$ $3x - 2mx - 2k = \mathrm{m}(4x - 3mx - 3k) + k$	(m1)		Substitution into $y' = mx' + k$ to obtain a quadratic in m – can be unsimplified
	Giving $3(m-1)^2x + 3(m-1)k = 0$			
	Hence $m = 1$ and k can take any value So $y' = x' + k$ is invariant	(A1)	3	Fully shown - **CSO**
	Total		**9**	

AQA FP4 JUNE 2013

6 The plane transformation T is defined by

$$\mathrm{T}: \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(a) A shape has an area of 3 square units. Find the area of the shape after being transformed by T. *(2 marks)*

(b) (i) Find the equations of all the invariant lines of T. *(5 marks)*

(ii) State the equation of the line of invariant points of T. *(1 mark)*

Q	Solution	Marks	Total	Comments
6(a)	Determinant of matrix $= -8+9=1$	M1		Finding determinant and multiplying by area
	Area $= 3\times 1 = 3$ (square units)	A1	2	CAO – must show multiplication or refer to scale factor/invariant area or equivalent
(b)(i)	$\begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix}\begin{bmatrix} x \\ mx+c \end{bmatrix} = \begin{bmatrix} x' \\ y' \end{bmatrix}$ $\Rightarrow$ $(x') = 4x+3(mx+c)$ $(y') = -3x-2(mx+c)$	M1		x', y' in terms of x, y, m, c
	Invariant lines $\Rightarrow y' = mx' + c$ $\Rightarrow -3x-2mx-2c = 4mx+3m^2x+3mc+c$ $\Rightarrow 0 = (3m^2+6m+3)x + 3mc + 3c$	A1		Use of $y' = mx' + c$
	$\Rightarrow 3m^2+6m+3 = 0 \quad 3mc+3c=0$ $3(m+1)^2 = 0 \quad 3c(m+1)=0$	M1		Attempt at solving equations where coefficients = 0 or compares coefficients
	$\Rightarrow m=-1 \quad c$ can be any value	A1		Finding the correct value of m
	$\Rightarrow$ lines are $y = -x + c$	A1	5	Fully correct line – no restriction on c

(ii)	When $c = 0$, $y = -x$ is a line of invariant points **SPECIAL CASES – (b)(i)** $\begin{pmatrix} 4 & 3 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} x \\ -x+c \end{pmatrix} = \begin{pmatrix} x+3c \\ -x-2c \end{pmatrix}$ $x' = x + 3c$ $y' = -x - 2c$ Consider $-x'+c$ $= -(x+3c)+c$ $= -x-3c+c$ $= -x-2c$ $= y'$ Hence $y = -x + c$ is an invariant line	B1	1	Any equivalent form SC1 – Correct multiplication as shown SC2 – correct multiplication as shown above and full algebraic solution using $y' = -x' + c$
	Total		**8**	

AQA FP4 JAN 2013

6 The linear transformations T_1 and T_2 are represented by the matrices

$$\mathbf{M}_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ and } \mathbf{M}_2 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix}$$

respectively.

(a) Give a full geometrical description of the transformations:

(i) T_1 ; *(2 marks)*

(ii) T_2 . *(3 marks)*

(b) Find the matrix which represents the transformation T_1 followed by T_2 . *(2 marks)*

(c) The linear transformation T_3 is represented by the matrix

$$\mathbf{M}_3 = \begin{bmatrix} k & 2 & -1 \\ 1 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix}$$

where k is a constant.

For one particular value of k, T_3 has a line L of invariant points.

(i) Find k.

(ii) Find the Cartesian equations of L in the form $\frac{x}{p} = \frac{y}{q} = \frac{z}{r}$. *(7 marks)*

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Q	Solution	Marks	Total	Comments
6(a)(i)	Reflection In (the plane) $z=0$ (or in the x–y plane)	M1 A1	 2	Reflection stated for M1 Either version for A1
(ii)	Rotation About the y-axis through $\frac{\pi}{3}$ radians	M1 A1 B1	 3	Rotation stated. y-axis (or 60°)
(b)	$\mathrm{T}_2\ \mathrm{T}_1 = \begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{-\sqrt{3}}{2} & 0 & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{-\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{-\sqrt{3}}{2} & 0 & -\frac{1}{2} \end{bmatrix}$	M1A1	2	M1 correct order of matrices A1 fully correct [N.B $\begin{bmatrix} \frac{1}{2} & 0 & \frac{\sqrt{3}}{2} \\ 0 & 1 & 0 \\ \frac{\sqrt{3}}{2} & 0 & -\frac{1}{2} \end{bmatrix}$ scores M0A0]

(c)(i)	For line of invariants points $\begin{bmatrix} k & 2 & -1 \\ 1 & 1 & 1 \\ 3 & 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	M1		Set up equations – uses $\mathbf{M}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$
	$\Rightarrow kx+2y-z=x \Rightarrow (k-1)x+2y-z=0$ ①			
	$x+y+z=y \quad \Rightarrow x+z=0$ ②	A1		Two equations correct
	$3x+4y+z=z \Rightarrow 3x+4y=0$ ③	A1		All three equations correct
	From ② $z=-x$	M1		Defines variables in terms of one letter
	From ③ $y = \frac{-3x}{4}$			*or* 2 components in $\begin{bmatrix} 4 \\ -3 \\ -4 \end{bmatrix}$ correct
	Substitute in ① $(k-1)x - \frac{3}{2}x + x = 0$	A1		Substitution into other equations
	$x\left[k-1-\frac{3}{2}+1\right]=0$			*or* $\begin{bmatrix} 4 \\ -3 \\ -4 \end{bmatrix}$ correct
	$x\left[k-\frac{3}{2}\right]=0$			
	$x \neq 0 \quad \Rightarrow \quad k=\frac{3}{2}$	A1		k-value obtained.
(ii)	Line $x = \frac{-4}{3}y = -z$			
	$\frac{x}{4} = \frac{y}{-3} = \frac{z}{-4}$	B1cao	7	Or equivalent
	Total		**14**	

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AQA FP4 JAN 2011

8 The plane transformation T is represented by the matrix $\mathbf{M} = \begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix}$.

(a) The quadrilateral $ABCD$ has image $A'B'C'D'$ under T.

Evaluate $\det \mathbf{M}$ and describe the geometrical significance of both its sign and its magnitude in relation to $ABCD$ and $A'B'C'D'$. *(3 marks)*

(b) The line $y = px$ is a line of invariant points of T, and the line $y = qx$ is an invariant line of T.

Show that $p = \frac{1}{2}$ and determine the value of q. *(5 marks)*

(c) (i) Find the 2×2 matrix $\mathbf{R}$ which represents a reflection in the line $y = \frac{1}{2}x$. *(2 marks)*

(ii) Given that T is the composition of a shear, with matrix $\mathbf{S}$, followed by a reflection in the line $y = \frac{1}{2}x$, determine the matrix $\mathbf{S}$ and describe the shear as fully as possible.

(5 marks)

MFP4(cont)

Q	Solution	Marks	Total	Comments
8(a)	Det(**M**) = – 1	B1		
	Magnitude = 1 $\Rightarrow$ area invariant	B1√		FT area s.f.
	– ve sign $\Rightarrow$ cyclic order of vertices is reversed OR "reflection" involved	B1	3	
(b)	**Method 1**			
	Char. Eqn.: $\lambda^2 - 1 = 0 \Rightarrow \lambda = \pm 1$	M1 A1		Finding and solving attempt
	Subst[g]. back: $\lambda = 1 \Rightarrow y = \frac{1}{2}x$	M1 A1		
	and $\lambda = -1 \Rightarrow y = \frac{1}{4}x$	A1	5	
	Method 2			
	$\begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ mx \end{bmatrix} = \begin{bmatrix} (8m-3)x \\ (3m-1)x \end{bmatrix}$	(M1)		Attempted
	Use of $y' = mx'$: $3m - 1 = 8m^2 - 3m$	(M1)		
	Solving a quadratic eqn. in $m = \frac{1}{4}, \frac{1}{2}$	(M1A1)		From $(4m-1)(2m-1) = 0$
	$p = \frac{1}{2}$ and $q = \frac{1}{4}$	(A1)		

(c)	(i) $p = \frac{1}{2} = \tan\theta$ $\Rightarrow \cos 2\theta = \frac{3}{5}$ and $\sin 2\theta = \frac{4}{5}$ $\mathbf{R} = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}$	M1 A1	2	For these attempted and used in a reflection matrix
	(ii) Use $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}\mathbf{S} = \begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix}$	M1		FT their **R**
	S found using inverse matrix	M1		Or equivalent method
	$= \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}\begin{bmatrix} -3 & 8 \\ -1 & 3 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} -13 & 36 \\ -9 & 23 \end{bmatrix}$	A1		
	Shear, parallel to $y = \frac{1}{2}x$	B1		CAO
	mapping (e.g.) $(1, 1) \rightarrow (4.6, 2.8)$	B1√	5	FT any pt. and its image
	Total		**15**	
	TOTAL		**75**	

EDEXCEL IAL F1

- Similar to Edexcel FP1, up to 2x2 matrices, inverses, transformations in 2D

EDEXCEL IAL F1 OCT 2020

(i)

$$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix **A**.

(2)

The matrix **B** represents a rotation of 45° clockwise about the origin.

(b) Write down the matrix **B**, giving each element of the matrix in exact form.

(1)

The transformation represented by matrix **A** followed by the transformation represented by matrix **B** is represented by the matrix **C**.

(c) Determine **C**.

(2)

(ii) The trapezium T has vertices at the points (–2, 0), (–2, k), (5, 8) and (5, 0), where k is a positive constant. Trapezium T is transformed onto the trapezium T' by the matrix

$$\begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix}$$

Given that the area of trapezium T' is 510 square units, calculate the exact value of k.

(5)

(Total for question = 10 marks)

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Question Number	Scheme	Notes	Marks
(i)(a)	Stretch scale factor 3 parallel to the y-axis	Stretch (<u>not</u> enlargement)	B1
		Scale factor 3 parallel to the y-axis. Allow, e.g. '3 times y values', 'y increased by 3 factor', or similar. Allow, e.g. 'direction of y', 'along y', 'vertical', or similar. Ignore any mention of the origin. If additional transformations are included, send to Review	B1
			(2)
(b)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$	Correct matrix. $\frac{1}{\sqrt{2}}$ may be seen rather than $\frac{\sqrt{2}}{2}$	B1
			(1)
(c)	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$	Attempt to multiply the right way round, i.e. **BA**, not **AB** At least two correct terms (for their matrix **B**) are needed to indicate a correct multiplication attempt	M1
	$\begin{pmatrix} \frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{3\sqrt{2}}{2} \end{pmatrix}$ or equiv. e.g. $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 3 \\ -1 & 3 \end{pmatrix}$	Correct matrix	A1
			(2)

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(ii)	Trapezium area$=\frac{1}{2}(5+2)(k+8)$	Correct method for the area of the trapezium	M1
	$\begin{vmatrix} 5 & 1 \\ -2 & 3 \end{vmatrix} = 5\times 3-(-2)\times 1 = 17$	Correct method for the determinant	M1
		17 (Allow $\pm$ 17)	A1
	$\frac{1}{2}(5+2)(k+8)\times 17 = 510 \Rightarrow k = \ldots$	Multiplies their trapezium area by their determinant, sets equal to 510 and solves for k. Or equivalently: Equates their trapezium area to (510 ÷ determinant) and solves for k	M1
	$k=\frac{4}{7}$	$\frac{4}{7}$ or exact equivalent. If additional answers such as $-\frac{4}{7}$ are given and not rejected, this is A0	A1
			(5)
(ii) Way 2	$\begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} -2 & -2 & 5 & 5 \\ 0 & k & 8 & 0 \end{pmatrix} = \begin{pmatrix} -10 & -10+k & 33 & 25 \\ 4 & 4+3k & 14 & -10 \end{pmatrix}$	Multiplies correct matrices to find the coordinates for T'	2[nd] M
		Correct coordinates (can be left in matrix form)	A1
	$\frac{1}{2}[-10(4+3k)+14(-10+k)-330+100-4(-10+k)-33(4+3k)-350-100]$	Correct method for area of T' ('shoelace rule' with or without a modulus), using their coordinates for T'	1[st] M
	$\pm\frac{1}{2}(952+119k) = 510$, $k = \ldots$	Sets area of T' equal to 510 and solves for k	M1
	$k=\frac{4}{7}$	$\frac{4}{7}$ or exact equivalent. If additional answers such as $-\frac{4}{7}$ are given and not rejected, this is A0	A1
			Total 10

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EDEXCEL IAL F1 JAN 2020

$$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 1 & -4 \end{pmatrix}$$

The transformation represented by **A** maps the point $R(3p - 13, p - 4)$, where p is a constant, onto the point $R'(7, -2)$

(a) Determine the value of p

(3)

The point S has coordinates $(0, 7)$

Given that O is the origin,

(b) determine the area of triangle ORS

(2)

The transformation represented by **A** maps the triangle ORS onto the triangle $OR'S'$

(c) Hence, using your answer to part (b), determine the area of triangle $OR'S'$

(2)

(Total for question = 7 marks)

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	$\mathbf{A}=\begin{pmatrix}2&3\\1&-4\end{pmatrix}$; $A: R(3p-13,\ p-4) \mapsto R'(7,-2)$		
(a) **Way 1**	$\left\{\begin{pmatrix}x_{R'}\\y_{R'}\end{pmatrix} = \begin{pmatrix}2&3\\1&-4\end{pmatrix}\begin{pmatrix}3p-13\\p-4\end{pmatrix} = \right\}$ $= \begin{pmatrix}2(3p-13)+3(p-4)\\1(3p-13)-4(p-4)\end{pmatrix}$	Correct method of multiplying out either $\begin{pmatrix}2&3\end{pmatrix}\begin{pmatrix}3p-13\\p-4\end{pmatrix}$ or $\begin{pmatrix}1&-4\end{pmatrix}\begin{pmatrix}3p-13\\p-4\end{pmatrix}$ to give a linear expression in terms of p for either $x_{R'}$ or $y_{R'}$ **Note:** Allow one slip in their multiplication	M1
	• $2(3p-13)+3(p-4)=7 \Rightarrow p = ...$ • $1(3p-13)-4(p-4)=-2 \Rightarrow p = ...$	**dependent on the previous M mark** Solves either their $x_{R'}=7$ or their $y_{R'}=-2$ to give $p = ...$	dM1
	$\{9p-38=7$ or $-p+3=-2 \Rightarrow\}$ $p=5$	$p=5$	A1
			(3)
(a) **Way 2**	$\{\mathbf{AR}=\mathbf{R}' \Rightarrow \mathbf{R}=\mathbf{A}^{-1}\mathbf{R}' \Rightarrow\}$		
	$\mathbf{R}=\dfrac{1}{-8-3}\begin{pmatrix}-4&-3\\-1&2\end{pmatrix}\begin{pmatrix}7\\-2\end{pmatrix}=\begin{pmatrix}2\\1\end{pmatrix}$	Applies $\mathbf{A}^{-1}\begin{pmatrix}7\\-2\end{pmatrix}$ to find the value for either x_R or y_R **Note:** Allow one slip in finding $\mathbf{A}^{-1}$	M1
	• $3p-13=2 \Rightarrow p = ...$ • $p-4=1 \Rightarrow p = ...$	**dependent on the previous M mark** Solves either $3p-13=$ their x_R or $p-4=$ their y_R to give $p = ...$	dM1
	$p=5$	$p=5$	A1
			(3)
(a) **Way 3**	$\{\mathbf{AR}=\mathbf{R}' \Rightarrow\}$ $\begin{pmatrix}2&3\\1&-4\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}7\\-2\end{pmatrix}$ $\begin{matrix}2a+3b=7\\a-4b=-2\end{matrix}$ $\Rightarrow a=2$ or $b=1$	Correct method of applying $\begin{pmatrix}2&3\\1&-4\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix}=\begin{pmatrix}7\\-2\end{pmatrix}$ to form a pair of simultaneous equations and attempts to find either $a = ...$ or $b = ...$ **Note:** Allow one slip in their multiplication	M1
	• $3p-13=2 \Rightarrow p = ...$ • $p-4=1 \Rightarrow p = ...$	**dependent on the previous M mark** Solves either $3p-13=$ their a or $p-4=$their b to give $p = ...$	dM1
	$p=5$	$p=5$	A1
			(3)

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(b) **Way 1**	$\{R(3(5)-13,\ 5-4) = R(2,1)\}$ $\{\text{Area}(ORS) = \}\ \frac{1}{2}(7)("2")$	A correct method for finding their x_R and applies $\frac{1}{2}(7)(\text{their } x_R)$	M1
	$= 7\ (\text{units})^2$	7	A1 cao
			(2)
(c)	$\{\text{Area}(OR'S') = \}\ \lvert 2(-4)-3(1) \rvert \times (7)$	$\pm(2(-4)-3(1)) \times (\text{their area}(ORS))$	M1
	$= \quad 77$	Correct answer of 77, which must be positive Only allow follow through of the value for $11\times$ their positive answer to (b)	A1 ft
			(2)
			7

Question Number	Scheme	Notes	Marks
(b) **Way 2**	$\{\text{Area } (ORS)\}$ $= \frac{1}{2}\begin{vmatrix} 0 & 2 & 0 & 0 \\ 0 & 1 & 7 & 0 \end{vmatrix} = \frac{1}{2}\lvert(0+14+0)-(0+0+0)\rvert$	A correct method for finding their $R(2,\ 1)$ with a complete applied method for finding area(ORS) using $S(0,\ 7)$ and their $R(2,\ 1)$	M1
	$= 7\ (\text{units})^2$	7	A1 cao
			(2)

Question Notes

	Note	$ORS \mapsto OR'S' \Rightarrow \begin{pmatrix} 0 & 2 & 0 \\ 0 & 1 & 7 \end{pmatrix} \mapsto \begin{pmatrix} 0 & 7 & 21 \\ 0 & -2 & -28 \end{pmatrix}$
(b) **Way 1**	**Note**	A correct method for finding their x_R includes any of • $x_R = 3("5")-13 = 2$, where $p = "5"$ is found using part (a), Way 1 • their x_R found by applying $\mathbf{A}^{-1}\mathbf{R}'$ using part (a), Way 2 • $x_R =$ their a found using part (a), Way 3
(b) **Way 2**	**Note**	Give M1 A1 for $\frac{1}{2}\begin{vmatrix} 2 & 0 \\ 1 & 7 \end{vmatrix} = \frac{1}{2}\lvert 14-0 \rvert = 7$
	Note	Give M0 A0 for $\begin{vmatrix} 0 & 2 & 0 & 0 \\ 0 & 1 & 7 & 0 \end{vmatrix} = \lVert(0+14+0)-(0+0+0)\rVert = 14$
	Note	There are other ways to find Area(ORS). All ways require a complete correct method for the M mark and a correct area of 7 for the A mark.
	Note	Give M1 for $\frac{1}{2}(1)("2") + \frac{1}{2}(6)("2")$ as this method is equivalent to writing $\frac{1}{2}(7)("2")$
	Note	Give M0 for the calculation $\frac{1}{2}(7)(7)\ \left\{= \frac{49}{2}\right\}$

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(c)	**Note**	Give M1 A0 for applying $(2(-4)-3(1))\times(7)$ to give -77 with no reference to 77
	Note	Part (c) requires the use of the answer to part (b). So give M0 A0 for • Area $(OR'S') = \frac{1}{2}\begin{vmatrix} 0 & 7 & 21 & 0 \\ 0 & -2 & -28 & 0 \end{vmatrix} = \frac{1}{2}\lvert(0-196+0)-(0-42+0)\rvert = \frac{1}{2}(154) = 77$ • Area $(OR'S') = \frac{1}{2}\begin{vmatrix} 7 & 21 \\ -2 & -28 \end{vmatrix} = \frac{1}{2}\lvert(-196)-(-42)\rvert = \frac{1}{2}(154) = 77$ • Area $(OR'S') = (28)(21) - \frac{1}{2}(21)(28) - \frac{1}{2}(7)(2) - \frac{1}{2}(2+28)(14)$ $= 588 - 294 - 7 - 210 = 77$
	Note	Allow M1 A1 for • $\dfrac{\begin{vmatrix} 7 & 21 \\ -2 & -28 \end{vmatrix}}{\begin{vmatrix} 2 & 0 \\ 1 & 7 \end{vmatrix}} \times 7 = \dfrac{\lvert(-196)-(-42)\rvert}{\lvert 14-0 \rvert} \times 7 = \dfrac{154}{14} \times 7 = 11 \times 7 = 77$

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$$\mathbf{P} = \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$$

(a) Show that $\mathbf{P}^3 = 8\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix.

(3)

(b) Describe fully the transformation represented by the matrix **P** as a combination of two simple geometrical transformations.

(4)

(c) Find the matrix $\mathbf{P}^{35}$, giving your answer in the form

$$\mathbf{P}^{35} = 2^k \begin{pmatrix} -1 & a \\ b & -1 \end{pmatrix}$$

where k is an integer and a and b are surds to be found.

(2)

(Total for question = 9 marks)

Question Number	Scheme	Notes	Marks
	$\mathbf{P}=\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}$; (a) $\mathbf{P}^3=8\mathbf{I}$; (c) $\mathbf{P}^{35}=2^k\begin{pmatrix}-1 & a\\ b & -1\end{pmatrix}$		
(a)	$\{\mathbf{P}^2=\}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}=\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$	Finds $\mathbf{P}^2$ (which can be un-simplified) with at least 3 correct elements for $\mathbf{P}^2$	M1
	$\{\mathbf{P}^3=\}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}=\begin{pmatrix}8 & 0\\ 0 & 8\end{pmatrix}=8\mathbf{I}$ * or $\{\mathbf{P}^3=\}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}=\begin{pmatrix}8 & 0\\ 0 & 8\end{pmatrix}=8\mathbf{I}$ *	**dependent on the previous M mark** Multiplies $\mathbf{P}^2$ by $\mathbf{P}$ or multiplies $\mathbf{P}$ by $\mathbf{P}^2$ to give a 2×2 matrix of 4 elements for $\mathbf{P}^3$ with at least 2 correct elements	dM1
		Correct proof with no errors	A1 *
			(3)
(b)	Enlargement	Enlargement or enlarge or dilation	M1
	Centre $(0, 0)$ with scale factor 2	about $(0, 0)$ or about O or about the origin **and** scale or factor or times **and** 2	A1
	Rotation	Rotation or rotate (condone turn)	M1
	120 degrees (anticlockwise) about $(0, 0)$	**Both** 120 degrees or $\frac{2\pi}{3}$ or 240 degrees clockwise or $\frac{4\pi}{3}$ clockwise **and** about $(0, 0)$ or about O or about the origin	A1
			(4)

(c) Way 1	$\mathbf{P}^{35} = (\mathbf{P}^3)^{11} \times \mathbf{P}^2$ or	$\mathbf{P}^{35} = \mathbf{P}^{33} \times \mathbf{P}^2$		
	$= (8\mathbf{I})^{11} \times \begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix}$	$= (2\mathbf{I})^{33} \times \begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix}$	$((8\mathbf{I})^{11}$ or $(8)^{11}) \times$ (their $\mathbf{P}^2$) or $((2\mathbf{I})^{33}$ or $(2)^{33}) \times$ (their $\mathbf{P}^2$)	M1
	$= 2^{34}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$		Correct answer **Note:** $k = 34,\ a = \sqrt{3},\ b = -\sqrt{3}$	A1
				(2)
(c) Way 2	$\mathbf{P}^{35} = (\mathbf{P}^3)^{12} \times \mathbf{P}^{-1}$ or $\mathbf{P}^{35} = \mathbf{P}^{36} \times \mathbf{P}^{-1}$			
	$= (8\mathbf{I})^{12} \times \dfrac{1}{(-1)(-1) - (-\sqrt{3})(\sqrt{3})}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$ or $= (2\mathbf{I})^{36} \times \dfrac{1}{(-1)(-1) - (-\sqrt{3})(\sqrt{3})}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$		$((8\mathbf{I})^{12}$ or $(8)^{12}) \times \dfrac{1}{\text{their det}(\mathbf{P})}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$ or $((2\mathbf{I})^{36}$ or $(2)^{36}) \times \dfrac{1}{\text{their det}(\mathbf{P})}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$ where their $\det(\mathbf{P}) > 1$	M1
	$\left\{= (2^{36})\left(\dfrac{1}{4}\right)\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}\right\} = 2^{34}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$		Correct answer **Note:** $k = 34,\ a = \sqrt{3},\ b = -\sqrt{3}$	A1
				(2)
				9

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	Question Notes	
(a)	**Note**	Proof must contain the final steps of $= \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ **and** $= 8\mathbf{I}$ **or** $= \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix}$ **and** $=$ RHS
	Note	Other acceptable proofs for M1 dM1 A1 include • $\mathbf{P}^3 = \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$ or $\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}^3$ $= \begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = 8\mathbf{I}$ * • $\mathbf{P}^3 = \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$ or $\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}^3$ $= \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = 8\mathbf{I}$ * • $\mathbf{P}^3 = \begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = 8\mathbf{I}$ * • $\mathbf{P}^3 = \begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}\begin{pmatrix} -2 & 2\sqrt{3} \\ -2\sqrt{3} & -2 \end{pmatrix} = \begin{pmatrix} 8 & 0 \\ 0 & 8 \end{pmatrix} = 8\mathbf{I}$ *

(b)	**Note**	"original point" is not acceptable in place of the word "origin".
	Note	"expand" is 1st M0
	Note	"enlarge x by 2 and no change in y" is 1st M0 1st A0
	Note	Writing "120 degrees" by itself implies by convention "120 degrees anti-clockwise". So • "Rotation 120 degrees about O" is 2nd M1 2nd A1 • "Rotation 120 degrees clockwise about O" is 2nd M1 2nd A0
	Note	Writing down "centre $(0, 0)$ with scale factor 2" with no reference to "enlargement" or "enlarge" or "dilation" is 1st M0 1st A0
	Note	Writing down "120 degrees anti-clockwise about O" with no reference to "rotation" or "turn" is 2nd M0 2nd A0
	Note	Give 1st M1 1st A0 for writing "stretch parallel to x-axis and y-axis"
	Note	Give 1st M1 1st A0 for writing "stretch scale factor 2 parallel to x-axis and stretch scale factor 2 parallel to y-axis {with centre $(0, 0)$ }"
	Note	If a candidate would score M1 A1 M1 A1 in part (b) and there is an error in their solution (e.g. a third transformation given) then give M1 A1 M1 A0
(c)	**Note**	$8^{11} = 2^{33} = 8589934592$
	Note	$8^{12} = 2^{36} = 68719476736$
	Note	(their $\mathbf{P}^2$) must be a genuine attempt at $\mathbf{P}^2$ or must be for (their $\mathbf{P}^2$) seen in part (a)
	Note	Allow M1 A1 for writing $\mathbf{P}^{35} = 2^{34}\begin{pmatrix} -1 & \sqrt{3} \\ -\sqrt{3} & -1 \end{pmatrix}$
	Note	Stating $k = 34,\ a = \sqrt{3},\ b = -\sqrt{3}$ from no working is M1 A1
	Note	Give M0 A0 for $\mathbf{P}^4 = 2^3\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix} \Rightarrow \mathbf{P}^{35} = 2^{34}\begin{pmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{pmatrix}$

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	Question	Notes Continued
(c)	Note	Writing down $(8\mathbf{I})^{11}\times\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}$ or $(2\mathbf{I})^{33}\times\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}$ or $(8\mathbf{I})^{11}\times\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}^2$ or $(2\mathbf{I})^{33}\times\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}^2$ **with no attempt** to evaluate $\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}\begin{pmatrix}-1 & -\sqrt{3}\\ \sqrt{3} & -1\end{pmatrix}$ is M0
	Note	Allow M1 for **applying** $\mathbf{P}^{35}=(\mathbf{P}^3)^{11}\times\mathbf{P}^2$ or $\mathbf{P}^{35}=\mathbf{P}^{33}\times\mathbf{P}^2$ E.g. Allow M1 for $\begin{pmatrix}8 & 0\\ 0 & 8\end{pmatrix}^{11}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$ or $\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}^{33}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$ or $\begin{pmatrix}8^{11} & 0\\ 0 & 8^{11}\end{pmatrix}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$ or $\begin{pmatrix}2^{33} & 0\\ 0 & 2^{33}\end{pmatrix}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$ or $(8)^{11}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$ or $(2)^{33}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}-2 & 2\sqrt{3}\\ -2\sqrt{3} & -2\end{pmatrix}$
	Note	Allow M1 for $(2)^{35}\begin{pmatrix}\cos 240 & -\sin 240\\ \sin 240 & \cos 240\end{pmatrix}$ **or** $(2)^{35}\begin{pmatrix}\cos 4200 & -\sin 4200\\ \sin 4200 & \cos 4200\end{pmatrix}$ **or** $(2)^{35}\begin{pmatrix}-0.5 & \frac{\sqrt{3}}{2}\\ -\frac{\sqrt{3}}{2} & -0.5\end{pmatrix}$ **or** equivalent in radians
	Note	Give M0 for $\mathbf{P}^{35}=(\mathbf{P}^3)^{11}\times\mathbf{P}^2$ by itself
	Note	Give M0 for $\mathbf{P}^{35}=\mathbf{P}^{33}\times\mathbf{P}^2$ by itself

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$$\mathbf{A} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation represented by the matrix **A**.

(3)

(b) Hence write down the matrix $\mathbf{A}^6$

(1)

The transformation represented by the matrix **C** followed by the transformation represented by the matrix **B** is equivalent to the transformation represented by the matrix **A**.

Given that

$$\mathbf{B} = \begin{pmatrix} 2\sqrt{3} & -7 \\ -4 & 5\sqrt{3} \end{pmatrix}$$

(c) find the matrix **C**, giving your answer in simplest form.

(4)

(Total for question = 8 marks)

Question Number	Scheme	Notes	Marks
	$\mathbf{A} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2\sqrt{3} & -7 \\ -4 & 5\sqrt{3} \end{pmatrix}$		
(a)	Rotation	Rotation or rotate (condone turn)	B1
	60 degrees {anti-clockwise}	60 degrees or $\frac{\pi}{3}$ or 300 degrees clockwise or $\frac{5\pi}{3}$ clockwise	B1 o.e.
	about $(0, 0)$	**This mark is dependent on at least one of the previous B marks being given.** about $(0, 0)$ or about O or about the origin	dB1
	Note: Give 2nd B0 for 60 degrees clockwise o.e.		**(3)**
(b)	$\{\mathbf{A}^6 =\} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	Correct matrix	B1
			(1)
(c) Way 1	$\mathbf{B}^{-1} = \frac{1}{2}\begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix}$	Correct matrix for $\mathbf{B}^{-1}$, which can be simplified or un-simplified	B1
	$\{\mathbf{C} = \mathbf{B}^{-1}\mathbf{A}\} = \frac{1}{2}\begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \ldots$	Applies (their $\mathbf{B}^{-1}$)$\mathbf{A}$, where (their $\mathbf{B}^{-1}$) $\neq \mathbf{B}$, and finds at least one element (or at least one element calculation) of their matrix $\mathbf{C}$ **Note:** Allow one slip in copying down $\mathbf{A}$	M1
	$= \frac{1}{2}\begin{pmatrix} 6\sqrt{3} & -4 \\ 5 & -\sqrt{3} \end{pmatrix}$ or $= \begin{pmatrix} 3\sqrt{3} & -2 \\ \frac{5}{2} & -\frac{1}{2}\sqrt{3} \end{pmatrix}$	**dependent on the previous B1M1 marks** At least 2 elements in $\mathbf{C}$ are correct	A1
		All elements in $\mathbf{C}$ are correct	A1
			(4)
(c) Way 2	$\{\mathbf{BC} = \mathbf{A} \Rightarrow\}$ $\begin{pmatrix} 2\sqrt{3} & -7 \\ -4 & 5\sqrt{3} \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	Correct statement using 2×2 matrices. All 3 matrices must contain four elements. **Can be implied by the 4 correct equations that are below.**	B1
	$2\sqrt{3}a - 7c = \frac{1}{2}$, $2\sqrt{3}b - 7d = -\frac{\sqrt{3}}{2}$ $-4a + 5\sqrt{3}c = \frac{\sqrt{3}}{2}$, $-4b + 5\sqrt{3}d = \frac{1}{2}$ and finds at least one of either a, b, c or d	Applies $\mathbf{BC} = \mathbf{A}$ and attempts to solve simultaneous equations in a and c **or** b and d **and** finds at least one of either a, b, c or d	M1
	$= \frac{1}{2}\begin{pmatrix} 6\sqrt{3} & -4 \\ 5 & -\sqrt{3} \end{pmatrix}$ or $= \begin{pmatrix} 3\sqrt{3} & -2 \\ \frac{5}{2} & -\frac{1}{2}\sqrt{3} \end{pmatrix}$	**dependent on the previous B1M1 marks** At least 2 elements in $\mathbf{C}$ are correct	A1
	or $a = 3\sqrt{3}, b = -2, c = \frac{5}{2}, d = -\frac{1}{2}\sqrt{3}$	All elements in $\mathbf{C}$ are correct	A1
			(4)
			8

	Question Notes	
(a)	Note	Writing "60 degrees" by itself implies by convention "60 degrees anti-clockwise". So, • "Rotation 60 degrees about O" is B1 B1 B1 • "Rotation 60 degrees clockwise about O" is B1 B0 B1
	Note	Writing down "60 degrees anti-clockwise about O" with no reference to "rotation" or "turn" is B0 B1 B1
	Note	"original point" is not acceptable in place of the word "origin".
	Note	Give B0 B0 B0 for a combination of 2 or more transformations.
(b)	Note	Give B0 for writing down $\mathbf{I}$ without reference to $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
	Note	Allow B1 for writing down $\mathbf{I}_2$ without reference to $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
(c)	Note	Allow B1 for $\dfrac{1}{(2\sqrt{3})(5\sqrt{3})-(-7)(-4)}\begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix}$ or $\dfrac{1}{30-28}\begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix}$
	Note	Allow B1 for $\begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix}\dfrac{1}{(2\sqrt{3})(5\sqrt{3})-(-7)(-4)}$ or $\begin{pmatrix} 5\sqrt{3} & 7 \\ 4 & 2\sqrt{3} \end{pmatrix}\dfrac{1}{30-28}$
	Note	You can ignore previous working prior to their finding $\mathbf{B}^{-1}\mathbf{A}$ (i.e. you can ignore an incorrect statement such as $\mathbf{A}=\mathbf{CB}$)

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The transformation represented by the 2×2 matrix **P** is an anticlockwise rotation about the origin through 45 degrees.

(a) Write down the matrix **P**, giving the exact numerical value of each element.

(1)

$$\mathbf{Q} = \begin{pmatrix} k\sqrt{2} & 0 \\ 0 & k\sqrt{2} \end{pmatrix}, \text{ where } k \text{ is a constant and } k > 0$$

(b) Describe fully the single geometrical transformation represented by the matrix **Q**.

(2)

The combined transformation represented by the matrix **PQ** transforms the rhombus R_1 onto the rhombus R_2.

The area of the rhombus R_1 is 6 and the area of the rhombus R_2 is 147

(c) Find the value of the constant k.

(4)

(Total for question = 7 marks)

Question Number	Scheme	Notes	Marks
	P represents an anti-clockwise rotation about the origin through 45 degrees		
(a)	$\{\mathbf{P} =\}\ \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$ or $\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$ or e.g. $\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$	Correct matrix which is expressed in exact surds	B1
			(1)
(b)	Enlargement	Enlargement or enlarge	M1
	Centre $(0, 0)$ with scale factor $k\sqrt{2}$	About $(0, 0)$ or about O or about the origin and scale or factor or times and $k\sqrt{2}$ **Note:** Allow $\sqrt{2k^2}$ in place of $k\sqrt{2}$	A1
	Note: Give M0A0 for combinations of transformations		**(2)**

(c) Way 1	$\{\mathbf{PQ} =\}\begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}\begin{pmatrix} k\sqrt{2} & 0 \\ 0 & k\sqrt{2} \end{pmatrix} = \begin{pmatrix} k & -k \\ k & k \end{pmatrix}$	Multiplies their matrix from part (a) by **Q [either way round]** and applies "$ad - bc$" to the resulting matrix	M1
	$\{\det \mathbf{PQ} =\}\ (k)(k) - (-k)(k) = 2k^2$	to give $2k^2$ or states $\lvert$their $\det \mathbf{PQ}\rvert = 2k^2$ Condone $\{\det \mathbf{PQ} =\}\ k^2 + k^2$	A1
	$6(2k^2) = 147$ or $2k^2 = \frac{147}{6}$	6(their determinant) $= 147$ or puts their determinant **equal to** $\frac{147}{6}$	M1
	$\left\{\Rightarrow k^2 = \frac{49}{4} \Rightarrow\right\}\ k = \frac{7}{2}$	Obtains $k = 3.5$, o.e.	A1
			(4)
(c) Way 2	$\det \mathbf{Q} = \left(k\sqrt{2}\right)\left(k\sqrt{2}\right) - (0)(0)$ or $\det \mathbf{Q} = \left(k\sqrt{2}\right)\left(k\sqrt{2}\right)$	applies "$ad - bc$" to **Q** or applies $\left(k\sqrt{2}\right)^2$	M1
	$\{\det \mathbf{P} = 1 \Rightarrow\}\ \det \mathbf{PQ} = (1)\left(2k^2\right) = 2k^2$ or $\det \mathbf{Q} = 2k^2$	and deduces that $\det \mathbf{PQ} = 2k^2$ or states $\lvert$their $\det \mathbf{PQ}\rvert = 2k^2$ or $\det \mathbf{Q} = 2k^2$	A1
	$6(2k^2) = 147$ or $2k^2 = \frac{147}{6}$	$6(\text{their } \det(\mathbf{PQ})) = 147$ or $(\text{their } \det(\mathbf{PQ})) = \frac{147}{6}$ or $6(\text{their } \det(\mathbf{Q})) = 147$ or $(\text{their } \det(\mathbf{PQ})) = \frac{147}{6}$	M1
	$\left\{\Rightarrow k^2 = \frac{49}{4} \Rightarrow\right\}\ k = \frac{7}{2}$	Obtains $k = 3.5$, o.e.	A1
			(4)
			7

	Question Notes	
(b)	**Note**	"original point" is not acceptable in place of the word "origin".
	Note	"expand" is not acceptable for M1
	Note	"enlarge x by $k\sqrt{2}$ and no change in y" is M0A0
(c)	**Note**	Obtaining $k = \pm 3.5$ with no evidence of $k = 3.5$ {only} is A0
	Way 2 Note 1	Give M1A1M0A0 for writing down $147(2k^2) = 6$ **or** $\dfrac{1}{2k^2} = \dfrac{147}{6}$ **or** $6\left(\dfrac{1}{2k^2}\right) = 147$, o.e. with no other supporting working.
	Way 2 Note 2	Give M0A0M1A0 for writing $\det \mathbf{Q} = \dfrac{1}{k^2 - (-k^2)}$ or $\dfrac{1}{2k^2}$, followed by $6\left(\dfrac{1}{2k^2}\right) = 147$
	Note	Allow M1A1 for an incorrect rotation matrix **P**, leading to $\det \mathbf{PQ} = 2k^2$
	Note	Allow M1A1M1A1 for an incorrect rotation matrix **P**, leading to $\det \mathbf{PQ} = 2k^2$ and $k = 3.5$, o.e.
	Note	Using the scale factor of enlargement to write down $k\sqrt{2} = \sqrt{\dfrac{147}{6}} \Rightarrow k = 3.5$ is M1A1dM1A1
	Note	Using the scale factor of enlargement to write down $k\sqrt{2} = \sqrt{\dfrac{6}{147}}$ is M1A1dM0

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$$\mathbf{A} = \begin{pmatrix} 2p & 3q \\ 3p & 5q \end{pmatrix}$$

where p and q are non-zero real constants.

(a) Find $\mathbf{A}^{-1}$ in terms of p and q.

(3)

Given $\mathbf{XA} = \mathbf{B}$, where

$$\mathbf{B} = \begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix}$$

(b) find the matrix $\mathbf{X}$, giving your answer in its simplest form.

(4)

(Total for question = 7 marks)

Question Number	Scheme	Notes	Marks
	$\mathbf{A} = \begin{pmatrix} 2p & 3q \\ 3p & 5q \end{pmatrix}$; $\mathbf{XA} = \mathbf{B}$; $\mathbf{B} = \begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix}$		
(a)	$\{\det(\mathbf{A}) =\}\ 2p(5q) - (3p)(3q)\ \ \{= pq\}$	$2p(5q) - (3p)(3q)$ which can be un-simplifed or simplifed	B1
	$\{\mathbf{A}^{-1} =\}\ \dfrac{1}{pq}\begin{pmatrix} 5q & -3q \\ -3p & 2p \end{pmatrix}$ or $\begin{pmatrix} \dfrac{5}{p} & -\dfrac{3}{p} \\ -\dfrac{3}{q} & \dfrac{2}{q} \end{pmatrix}$	$\begin{pmatrix} 5q & -3q \\ -3p & 2p \end{pmatrix}$	M1
		Correct $\mathbf{A}^{-1}$	A1
			(3)

(b) **Way 1**	$\{\mathbf{X}=\mathbf{BA}^{-1}=\}$ $\begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix}\frac{1}{pq}\begin{pmatrix} 5q & -3q \\ -3p & 2p \end{pmatrix}=\ldots$	Attempts $\mathbf{BA}^{-1}$ and finds at least one element (or at least one element calculation) of their matrix $\mathbf{X}$ **Note:** Allow one slip in copying down $\mathbf{B}$ **Note:** Allow one slip in copying down $\mathbf{A}^{-1}$	M1
	$=\frac{1}{pq}\begin{pmatrix} 2pq & -pq \\ -3pq & 4pq \\ pq & pq \end{pmatrix}$	At least 4 correct elements (need not be in a matrix)	A1
		dependent on the first M mark Finds a 3×2 matrix of 6 elements	dM1
	$=\begin{pmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 1 \end{pmatrix}$	Correct simplified matrix for $\mathbf{X}$	A1
			(4)
(b) **Way 2**	$\{\mathbf{XA}=\mathbf{B}\Rightarrow\}\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}\begin{pmatrix} 2p & 3q \\ 3p & 5q \end{pmatrix}=\begin{pmatrix} p & q \\ 6p & 11q \\ 5p & 8q \end{pmatrix}$ $2pa+3pb=p,\ \ 3qa+5qb=q$ **or** $2pc+3pd=6p,\ \ 3qc+5qd=11q$ **or** $2pe+3pf=5p,\ \ 3qe+5qf=8q$ **and finds at least one of** a, b, c, d, e **or** f	Applies $\mathbf{XA}=\mathbf{B}$ for a 3×2 matrix $\mathbf{X}$ and attempts simultaneous equations in a and b **or** c and d **or** e and f to find at least one of a, b, c, d, e or f **Note:** Allow one slip in copying down $\mathbf{A}$ **Note:** Allow one slip in copying down $\mathbf{B}$	M1
	$\begin{Bmatrix} 2a+3b=1, & 3a+5b=1 \\ 2c+3d=6, & 3c+5d=11 \\ 2e+3f=5, & 3e+5f=8 \end{Bmatrix}\Rightarrow\begin{matrix} a=2,\ b=-1 \\ c=-3,\ d=4 \\ e=1,\ f=1 \end{matrix}$	At least 4 correct elements	A1
		dependent on the first M mark Finds all 6 elements for the 3×2 matrix $\mathbf{X}$	dM1
	$\Rightarrow\mathbf{X}=\begin{pmatrix} 2 & -1 \\ -3 & 4 \\ 1 & 1 \end{pmatrix}$	Correct simplified matrix for $\mathbf{X}$	A1
			(4)

	Question Notes	
(a)	**Note**	Condone $\dfrac{1}{10pq-9pq}\begin{pmatrix} 5q & -3q \\ -3p & 2p \end{pmatrix}$ or $\dfrac{1}{2p(5q)-(3p)(3q)}\begin{pmatrix} 5q & -3q \\ -3p & 2p \end{pmatrix}$ for A1
	Note	Condone $\begin{pmatrix} 5q & -3q \\ -3p & 2p \end{pmatrix}\dfrac{1}{pq}$ or $\begin{pmatrix} 5q & -3q \\ -3p & 2p \end{pmatrix}\dfrac{1}{2p(5q)-(3p)(3q)}$ for A1
	Note	Condone $\begin{pmatrix} \dfrac{5q}{pq} & -\dfrac{3q}{pq} \\ -\dfrac{3p}{pq} & \dfrac{2p}{pq} \end{pmatrix}$ for A1
(b)	**Note**	**Way 1:** Allow SC 1st A1 for at least 4 correct elements in $\begin{pmatrix} \dfrac{2pq}{\text{their det }\mathbf{A}} & \dfrac{-pq}{\text{their det }\mathbf{A}} \\ \dfrac{-3pq}{\text{their det }\mathbf{A}} & \dfrac{4pq}{\text{their det }\mathbf{A}} \\ \dfrac{pq}{\text{their det }\mathbf{A}} & \dfrac{pq}{\text{their det }\mathbf{A}} \end{pmatrix}$ or for at least 4 of these elements seen in their calculations

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(i)

$$\mathbf{A} = \begin{pmatrix} 6 & k \\ -3 & -4 \end{pmatrix}, \text{ where } k \text{ is a real constant, } k \neq 8$$

Find, in terms of k,

(a) $\mathbf{A}^{-1}$

(3)

(b) $\mathbf{A}^2$

(1)

Given that $\mathbf{A}^2 + 3\mathbf{A}^{-1} = \begin{pmatrix} 5 & 9 \\ -3 & -5 \end{pmatrix}$

(c) find the value of k.

(3)

(ii)

(ii)

$$\mathbf{M} = \begin{pmatrix} -\frac{1}{2} & -\sqrt{3} \\ \frac{\sqrt{3}}{2} & -1 \end{pmatrix}$$

The matrix **M** represents a one way stretch, parallel to the y-axis, scale factor p, where $p > 0$, followed by a rotation anticlockwise through an angle θ about (0, 0).

(a) Find the value of p.

(2)

(b) Find the value of θ.

(2)

(Total for question = 11 marks)

Question Number	Scheme	Notes	Marks
	$\mathbf{A}=\begin{pmatrix}6 & k\\ -3 & -4\end{pmatrix}, k\neq 8;\ \mathbf{A}^2+3\mathbf{A}^{-1}=\begin{pmatrix}5 & 9\\ -3 & -5\end{pmatrix};\ \mathbf{M}=\begin{pmatrix}-\frac{1}{2} & -\sqrt{3}\\ \frac{\sqrt{3}}{2} & -1\end{pmatrix}$		
(i)(a)	$\det(\mathbf{A})=6(-4)-(k)(-3)\ \ \{=-24+3k\}$	Correct $\det(\mathbf{A})$ which can be un-simplifed or simplifed	B1
	$\{\mathbf{A}^{-1}=\}\ \dfrac{1}{3k-24}\begin{pmatrix}-4 & -k\\ 3 & 6\end{pmatrix}$	$\begin{pmatrix}-4 & -k\\ 3 & 6\end{pmatrix}$	M1
		Correct $\mathbf{A}^{-1}$	A1
			(3)
(b)	$\{\mathbf{A}^2=\}\begin{pmatrix}36-3k & 6k-4k\\ -18+12 & -3k+16\end{pmatrix}\left\{=\begin{pmatrix}36-3k & 2k\\ -6 & -3k+16\end{pmatrix}\right\}$	Correct $\mathbf{A}^2$ which can be un-simplifed or simplifed	B1
			(1)

		(1)
(c)	• $\begin{pmatrix} 36-3k & 2k \\ -6 & -3k+16 \end{pmatrix} + \frac{3}{3k-24}\begin{pmatrix} -4 & -k \\ 3 & 6 \end{pmatrix} = \begin{pmatrix} 5 & 9 \\ -3 & -5 \end{pmatrix}$ • $36-3k-\frac{12}{3k-24}=5$ • $2k-\frac{3k}{3k-24}=9$ • $-6+\frac{9}{3k-24}=-3$ • $-3k+16+\frac{18}{3k-24}=-5$ **Either** • attempts to form an equation for (their $\mathbf{A}^2$) + 3(their $\mathbf{A}^{-1}$) $=\begin{pmatrix} 5 & 9 \\ -3 & -5 \end{pmatrix}$ in k • or attempts to add an element of (their $\mathbf{A}^2$) to the corresponding element of 3(their $\mathbf{A}^{-1}$) and equates to the corresponding element of the given matrix to form an equation in k	M1
	$\left\{\text{e.g. } -6+\frac{9}{3k-24}=-3\right\} \Rightarrow k=9$ — **dependent on the previous M mark** Solves their equation to give $k = \ldots$	dM1
	Final answer of $k=9$ *only*	A1
		(3)
	Note: Parts (ii)(a) and (ii)(b) can be marked together **Please refer to the notes on the next page when marking (ii)(a) and (ii)(b)**	

(ii)(a)	• $p=\left(-\frac{1}{2}\right)(-1)-\left(-\sqrt{3}\right)\left(\frac{\sqrt{3}}{2}\right)=2$ • $-p\sin\theta=-\sqrt{3}$, $p\cos\theta=-1$ ○ $p=\sqrt{(\pm\sqrt{3})^2+(-1)^2}=2$ ○ $p=\frac{-\sqrt{3}}{-\sin"120°"}=2$ or $p=\frac{-1}{\cos"120°"}=2$	Attempts $p=\pm\frac{1}{2}\pm\left(\sqrt{3}\right)\left(\frac{\sqrt{3}}{2}\right)$ or uses a full method of trigonometry to find $p=\ldots$	M1
		$p=2$ only	A1
			(2)
(b)	$\cos\theta=-\frac{1}{2}$, $\sin\theta=\frac{\sqrt{3}}{2}$, $\tan\theta=-\sqrt{3}$ E.g. • $\Rightarrow\theta=120°$ • $\Rightarrow\theta=180-\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)=120°$ • $\Rightarrow\theta=180-\tan^{-1}\left(\sqrt{3}\right)=120°$	Uses trigonometry to find an expression or value for θ which is in the range (1.57..., 3.14...) or (90°, 180°) (−3.14..., −4.71...) or (−180°, −270°)	M1
		$120°$ or $-240°$ or $\frac{2\pi}{3}$ or $-\frac{4\pi}{3}$ or awrt 2.09 or awrt −4.19	A1
			(2)
			11

	Question Notes	
(i)(c)	Note	Give 1st M1 for $\begin{pmatrix} 36-3k-\dfrac{12}{3k-24} & 2k-\dfrac{3k}{3k-24} \\ -6+\dfrac{9}{3k-24} & -3k+16-\dfrac{18}{3k-24} \end{pmatrix} = \begin{pmatrix} 5 & 9 \\ -3 & -5 \end{pmatrix}$
	Note	• $36-3k-\dfrac{12}{3k-24}=5 \rightarrow 3k^2-55k+252=0 \rightarrow (k-9)(3k-28)=0 \rightarrow k=9, \dfrac{28}{3}$ • $2k-\dfrac{3k}{3k-24}=9 \rightarrow k^2-13k+36=0 \rightarrow (k-9)(k-4)=0 \rightarrow k=9, 4$ • $-6+\dfrac{9}{3k-24}=-3 \rightarrow k=9$ • $-3k+16-\dfrac{18}{3k-24}=-5 \rightarrow k^2-15k+54=0 \rightarrow (k-9)(k-6)=0 \rightarrow k=9, 6$
	Note	Uses a correct element equation in part (c) leading to $k=9$ is M1 dM1 A1 even if they have followed through an incorrect $\mathbf{A}^{-1}$ in (i)(a) or an incorrect $\mathbf{A}^2$ in (ii)(b).
	Note	Give M0 dM0 A0 for an incorrect method of $36-3k-4=5 \Rightarrow k=9$
(ii)	Note	$\mathbf{M}=\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix}=\begin{pmatrix} -\frac{1}{2} & -\sqrt{3} \\ \frac{\sqrt{3}}{2} & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} \cos\theta & -p\sin\theta \\ \sin\theta & p\cos\theta \end{pmatrix}=\begin{pmatrix} -\frac{1}{2} & -\sqrt{3} \\ \frac{\sqrt{3}}{2} & -1 \end{pmatrix}$
	Note	**IMPORTANT NOTE** Give (ii)(a) M0A0 (b) M0A0 for a method of $\mathbf{M}=\begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix}\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}=\begin{pmatrix} -\frac{1}{2} & -\sqrt{3} \\ \frac{\sqrt{3}}{2} & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} \cos\theta & -\sin\theta \\ p\sin\theta & p\cos\theta \end{pmatrix}=\begin{pmatrix} -\frac{1}{2} & -\sqrt{3} \\ \frac{\sqrt{3}}{2} & -1 \end{pmatrix}$ leading to (ii)(a) $p=...$, (ii)(b) $\theta=...$
(ii)(a)	Note	$\det(\mathbf{M})=\left(-\frac{1}{2}\right)(-1)-(-\sqrt{3})\left(\frac{\sqrt{3}}{2}\right)=2$ followed by $p=\sqrt{2}$ is M0 A0
	Note	$p=\det(\mathbf{M})=\left(-\frac{1}{2}\right)(-1)-(-\sqrt{3})\left(\frac{\sqrt{3}}{2}\right)=2$ is M1 A1
	Note	$p=\dfrac{\sqrt{(\pm\sqrt{3})^2+(-1)^2}}{\sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}}=2$ is M1 A1

3. Table of contents

NEW SPECIFICATION
OCR/MEI/EDEXCEL/AQA QUESTIONS

OCR 2019 AS PURE CORE

OCR
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Monday 13 May 2019 – Afternoon

AS Level Further Mathematics A

Y531/01 Pure Core

2 Matrices **P** and **Q** are given by $\mathbf{P} = \begin{pmatrix} 1 & k & 0 \\ -2 & 1 & 3 \end{pmatrix}$ and $\mathbf{Q} = \begin{pmatrix} (1+k) & -1 \end{pmatrix}$ where k is a constant.

Exactly one of statements A and B is true.

Statement A: **P** and **Q** (in that order) are conformable for multiplication.
Statement B: **Q** and **P** (in that order) are conformable for multiplication.

(a) State, with a reason, which **one** of A and B is true. **[2]**

(b) Find either **PQ** or **QP** in terms of k. **[2]**

3. Table of contents

2	**(a)**		B (For matrices to be conformable for multiplication) the number of columns of the first must equal the number of rows in the second oe "the number of rows of P is equal to the number of columns of Q"	**B1** **E1** **[2]**	2.2a 1.2	Note "B" is that QP is conformable Statement can be general or specific. Allow eg $(1\times2)\times(2\times3)=(1\times3)$ provided that it is clear which two numbers must be the same	Since told exactly one is true it is sufficient to give a reason why one is true or why one is false
	(b)		$\mathbf{QP}=\begin{pmatrix}(1+k) & -1\end{pmatrix}\begin{pmatrix}1 & k & 0\\ -2 & 1 & 3\end{pmatrix}$ $=\begin{pmatrix}(1+k)+2 & k(1+k)-1 & -3\end{pmatrix}$ $\begin{pmatrix}(k+3) & (k^2+k-1) & -3\end{pmatrix}$	**M1** **A1** **[2]**	1.1 1.1	Correct method for multiplying matrices (can be implied by any one entry correct) Accept un-simplified elements	If **PQ** attempted then **M0A0** unless explicitly rejected

OCR 2019 AS PURE CORE

OCR
Oxford Cambridge and RSA

Monday 13 May 2019 – Afternoon

AS Level Further Mathematics A

Y531/01 Pure Core

6 A transformation T is represented by the matrix **T** where $\mathbf{T} = \begin{pmatrix} x^2+1 & -4 \\ 3-2x^2 & x^2+5 \end{pmatrix}$.

A quadrilateral Q, whose area is 12 units, is transformed by T to Q'.

Find the smallest possible value of the area of Q'. **[5]**

6			$(\Delta =)\begin{vmatrix} x^2+1 & -4 \\ 3-2x^2 & x^2+5 \end{vmatrix}$ $=(x^2+1)(x^2+5)-(-4)(3-2x^2)$	**M1**	1.1	Expanding the determinant of **T**. Condone sign error and/or one "x" rather than "x^2"	
			$=x^4-2x^2+17$	**A1**	1.1		
			$\Delta = (x^2-1)^2 + 16$	**M1***	3.1a	Attempt to find minimum value of quadratic in x^2	or from $\frac{\mathrm{d}\Delta}{\mathrm{d}x} = 4x^3 - 4x = 0$
			$(x^2 = 1) \Rightarrow \Delta_{min} = 16$	**A1**	1.1		
			So the smallest possible area is 192 (units)	**A1ft (dep*)** **[5]**	3.2a	Their $\Delta_{min} \times 12$	

OCR 2019 AS PURE CORE

OCR
Oxford Cambridge and RSA

Monday 13 May 2019 – Afternoon

AS Level Further Mathematics A

Y531/01 Pure Core

7 A transformation A is represented by the matrix **A** where $\mathbf{A} = \begin{pmatrix} -1 & x & 2 \\ 7-x & -6 & 1 \\ 5 & -5x & 2x \end{pmatrix}$.

The tetrahedron H has vertices at O, P, Q and R. The volume of H is 6 units.

P', Q', R' and H' are the images of P, Q, R and H under A.

(a) In the case where $x = 5$

- find the volume of H',
- determine whether A preserves the orientation of H. **[3]**

(b) Find the values of x for which O, P', Q' and R' are coplanar (i.e. the four points lie in the same plane). **[4]**

Question		Answer	Marks	AO	Guidance	
7	(a)	$x = 5 \Rightarrow \det\mathbf{A} = 2\times5^3 - 4\times5^2 - 58\times5 + 60$ $= 250 - 100 - 290 + 60 = -80$	B1	1.1		Could see e.g.- $\begin{vmatrix} -1 & 5 & 2 \\ 2 & -6 & 1 \\ 5 & -25 & 10 \end{vmatrix}$ $= -1(-60+25)$ $-5(20-5)+2(-50+30)$ $= 35-75-40 = -80$
		So vol of $H' = 6\times80 = 480$ cao	B1	1.1	not -480	
		...and A does not preserve the orientation because $\det\mathbf{A} < 0$.	E1 ft [3]	2.4	Follow through on the sign of their determinant	If positive then "A does preserve the orientation because det $\mathbf{A} > 0$"
7	(b)	Image coplanar $\Rightarrow \det\mathbf{A} = 0$ soi	B1	2.2a		
		$\det\mathbf{A} = -1(-6\times2x+5x) - x(2x(7-x)-5)$ $+2(-5x(7-x)+6\times5)$	M1	3.1a	Attempt to expand determinant	
		$2x^3 - 4x^2 - 58x + 60$	A1	1.1	Don't need "=0" here	
		1, 6, -5	A1 [4]	1.1	**BC**. All three	No working required for roots

OCR 2019 A2 PURE CORE

OCR
Oxford Cambridge and RSA

Monday 3 June 2019 – Morning

A Level Further Mathematics A

Y540/01 Pure Core 1

10 You are given the matrix $\mathbf{A}$ where $\mathbf{A} = \begin{pmatrix} a & 2 & 0 \\ 0 & a & 2 \\ 4 & 5 & 1 \end{pmatrix}$.

(a) Find, in terms of a, the determinant of $\mathbf{A}$, simplifying your answer. **[2]**

(b) Hence find the values of a for which $\mathbf{A}$ is singular. **[2]**

You are given the following equations which are to be solved simultaneously.

$$\begin{aligned} ax + 2y &= 6 \\ ay + 2z &= 8 \\ 4x + 5y + z &= 16 \end{aligned}$$

(c) For each of the values of a found in part **(b)** determine whether the equations have

- a unique solution, which should be found, or
- an infinite set of solutions or
- no solution. **[7]**

Question			Answer	Marks	AO	Guidance	
10	(a)		$\det\mathbf{A} = a^2 - 10a + 16$	M1 A1 [2]	1.1a 1.1	Attempt to work out the determinant	
	(b)		$a^2 - 10a + 16 = 0 \Rightarrow (a-2)(a-8) = 0 \Rightarrow a = 2,\ 8$	M1 A1 [2]	1.1a 1.1	Solving *their* quadratic soi	
	(c)		For both values there is no unique solution as $\det\mathbf{A} = 0$	B1	2.4	**Soi** by correct answers	"correct answers" means solns are either infinite or non-existent.
			For $a = 2$, equations are: $p_1 : 2x + 2y = 6$ $p_2 : 2y + 2z = 8$ $p_3 : 4x + 5y + z = 16$	M1	2.1	Substitute one of *their* values and solve	
			$2p_1 + \frac{1}{2}p_2 = p_3$ So there is an infinite set of solutions.	A1 A1	1.1 2.2a		
			For $a = 8$, equations are: $p_1 : 8x + 2y = 6$ $p_2 : 8y + 2z = 8$ $p_3 : 4x + 5y + z = 16$	M1	2.1	Substitute the other one of *their* values and solve	
			$\frac{1}{2}p_1 + \frac{1}{2}p_2 \neq p_3$ as it gives $4x + 5y + z = 7$	A1	1.1		
			and $16 \neq 7$ so no solution	A1 [7]	2.2a		

OCR 2019 A2 PURE CORE

OCR
Oxford Cambridge and RSA

Thursday 6 June 2019 – Afternoon

A Level Further Mathematics A

Y541/01 Pure Core 2

4 A 2-D transformation T is a shear which leaves the y-axis invariant and which transforms the object point $(2, 1)$ to the image point $(2, 9)$. $\mathbf{A}$ is the matrix which represents the transformation T.

(a) Find $\mathbf{A}$. **[3]**

(b) By considering the determinant of $\mathbf{A}$, explain why the area of a shape is invariant under T. **[2]**

Question			Answer	Marks	AO	Guidance	
4	(a)		$\mathbf{A}=\begin{pmatrix}1 & 0\\ k & 1\end{pmatrix}$	B1	1.2	Correct form of **A** seen	
			$\mathbf{A}=\begin{pmatrix}1 & 0\\ k & 1\end{pmatrix}\begin{pmatrix}2\\ 1\end{pmatrix}=\begin{pmatrix}2\\ 2k+1\end{pmatrix}=\begin{pmatrix}2\\ 9\end{pmatrix}$ $\begin{pmatrix}1 & 0\\ k & 1\end{pmatrix}\begin{pmatrix}2\\ 1\end{pmatrix}=\begin{pmatrix}2\\ 2k+1\end{pmatrix}=\begin{pmatrix}2\\ 9\end{pmatrix}$	M1	1.1	Correctly multiplying object vector into their **A** to find image and equating to given image	their **A** must have at least 1 unknown element
			$k=4$ so $\mathbf{A}=\begin{pmatrix}1 & 0\\ 4 & 1\end{pmatrix}$	A1	1.1		
				[3]			
	(b)		$\det\mathbf{A}=1-0=1$	B1	1.1	Correctly finding determinant	
			The determinant is the area scale factor so a determinant of 1 leaves the area unchanged	B1	2.4	Convincing explanation. Must include both ideas.	
				[2]			

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AS Level Further Mathematics A

Y531/01 Pure Core

4 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} 2 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 2 & a \end{pmatrix}$.

(i) Show that $\det\mathbf{A} = 6 - 3a$. **[2]**

(ii) State the value of a for which $\mathbf{A}$ is singular. **[1]**

(iii) Given that $\mathbf{A}$ is non-singular find $\mathbf{A}^{-1}$ in terms of a. **[4]**

3. Table of contents

4	(i)		e.g. $2(-a-2)-1(a-2)+2(2+2)$ (1st row) or $2(-a-2)-1(a-4)+2(1+2)$ (1st col)	**M1**	1.1a	Attempt to expand determinant. Could use any row or column or other method	
			$= -2a-4-a+2+8 = 6-3a$	**A1 (AG)**	1.1	Must be convincing	
				[2]			
	(ii)		2	**B1**	2.2a		
				[1]			
	(iii)		Matrix of cofactors: $\begin{pmatrix} -a-2 & 2-a & 4 \\ 4-a & 2a-4 & -2 \\ 3 & 0 & -3 \end{pmatrix}$	**M1***	1.1a	At least 4 co-factors correct, or correct apart from sign. Could be seen in separate calculations or in $\mathbf{A}^{-1}$. Could be transposed, even if stated as matrix of cofactors. If ambiguity use $\mathbf{A}^{-1}$. If $\mathbf{A}^{-1}$ not given then make whichever assumption, transposed or not, which results in most marks.	If not anywhere in matrix form only award M1 if it is clear where the cofactors come from. Cofactor must not be multiplied by anything Alternative method using cross product also ok. Matrix of cofactors is given by $(C_2 \times C_3, C_3 \times C_1\ C_1 \times C_2)$
				A1	1.1	6 cofactors correct	Must include correct sign.
				M1dep*	1.1	Transposing matrix of cofactors and dividing by determinant	

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6 The matrices **A** and **B** are given by $\mathbf{A} = \begin{pmatrix} t & 6 \\ t & -2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2t & 4 \\ t & -2 \end{pmatrix}$ where t is a constant.

(i) Show that $|\mathbf{A}| = |\mathbf{B}|$. **[2]**

(ii) Verify that $|\mathbf{AB}| = |\mathbf{A}||\mathbf{B}|$. **[3]**

(iii) Given that $|\mathbf{AB}| = -1$ explain what this means about the constant t. **[2]**

				[4]			
6	(i)		$\|\mathbf{A}\| = -2t - 6t$ or $\|\mathbf{B}\| = -4t - 4t$	M1	1.1a	Correct expression for either seen or implied	
			$\|\mathbf{B}\| = -8t = \|\mathbf{A}\|$	A1	2.2a	Both correct and statement of equality	Need to have an indication that candidate understands that they have shown that these are equal. Could be done by re-writing |**A**|= -8*t* immediately next to |**B**|= -8*t*. |**A**|=|**B**| is fine after having shown both are equal to -8*t*, but -8*t* = -8*t* is not ok for the A mark.
				[2]			
	(ii)		$\begin{pmatrix} t & 6 \\ t & -2 \end{pmatrix}\begin{pmatrix} 2t & 4 \\ t & -2 \end{pmatrix} = \begin{pmatrix} 2t^2+6t & 4t-12 \\ 2t^2-2t & 4t+4 \end{pmatrix}$	M1	3.1a	Must be attempt at proper matrix multiplication (i.e. columns into rows). Condone one error	
			$\|\mathbf{AB}\| = \begin{vmatrix} 2t^2+6t & 4t-12 \\ 2t^2-2t & 4t+4 \end{vmatrix} =$ $(2t^2+6t)(4t+4)-(2t^2-2t)(4t-12)$	M1	2.1	Correct expression of determinant of their matrix.	Condone one error
			$= 8t^3+8t^2+24t^2+24t-(8t^3-24t^2-8t^2+24t)$ $= 64t^2 = (-8t)(-8t) = \|\mathbf{A}\|\|\mathbf{B}\|$	A1	2.1	Convincing expansion, correct answer and conclusion	Similar to question above. Need candidate to conclude that |**AB**| = |**A**| |**B**| Condone not seeing (-8*t*)(-8*t*) explicitly
				[3]			
	(iii)		Set *their* $\|\mathbf{AB}\| = -1$ or $\|\mathbf{A}\|\|\mathbf{A}\| = \|\mathbf{A}\|^2 = -1$	M1	3.1a	Seen or implied	$64t^2 = -1$ or $(-8t)^2 = -1$
			$\Rightarrow 64t^2 = -1$ so t must be complex/imaginary/not real	A1ft	3.2a	Accept $t = -i/8$ or $t = i/8$	Allow follow through if their |**AB**| is of the form kt^2.
				[2]			

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8 The 2×2 matrix $\mathbf{A}$ represents a transformation T which has the following properties.

- The image of the point $(0, 1)$ is the point $(3, 4)$.
- An object shape whose area is 7 is transformed to an image shape whose area is 35.
- T has a line of invariant points.

(i) Find a possible matrix for $\mathbf{A}$. **[8]**

The transformation S is represented by the matrix $\mathbf{B}$ where $\mathbf{B} = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$.

(ii) Find the equation of the line of invariant points of S. **[2]**

(iii) Show that any line of the form $y = x + c$ is an invariant line of S. **[3]**

8	**(i)**		$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$	**B1**	3.1a	$\Rightarrow b = 3, d = 4$	
			Determinant $= ad - bc = 5$	**B1**	3.1a	$4a - 3c = 5$	0r det $= -5$ and follow through
			$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$	**B1**	1.2	Understanding of invariant point seen or implied	
			$(1 - a)x = by$ or $(1 - d)y = cx$	**M1**	2.2a	May have $b = 3$ and/or $d = 4$ already substituted	$(1 - a)x = 3y$ or $-3y = cx$
			$\frac{1-a}{c} = \frac{b}{1-d}$ Or $\frac{\frac{1-a}{b}}{} = \frac{\frac{c}{1-d}}{}$	**M1**	1.1	Eliminating x and y	
			$c = a - 1$	**A1**	1.1		
			e.g. $4a - 3(a - 1) = 5$	**M1**	1.1	Attempting to solve their simultaneous equations	If no working and incorrect then M0A0.
			$\mathbf{A} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$	**A1**	3.2a	Condone $a = 2$, etc as long as Matrix seen as $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$	
				[8]			

Question		Answer	Marks	AO	Guidance	
	(ii)	Need $\begin{pmatrix}3 & 1\\2 & 2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}3x+y\\2x+2y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}$	M1	1.1	Substituting a general point into their matrix, calculating an image point and equating it to the object point.	
		$2x+y=0$	A1	2.2a	Final form must be $y = -2x$ or $x = -\frac{1}{2}y$ or a numerical multiple of $2x + y = 0$.	Need to have considered both x and y coordinates.
			[2]			
	(iii)	$\begin{pmatrix}3 & 1\\2 & 2\end{pmatrix}\begin{pmatrix}x\\x+c\end{pmatrix}=\begin{pmatrix}3x+x+c\\2x+2x+2c\end{pmatrix}$	M1*	3.1a		
		So $\begin{pmatrix}X\\Y\end{pmatrix}=\begin{pmatrix}4x+c\\4x+2c\end{pmatrix}$ ...	M1dep*	2.2a		
		...and $Y=X+c$	A1	1.1	Could see the y component of the vector written as $4x + c + c$.	
	Alt	$\begin{pmatrix}3 & 1\\2 & 2\end{pmatrix}\begin{pmatrix}x\\mx+c\end{pmatrix}=\begin{pmatrix}X\\mX+c\end{pmatrix}$ $3x+mx+c=X$ $2x+2mx+2c=mX+c$ $2x+2mx+2c=m(3x+mx+c)+c$	M1		Need to eliminate X, i.e. an equation in x, m and c.	
		$x(m^2+m-2)+c(m-1)=0$ $x(m+2)(m-1)+c(m-1)=0$	M1			
		If $m=1$, $c\,(m-1)=0$ satisfied by any c	A1			
			[3]			

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AS Level Further Mathematics B (MEI)
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Question Paper

Monday 14 May 2018 – Afternoon

1 The matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ are defined as follows:

$$\mathbf{A} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 0 & 3 \\ 1 & -1 & 3 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 1 & 3 \end{pmatrix}.$$

Calculate all possible products formed from two of these three matrices. **[4]**

Question			Answer	Marks	AOs		Guidance
1			BA = $\begin{pmatrix} 2 & 0 & 3 \\ 1 & -1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 11 \\ 8 \end{pmatrix}$	M1 A1	1.1a 1.1	**BA**, **AC** or **CB** calculated with correct shape	allow one arith error for M1
			AC = $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\begin{pmatrix} 1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 6 \\ 3 & 9 \end{pmatrix}$	A1	1.1		
			CB = $\begin{pmatrix} 1 & 3 \end{pmatrix}\begin{pmatrix} 2 & 0 & 3 \\ 1 & -1 & 3 \end{pmatrix} = \begin{pmatrix} 5 & -3 & 12 \end{pmatrix}$	A1 [4]	1.1	deduct A1 for any incorrect products pursued	

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5 A transformation of the x-y plane is represented by the matrix $\begin{pmatrix} \cos\theta & 2\sin\theta \\ 2\sin\theta & -\cos\theta \end{pmatrix}$, where θ is a positive acute angle.

(i) Write down the image of the point (2, 3) under this transformation. **[2]**

(ii) You are given that this image is the point $(a, 0)$. Find the value of a. **[5]**

6 Find the invariant line of the transformation of the x-y plane represented by the matrix $\begin{pmatrix} 2 & 0 \\ 4 & -1 \end{pmatrix}$. **[4]**

				[5]			
5	**(i)**		$(2\cos\theta+6\sin\theta,\ 4\sin\theta-3\cos\theta)$	**B1B1** **[2]**	**1.1,1.1**	Accept in vector form	
5	**(ii)**		$4\sin\theta-3\cos\theta=0$ $\Rightarrow \tan\theta=\frac{3}{4}$ $\Rightarrow \theta = 36.9°$ or 0.644 rad $a=2\cos\theta+6\sin\theta=5.2$	**M1** **M1** **A1** **M1** **A1** **[5]**	**3.1a** **1.1** **1.1** **1.1** **1.1cao**	their $4\sin\theta-3\cos\theta=0$ tan θ = sin θ / cos θ used θ = 36.9° or 0.644 rad or better substituting their θ into their 2cos θ + 6 sin θ 5.2	 or $\sin^2\theta + \cos^2\theta = 1$ used or $\sin\theta=\frac{3}{5}, \cos\theta=\frac{4}{5}$ or sin θ and cos θ
6			$\begin{pmatrix}2 & 0\\ 4 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x'\\ y'\end{pmatrix}$ $\Rightarrow x' = 2x,\ y' = 4x - y$ $y = mx + c$ and $y' = mx' + c$ $\Rightarrow 4x - mx - c = m.2x + c$ $\Rightarrow m=\frac{4}{3},\ c = 0$ so invariant line is $y=\frac{4}{3}x$	 **M1** **M1** **A1** **A1** **[4]**	 **2.1** **1.1b** **2.4** **1.1b**	soi [condone $x = 2x$, $y=4x - y$] condition for invariance forming identity in x (may assume $c = 0$)	assuming $c = 0$: $y = mx$ and $y' = mx'$ $4x - mx = m.2x$ $m=\frac{4}{3}$

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Question Paper

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10 Three planes have equations

$$\begin{aligned} -x + 2y + z &= 0 \\ 2x - y - z &= 0 \\ x + y \qquad &= a \end{aligned}$$

where a is a constant.

(i) Investigate the arrangement of the planes:

- when $a = 0$;
- when $a \neq 0$. **[6]**

(ii) Chris claims that the position vectors $-\mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $2\mathbf{i} - \mathbf{j} - \mathbf{k}$ and $\mathbf{i} + \mathbf{j}$ lie in a plane. Determine whether or not Chris is correct. **[2]**

				[4]		checking both conditions	
10	(i)		$\det \mathbf{M} = 0$ [so no unique solution] no planes parallel [so prism or sheaf] when a = 0, they form a sheaf as the system has solutions when $a \neq 0$, they form a prismatic intersection as there are no solutions	B1 B1 B1 B1 B1 B1 [6]	3.1a 1.1 2.2a 2.2a 2.2a 3.1a	or **M** is singular allow 'intersect in a line' o.e. e.g. finding solutions allow 'prism'	or state no unique solution by direct solution of equations
10	(ii)		These are the normals to the three planes In either of the above cases, they must lie in the same plane	M1 A1dep	1.1a 2.3	 dep previous part correct	
			OR (e.g) $\mathbf{i} + \mathbf{j} = -\mathbf{i} + 2\mathbf{j} + \mathbf{k} + 2\mathbf{i} - \mathbf{j} - \mathbf{k}$ $\Rightarrow$ coplanar	 M1 A1 [2]		 showing linear dependence	

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3 **In this question you must show detailed reasoning.**

A and **B** are matrices such that $\mathbf{B}^{-1}\mathbf{A}^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$.

(a) Find **AB**. **[3]**

(b) Given that $\mathbf{A} = \begin{pmatrix} \frac{1}{3} & 1 \\ 0 & 1 \end{pmatrix}$, find **B**. **[3]**

Question			Answer	Marks	AOs	Guidance	
3	(a)		**DR**				
			$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}$ so $\mathbf{AB} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}^{-1}$	M1	3.1a	$\mathbf{AB} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}^{-1}$	
			$\begin{vmatrix} 2 & 1 \\ -1 & 1 \end{vmatrix} = 3$	B1	1.1	soi	
			$\Rightarrow \mathbf{AB} = \frac{1}{3}\begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}$	A1cao	1.1	or $\begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$	
				[3]			
3	(b)		**DR**				
			$\mathbf{A}^{-1} = 3\begin{pmatrix} 1 & -1 \\ 0 & \frac{1}{3} \end{pmatrix}$	B1	1.1		
			$\mathbf{B} = \mathbf{A}^{-1}\mathbf{AB} = 3 \times \frac{1}{3}\begin{pmatrix} 1 & -1 \\ 0 & \frac{1}{3} \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 1 & 2 \end{pmatrix}$	M1	3.1a	pre-multiply their **AB** by $\mathbf{A}^{-1}$	
			$= \begin{pmatrix} 0 & -3 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$	A1cao	1.1	Or $\frac{1}{3}\begin{pmatrix} 0 & -9 \\ 1 & 2 \end{pmatrix}$	

			Alternative solution				
			$\mathbf{B}^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{3} & 1 \\ 0 & 1 \end{pmatrix}$	**M1**		post-multiply $\mathbf{B}^{-1}\mathbf{A}^{-1}$ by $\mathbf{A}$	
			$= \begin{pmatrix} \frac{2}{3} & 3 \\ -\frac{1}{3} & 0 \end{pmatrix}$	**A1**			
			$\Rightarrow \mathbf{B} = \begin{pmatrix} 0 & -3 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$	**A1cao**		Or $\frac{1}{3}\begin{pmatrix} 0 & -9 \\ 1 & 2 \end{pmatrix}$	`

Question			Answer	Marks	AOs	Guidance	
			Alternative solution				
			$\begin{pmatrix} \frac{1}{3} & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & -\frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$				
			$\Rightarrow \frac{1}{3}a + c = \frac{1}{3}, \frac{1}{3}b + d = -\frac{1}{3}$	**M1**		forming equations (any two)	ft their **AB**
			$c = \frac{1}{3}, d = \frac{2}{3}$	**A1**			
			$\Rightarrow a = 0, b = -3 \ \Rightarrow \mathbf{B} = \begin{pmatrix} 0 & -3 \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$	**A1**			

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4 **(a)** Find $\mathbf{M}^{-1}$, where $\mathbf{M} = \begin{pmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ -2 & 1 & 2 \end{pmatrix}$. **[1]**

(b) Hence find, in terms of the constant k, the point of intersection of the planes

$$x + 2y + 3z = 19,$$
$$-x + y + 2z = 4,$$
$$-2x + y + 2z = k.$$ **[3]**

(c) **In this question you must show detailed reasoning.**

Find the acute angle between the planes $x + 2y + 3z = 19$ and $-x + y + 2z = 4$. **[4]**

Question			Answer	Marks	AOs	Guidance	
4	(a)		$\mathbf{M}^{-1}=\begin{pmatrix}0&1&-1\\2&-8&5\\-1&5&-3\end{pmatrix}$	**B1** **[1]**	**1.1**	**BC**	
4	(b)		$\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0&1&-1\\2&-8&5\\-1&5&-3\end{pmatrix}\begin{pmatrix}19\\4\\k\end{pmatrix}$	**M1**	**1.1a**		
			$x=4-k,\ y=6+5k,\ z=1-3k$	**A2,1,0** **[3]**	**1.1,1.1**	Allow **SCB2** for correct answer found using elimination	condone $\begin{pmatrix}4-k\\6+5k\\1-3k\end{pmatrix}$
4	(c)		**DR** normals are $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$ and $-\mathbf{i}+\mathbf{j}+2\mathbf{k}$	**B1**	**1.2**	soi	
			$\cos\theta=\dfrac{1\times(-1)+2\times1+3\times2}{\sqrt{1^2+2^2+3^2}\sqrt{(-1)^2+1^2+2^2}}$	**M1**	**3.1a**		
			$=\dfrac{7}{\sqrt{14}\sqrt{6}}$	**A1**	**1.1**	correct expression	oe eg $\dfrac{\sqrt{21}}{6}$
			$\Rightarrow\theta=40.2°$ or 0.702 rads	**A1** **[4]**	**1.1**	40° or 0.70 or better	mark final answer

6 A linear transformation T of the x-y plane has an associated matrix $\mathbf{M}$, where $\mathbf{M} = \begin{pmatrix} \lambda & k \\ 1 & \lambda - k \end{pmatrix}$, and λ and k are real constants.

(a) You are given that $\det\mathbf{M} > 0$ for all values of λ.

(i) Find the range of possible values of k. **[3]**

(ii) What is the significance of the condition $\det\mathbf{M} > 0$ for the transformation T? **[1]**

For the remainder of this question, take $k = -2$.

(b) Determine whether there are any lines through the origin that are invariant lines for the transformation T. **[4]**

(c) The transformation T is applied to a triangle with area 3 units^2. The area of the resulting image triangle is 15 units^2.
Find the possible values of λ. **[3]**

Question			Answer	Marks	AOs	Guidance	
6	(a)	(i)	$\det \mathbf{M} = \lambda(\lambda - k) - k$	B1	1.1		
			$\det \mathbf{M} > 0 \Rightarrow \lambda^2 - k\lambda - k > 0$ for all $\lambda \Rightarrow k^2 + 4k < 0$	M1	3.1a	attempt to find discriminant	or $(\lambda - k/2)^2 > k^2/4 + k$
			$\Rightarrow -4 < k < 0$	A1cao [3]	1.1		
6	(a)	(ii)	The transformation represented by **M** always preserves the orientation of shapes	B1 [1]	1.2	condone 'doesn't reflect'	
6	(b)		$\begin{pmatrix} \lambda & -2 \\ 1 & \lambda+2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \lambda x - 2y \\ x + (\lambda+2)y \end{pmatrix}$	B1	2.1	oe (e.g. with $y = mx$ [$+ c$])	
			invariant line if $x + \lambda mx + 2mx = m(\lambda x - 2mx)$	M1	2.1	subst $y = mx[+c]$ into $x+(\lambda+2)y = m(\lambda x - 2y)[+c]$	$x+(\lambda+2)(mx+c)$ $= m(\lambda x - 2mx - 2c)+c$
			$\Rightarrow 2m^2 + 2m + 1 = 0$	A1	1.1		
			discriminant is $2^2 - 2\times4 = -4 < 0$ so no real roots for m, i.e. there are no invariant lines	A1 [4]	2.3		
6	(c)		$\det \mathbf{M} = \lambda^2 + 2\lambda + 2 = 5$	M1	3.1a	det × area by 5 soi	
			$\Rightarrow \lambda^2 + 2\lambda - 3 = 0$	A1	1.1	correct equation in any form	
			$\Rightarrow \lambda = -3$ or 1	A1 [3]	1.1	BC	

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Monday 3 June 2019 – Morning

A Level Further Mathematics B (MEI)

Y420/01 Core Pure

3 Matrices **A** and **B** are defined by $\mathbf{A} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} k & 1 \\ 2 & 0 \end{pmatrix}$, where k is a constant.

(a) Verify the result $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ in this case. **[5]**

(b) Investigate whether **A** and **B** are commutative under matrix multiplication. **[2]**

3	(a)		$\mathbf{AB} = \begin{pmatrix} 3 & 1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} k & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 3k+2 & 3 \\ 2k+2 & 2 \end{pmatrix}$	B1	1.1b		
			$(\mathbf{AB})^{-1} = -\frac{1}{2}\begin{pmatrix} 2 & -3 \\ -2k-2 & 3k+2 \end{pmatrix}$	B1ft	1.1b	ft their **AB** provided det $\neq 0$	[isw]
			$\mathbf{A}^{-1} = \begin{pmatrix} 1 & -1 \\ -2 & 3 \end{pmatrix}$	B1	1.1b		
			$\mathbf{B}^{-1} = -\frac{1}{2}\begin{pmatrix} 0 & -1 \\ -2 & k \end{pmatrix}$	B1	1.1b		
			$\mathbf{B}^{-1}\mathbf{A}^{-1} = -\frac{1}{2}\begin{pmatrix} 2 & -3 \\ -2k-2 & 3k+2 \end{pmatrix}$	B1	2.2a		
			[so $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$]	[5]			

Question			Answer	Marks	AOs		Guidance
3	(b)		$\mathbf{BA} = \begin{pmatrix} 3k+2 & k+1 \\ 6 & 2 \end{pmatrix}$	B1	1.1b		
			AB = **BA** when $k = 2$ [and not otherwise]	B1 [2]	2.3		

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A Level Further Mathematics B (MEI)

Y420/01 Core Pure

11 **(a)** Specify fully the transformations represented by the following matrices.

- $\mathbf{M}_1 = \begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$

- $\mathbf{M}_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ **[4]**

(b) Find the equation of the mirror line of the reflection R represented by the matrix $\mathbf{M}_3 = \mathbf{M}_1\mathbf{M}_2$. **[5]**

(c) It is claimed that the reflection represented by the matrix $\mathbf{M}_4 = \mathbf{M}_2\mathbf{M}_1$ has the same mirror line as R. Explain whether or not this claim is correct. **[3]**

Question			Answer	Marks	AOs		Guidance
11	(a)		$\mathbf{M}_1$ rotation through $\cos^{-1}(3/5)$ or 53.1° or 0.927 rads anti-clockwise about O $\mathbf{M}_2$ reflection in x-axis	M1 A1 A1 B1 [4]	3.1a 1.1b 1.2 1.2	 oe e.g. $\sin^{-1}(4/5)$, $\tan^{-1}(4/3)$ or positive rotation about O or Ox or $y = 0$	 53° or 0.93 rads or better
11	(b)		$\mathbf{M}_3 = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{pmatrix}$	B1	1.1b		
			$\begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ $\Rightarrow \frac{3}{5}x + \frac{4}{5}y = x,\ \frac{4}{5}x - \frac{3}{5}y = y$ $\Rightarrow y = ½x$ so $y = ½x$ is mirror line	M1 A2 A1	3.1a 1.1b 2.2a	attempt to find invariant points either or both accept valid geometric args	or inv line $y = mx$ [$+c$] $2m^2+3m-2=0$ A1 $\Rightarrow m = ½, -2$ A1
			Alternative solution $\frac{1}{1+m^2}\begin{pmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{pmatrix}$ $\Rightarrow 2m^2 - 5m + 2 = 0$ $\Rightarrow m = ½$	 M1 A1 A2 [5]		 must discount $m = 2$	

11	**(c)**		$\mathbf{M}_4 = \begin{pmatrix} \frac{3}{5} & -\frac{4}{5} \\ -\frac{4}{5} & -\frac{3}{5} \end{pmatrix}$	**B1**	**1.1b**		
			$\mathbf{M}_4 \neq \mathbf{M}_3$ [so can't represent same reflection]	**M1**	**3.1a**	or attempt to find mirror line as in part **(b)**	$\mathbf{M}_4$ must be different
			so mirror line cannot be the same, and statement is incorrect	**A1**	**2.4**	$\Rightarrow y = -\frac{1}{2}x$, so statement is incorrect	

EDEXCEL 2018 AS CORE PURE

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Further Mathematics

Advanced Subsidiary

Paper 1: Core Pure Mathematics

Monday 14 May 2018 – Afternoon

Time: 1 hour 40 minutes

Paper Reference **8FM0/01**

1.

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & -3 \\ 4 & -2 & 1 \\ 3 & 5 & -2 \end{pmatrix}$$

(a) Find $\mathbf{M}^{-1}$ giving each element in exact form.

(2)

(b) Solve the simultaneous equations

$$\begin{aligned} 2x + y - 3z &= -4 \\ 4x - 2y + z &= 9 \\ 3x + 5y - 2z &= 5 \end{aligned}$$

(2)

(c) Interpret the answer to part (b) geometrically.

(1)

3. Table of contents

Question	Scheme	Marks	AOs
1(a)	$\mathbf{M}^{-1} = \frac{1}{69}\begin{pmatrix} 1 & 13 & 5 \\ -11 & -5 & 14 \\ -26 & 7 & 8 \end{pmatrix}$	B1 B1	1.1b 1.1b
		(2)	
(b)	$\frac{1}{69}\begin{pmatrix} 1 & 13 & 5 \\ -11 & -5 & 14 \\ -26 & 7 & 8 \end{pmatrix}\begin{pmatrix} -4 \\ 9 \\ 5 \end{pmatrix} = \ldots$	M1	1.1b
	$x = 2, y = 1, z = 3$ or $(2,1,3)$ or $2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$ or $\begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$	A1	1.1b
		(2)	
(c)	The **point** where three **planes** meet	B1ft	2.2a
		(1)	
			(5 marks)

(a)

B1: Evidence that the determinant is ± 69 (may be implied by their matrix e.g. where entries are not in exact form: $\pm\begin{pmatrix} 0.014 & 0.188 & 0.072 \\ -0.159 & -0.072 & 0.203 \\ -0.377 & 0.101 & 0.116 \end{pmatrix}$)(Should be mostly correct)

Must be seen in part (a).

B1: Fully correct inverse with all elements in **exact** form

(b)

M1: Any complete method to find the values of x, y and z (Must be using **their inverse** if using the method in the main scheme)

A1: Correct coordinates

A solution not using the inverse requires a complete method to find values for x, y and z for the method mark.

Correct coordinates only scores both marks.

(c)

B1: Describes the correct geometrical configuration.

Must include the two ideas of **planes** and **meet in a point** with no contradictory statements.

This is dependent on having obtained a unique point in part (b)

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Further Mathematics

Advanced Subsidiary

Paper 1: Core Pure Mathematics

Monday 14 May 2018 – Afternoon

Time: 1 hour 40 minutes

Paper Reference **8FM0/01**

5.

$$\mathbf{A} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$

(a) Describe fully the single geometrical transformation U represented by the matrix **A**.

(3)

The transformation V, represented by the 2×2 matrix **B**, is a reflection in the line $y = -x$

(b) Write down the matrix **B**.

(1)

Given that U followed by V is the transformation T, which is represented by the matrix **C**,

(c) find the matrix **C**.

(2)

(d) Show that there is a real number k for which the point $(1, k)$ is invariant under T.

(4)

3. Table of contents

Question	Scheme	Marks	AOs
5(a)	Rotation	B1	1.1b
	120 degrees (anticlockwise) or $\frac{2\pi}{3}$ radians (anticlockwise) Or 240 degrees clockwise or $\frac{4\pi}{3}$ radians clockwise	B1	2.5
	About (from) the origin. Allow (0, 0) or O for origin.	B1	1.2
		(3)	
(b)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1	1.1b
		(1)	
(c)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} \ldots & \ldots \\ \ldots & \ldots \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}$	A1ft	1.1b
		(2)	

(d)	$\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix}1\\k\end{pmatrix}=\begin{pmatrix}1\\k\end{pmatrix}=\ldots$ or $\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}x\\y\end{pmatrix}=\ldots$ Note: $\begin{pmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{pmatrix}\begin{pmatrix}1\\k\end{pmatrix}=\begin{pmatrix} -\frac{\sqrt{3}}{2}+\frac{1}{2}k \\ \frac{1}{2}+\frac{\sqrt{3}}{2}k \end{pmatrix}=\begin{pmatrix}1\\k\end{pmatrix}$ can score M1 (for the matrix equation) but needs an equation to be "extracted" to score the next A1	M1	3.1a
	$-\frac{\sqrt{3}}{2}+\frac{1}{2}k=1$ or $\frac{1}{2}+\frac{\sqrt{3}}{2}k=k$ or $x=-\frac{\sqrt{3}}{2}x+\frac{1}{2}y$ or $y=\frac{1}{2}x+\frac{\sqrt{3}}{2}y$ (Note that candidates may then substitute x = 1 which is acceptable)	A1ft	1.1b
	$-\frac{\sqrt{3}}{2}+\frac{1}{2}k=1$ or $x=-\frac{\sqrt{3}}{2}x+\frac{1}{2}y \Rightarrow k=2+\sqrt{3}\left(\text{or } \frac{1}{2-\sqrt{3}}\right)$	A1	1.1b
	$\frac{1}{2}+\frac{\sqrt{3}}{2}k=k$ or $y=\frac{1}{2}x+\frac{\sqrt{3}}{2}y \Rightarrow k=2+\sqrt{3}\left(\text{or } \frac{1}{2-\sqrt{3}}\right)$	B1	1.1b
		(4)	
			(10 marks)

Notes
(a) B1: Identifies the transformation as a rotation B1: Correct angle. Allow equivalents in degrees or radians. B1: Identifies the origin as the centre of rotation These marks can only be awarded as the elements of a **<u>single transformation</u>** (b) B1: Shows the correct matrix in the correct form (c) M1: Multiplies the matrices in the correct order (evidence of multiplication can be taken from 3 correct or 3 correct ft elements) A1ft: Correct matrix (follow through their matrix from part (b)) A correct matrix or a correct follow through matrix implies both marks. (d) M1: Translates the problem into a matrix multiplication to obtain at least one equation in k or in x and y A1ft: Obtains one correct equation (follow through their matrix from part (c)) A1: Correct value for k in any form B1: Checks their answer by independently solving both equations **correctly** to obtain $2+\sqrt{3}$ both times or substitutes $2+\sqrt{3}$ into the other equation to confirm its validity

EDEXCEL 2019 AS CORE PURE

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Monday 13 May 2019

Afternoon (Time: 1 hour 40 minutes) | Paper Reference **8FM0-01**

Further Mathematics

Advanced Subsidiary

Paper 1: Core Pure Mathematics

1.

$$\mathbf{M} = \begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}$$

(a) Show that the matrix **M** is non-singular.

(2)

The transformation T of the plane is represented by the matrix **M**.

The triangle R is transformed to the triangle S by the transformation T.

Given that the area of S is 63 square units,

(b) find the area of R.

(2)

(c) Show that the line $y = 2x$ is invariant under the transformation T.

(2)

3. Table of contents

Question	Scheme	Marks	AOs
1. (a)	$(\det(\mathbf{M}) =)\ (4)(-7)-(2)(-5)$	M1	1.1a
	$\mathbf{M}$ is non-singular because $\det(\mathbf{M}) = -18$ and so $\det(\mathbf{M}) \neq 0$	A1	2.4
		(2)	
(b)	$\text{Area } R = \dfrac{\text{Area } S}{(\pm)\lvert\det \mathbf{M}\rvert} = \ldots$	M1	1.2
	$\text{Area}(R) = \dfrac{63}{\lvert -18 \rvert} = \dfrac{7}{2}$ oe	A1ft	1.1b
		(2)	
(c)	$\begin{pmatrix} 4 & -5 \\ 2 & -7 \end{pmatrix}\begin{pmatrix} x \\ 2x \end{pmatrix} = \begin{pmatrix} 4x-10x \\ 2x-14x \end{pmatrix}$	M1	1.1b
	$= \begin{pmatrix} -6x \\ -12x \end{pmatrix}$ and so all points on $y = 2x$ map to points on $y = 2x$, hence the line is invariant. OR $= -6\begin{pmatrix} x \\ 2x \end{pmatrix}$ hence $y = 2x$ is invariant.	A1	2.1
		(2)	
	(6 marks)		

3. Table of contents

Notes		
(a)	M1	An attempt to find $\det(\mathbf{M})$. Just the calculation is sufficient. Site of −18 implies this mark, which may be embedded in an attempt at the inverse..
	A1	$\det(\mathbf{M}) = -18$ **and** reference to zero, e.g. $-18 \neq 0$ **and** conclusion.
		The conclusion may precede finding the determinant (e.g. "Non-singular if $\det(\mathbf{M}) \neq 0$, $\det(\mathbf{M}) = -18 \neq 0$" is sufficient or accept "Non-singular if $\det(\mathbf{M}) \neq 0$, $\det(\mathbf{M}) = -18$, therefore non-singular" or some other indication of conclusion.) Need not mention "$\det(\mathbf{M})$" to gain both marks here, a correct calculation, statement $-18 \neq 0$, and conclusion hence **M** is non-singular can gain M1A1.
(b)	M1	Recalls determinant is needed for area scale factor by dividing 63 by ±their determinant.
	A1ft	$\frac{7}{2}$ or follow through $\frac{63}{\lvert \text{their det} \rvert}$. Must be positive and should be simplified to single fraction or exact decimal. (Allow if made positive following division by a negative determinant.)
(c)	M1	Attempts the matrix multiplication shown or with equivalent, e.g $\begin{pmatrix} \frac{1}{2}y \\ y \end{pmatrix}$. May use $\begin{pmatrix} x \\ y \end{pmatrix}$ **and** substitute $y = 2x$ later and this is fine for the method.
	A1	Correct multiplication and working leading to conclusion that the line is invariant. If the −6 is not extracted, they must make reference to image points being on line $y = 2x$. If the −6 is extracted to show it is a multiple of $\begin{pmatrix} x \\ 2x \end{pmatrix}$ followed by a conclusion "invariant" as minimum.

3. Table of contents

EDEXCEL 2019 AS CORE PURE

10. The population of chimpanzees in a particular country consists of juveniles and adults. Juvenile chimpanzees do not reproduce.

In a study, the numbers of juvenile and adult chimpanzees were estimated at the start of each year. A model for the population satisfies the matrix system

$$\begin{pmatrix} J_{n+1} \\ A_{n+1} \end{pmatrix} = \begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix} \begin{pmatrix} J_n \\ A_n \end{pmatrix} \qquad n = 0, 1, 2, \ldots$$

where a is a constant, and J_n and A_n are the respective numbers of juvenile and adult chimpanzees n years after the start of the study.

(a) Interpret the meaning of the constant a in the context of the model.

(1)

At the start of the study, the total number of chimpanzees in the country was estimated to be 64 000

According to the model, after one year the number of juvenile chimpanzees is 15 360 and the number of adult chimpanzees is 43 008

Pearson Edexcel Level 3 GCE

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Further Mathematics

Advanced Subsidiary

Paper 1: Core Pure Mathematics

(b) (i) Find, in terms of a

$$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1}$$

(3)

(ii) Hence, or otherwise, find the value of a.

(3)

(iii) Calculate the change in the number of juvenile chimpanzees in the first year of the study, according to this model.

(2)

Given that the number of juvenile chimpanzees is known to be in decline in the country,

(c) comment on the short-term suitability of this model.

(1)

A study of the population revealed that adult chimpanzees stop reproducing at the age of 40 years.

(d) Refine the matrix system for the model to reflect this information, giving a reason for your answer.

(There is no need to estimate any unknown values for the refined model, but any known values should be made clear.)

(2)

3. Table of contents

Question	Scheme	Marks	AOs
10. (a)	a represents the proportion of juvenile chimpanzees that (survive and) **remain** juvenile chimpanzees the next year.	B1	3.4
		(1)	
(b)(i)	Determinant $= 0.82a - 0.08\times 0.15$	M1	1.1b
	$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1} = \ldots\begin{pmatrix} 0.82 & -0.15 \\ -0.08 & a \end{pmatrix}$	M1	1.1b
	$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1} = \dfrac{1}{0.82a - 0.012}\begin{pmatrix} 0.82 & -0.15 \\ -0.08 & a \end{pmatrix}$	A1	1.1b
		(3)	
(ii)	$\begin{pmatrix} a & 0.15 \\ 0.08 & 0.82 \end{pmatrix}^{-1}\begin{pmatrix} 15360 \\ 43008 \end{pmatrix} = \dfrac{1}{0.82a - 0.012}\begin{pmatrix} 0.82\times 15360 - 0.15\times 43008 \\ (-0.08)\times 15360 + 43008a \end{pmatrix}$ OR forms equations $\begin{matrix} 15360 = aJ_0 + 0.15\times A_0 \\ 43008 = 0.08\times J_0 + 0.82\times A_0 \end{matrix}$	M1	3.1a
	$\dfrac{1}{0.82a - 0.012}\left[6144 + (43008a - 1228.8)\right] = 64000$ $\Rightarrow 4915.2 + 43008a = 64000(0.82a - 0.012) \Rightarrow a = \ldots$ OR $A_0 = 64000 - J_0 \Rightarrow 43008 = 0.08\times J_0 + 0.82\times(64000 - J_0) = J_0 = \ldots$ $\Rightarrow a = \dfrac{15360 - (64000 - J_0)}{J_0} = \ldots$	M1	3.1a
	$a = \dfrac{5683.2}{9472} = 0.60$	A1	1.1b
		(3)	

(iii)	Initial juvenile population $= \dfrac{"6144"}{"0.48"} = 12800$	M1	3.4
	So change of 2560 juvenile chimpanzees	A1	1.1b
		(2)	
(c)	As the number of juveniles has increased, the model is not initially predicting a decline, so is not suitable in the short term. (Follow through their answer to (b) – but they must have made an attempt at it to find at least a value for J_0)	B1ft	3.5a
		(1)	
(d)	Third category needs to be introduced for chimpanzees aged 40 and above, mature chimpanzees M_n, and a matrix multiplication of increased dimension set up. Accept $3\times3, 3\times2$ or 2×3 matrices including all three categories in the column vector.	M1	3.5c
	The corresponding matrix model will have the form $\begin{pmatrix} J_{n+1} \\ A_{n+1} \\ M_{n+1} \end{pmatrix} = \begin{pmatrix} a & b & \underline{\mathbf{0}} \\ 0.08 & c & 0 \\ 0 & d & e \end{pmatrix} \begin{pmatrix} J_n \\ A_n \\ M_n \end{pmatrix}$ (The underlined zero must be correct but do not be concerned about any values used in the other entries.)	A1	3.3
		(2)	
	(12 marks)		

Notes		
(a)	**B1**	Correct interpretation. Need not mention survival but must be clear it is the (proportion of) **juveniles that remain as juveniles** the next year (ie those that survive but don't progress to adulthood). E.g. accept "(number of) juveniles who do not become adults" but do not accept "surviving juveniles".
		Mark part (b) as a whole.
(b)(i)	**M1**	Attempts the determinant in terms of a Allow miscopies for the attempt. Allow $0.82a - 0.12$ as a slip.
	M1	Attempts the form of the inverse, swapped leading diagonals and sign changed on both off diagonals. Allow miscopies of the numbers but the signs must be correct.
	A1	Correct inverse matrix
(ii)	**M1**	Use the inverse matrix and attempts to find the initial juvenile and adult populations. . (May have determinant 1 for this mark.) Alternatively, sets up simultaneous equations from the original system, $15360 = aJ_0 + 0.15 \times A_0$ and $43008 = 0.08 \times J_0 + 0.82 \times A_0$ Accept with J_n and A_n or other appropriate variables.
	M1	Uses the sum of initial populations equals 64000 in an attempt to find a. (May have determinant 1 for this mark.) If using alternative, use of e.g. $A_0 = 64000 - J_0$ in second equation to find J_0, followed by attempt to find a. Award for an attempt to solve the equations, but don't be too concerned with the algebraic process as long as they are attempting to use all three equations.
	A1	Correct value, $a = 0.6$ (or 0.60 or $\frac{3}{5}$).

(iii)	**M1**	Uses their a to find the value of J_0. This mark may be gained for work done in (ii) if the alternative has been used but must have come from a correct method.
	A1	Correct difference found, as long as there is no contradictory statement – so "decrease of 2560" is A0.
(c)	**B1ft**	Comments that the change is an increase so does not fit the model. Follow through their answer to (b) as long as at least a value for J_0 has been found. If a decrease has been found allow for commenting the model is suitable. If an answer is given to (b)(iii), follow through on whatever their answer is. If no answer has been given, but an initial population found, a comparison should be made between this value and 153600 with conclusion must be consistent with their answer for J_0
(d)	**M1**	Introduces a third category (may be *M*ature, *E*lderly or any suitable letter used) and sets up a matrix multiplication (the left hand side may be missing for this mark) with all three categories in the column vector. The dimension of the matrix should be 3 in at least either row or column, and there should be a 3×1 vector.
	A1	Sets up the new matrix equation, including both sides and making clear the zero (underlined) so that the correct progression that no new juveniles arise from the mature chimpanzees is clear. Overlook other values, though ideally the other two zeroes are shown too, to indicate mature chimpanzees do not regress to adulthood, and juveniles cannot proceed directly to mature chimpanzees.

7.

$$\mathbf{M} = \begin{pmatrix} 2 & -1 & 1 \\ 3 & k & 4 \\ 3 & 2 & -1 \end{pmatrix} \qquad \text{where } k \text{ is a constant}$$

(a) Find the values of k for which the matrix **M** has an inverse.

(2)

(b) Find, in terms of p, the coordinates of the point where the following planes intersect

$$\begin{aligned} 2x - y + z &= p \\ 3x - 6y + 4z &= 1 \\ 3x + 2y - z &= 0 \end{aligned}$$

(5)

(c) (i) Find the value of q for which the set of simultaneous equations

$$\begin{aligned} 2x - y + z &= 1 \\ 3x - 5y + 4z &= q \\ 3x + 2y - z &= 0 \end{aligned}$$

can be solved.

(ii) For this value of q, interpret the solution of the set of simultaneous equations geometrically.

(4)

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Thursday 6 June 2019

Afternoon (Time: 1 hour 30 minutes) | Paper Reference **9FM0/02**

Further Mathematics

Advanced

Paper 2: Core Pure Mathematics 2

3. Table of contents

Question	Scheme	Marks	AOs
7(a)	$\lvert\mathbf{M}\rvert = 2(-k-8)+1(-3-12)+1(6-3k)=0 \Rightarrow k = ...$	M1	1.1b
	$k \neq -5$	A1	2.4
		(2)	
(b) Way 1	$\mathbf{M} = \begin{pmatrix} 2 & -1 & 1 \\ 3 & -6 & 4 \\ 3 & 2 & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \mathbf{M}^{-1}\begin{pmatrix} p \\ 1 \\ 0 \end{pmatrix}$	M1	3.1a
	$\mathbf{M}^{-1} = \frac{1}{5}\begin{pmatrix} -2 & 1 & 2 \\ 15 & -5 & -5 \\ 24 & -7 & -9 \end{pmatrix}$	B1	1.1b
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -2 & 1 & 2 \\ 15 & -5 & -5 \\ 24 & -7 & -9 \end{pmatrix}\begin{pmatrix} p \\ 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \\ z \end{pmatrix} = ...$	M1	2.1
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5}\begin{pmatrix} -2p+1 \\ 15p-5 \\ 24p-7 \end{pmatrix}$	A1	1.1b
	$\left(\frac{-2p+1}{5},\ 3p-1,\ \frac{24p-7}{5}\right)$	A1ft	2.5
		(5)	

3. Table of contents

(b) Way 2	$\begin{array}{l} 2x - y + z = p \\ 3x - 6y + 4z = 1 \\ 3x + 2y - z = 0 \end{array} \Rightarrow \text{e.g.} \begin{array}{l} 8y - 5z = -1 \\ 9y - 5z = 3p - 2 \end{array} \Rightarrow y = \ldots$ $\Rightarrow x = \ldots, z = \ldots$	M1	3.1a
	$y = 3p - 1$ (or $x = \dfrac{-2p+1}{5}$ or $z = \dfrac{24p-7}{5}$)	B1	1.1b
	$8(3p-1) - 5z = -1 \Rightarrow z = \ldots \Rightarrow x = \ldots$	M1	2.1
	$z = \dfrac{24p-7}{5}, \; x = \dfrac{-2p+1}{5}$	A1	1.1b
	$\left(\dfrac{-2p+1}{5}, \; 3p-1, \dfrac{24p-7}{5} \right)$	A1ft	2.5

(c)(i)	For consistency: E.g. $5x+y=4-q$ and $15x+3y=q$	M1	3.1a
	$4-q=\frac{q}{3} \Rightarrow q=\ldots$	M1	2.1
	$q=3$	A1	1.1b
	Alternative for (c)(i): $x=1 \Rightarrow 2-y+z=1,\ 3+2y-z=0 \Rightarrow y=\ldots, z=\ldots$ M1 for allocating a number to one variable and solves for the other 2 $x=1, y=-4, z=-5 \Rightarrow 3+20-20=q$ M1 substitutes into the second equation and solves for q A1: $q=3$		
(ii)	Three **planes** that intersect in a **line** Or Three **planes** that form a **sheaf** allow **sheath**!	B1	2.4
		(4)	
			(11 marks)

(11 marks)

Notes

(a)

M1: Attempts determinant, equates to zero and attempts to solve for k in order to establish the restriction for k. For the determinant, at least 2 of the 3 "elements" should be correct.

May see rule of Sarrus used for determinant e.g.

$$|\mathbf{M}| = (2)(k)(-1)+(4)(3)(-1)+(3)(2)(1)-(3)(k)(-1)-(2)(4)(2)-(-1)(3)(-1)=0 \Rightarrow k = ...$$

A1: Describes the correct condition for k with no contradictions. Allow e.g. $k < -5$, $k > -5$

(b)**Way 1**

M1: A complete strategy for solving the given equations. Need to see an attempt at the inverse followed by a correct method for finding x, y and z

B1: Correct inverse matrix

M1: Uses their inverse and attempts the multiplication with the correct vector

A1: Correct values for x, y and z in any form

A1ft: Correct values given in coordinate form only. **Follow through their *x*, *y* and *z*.**

Way 2

M1: A complete strategy for solving the given equations. Need to see an attempt at eliminating one variable followed by a correct method for finding x, y and z

B1: One correct value

M1: Uses the equations to find values for the other 2 variables

A1: Correct values for x, y and z in any form

A1ft: Correct values given in coordinate form only. **Follow through their *x*, *y* and *z*.**

3. Table of contents

(c)(i)
M1: Uses a correct strategy that will lead to establishing a value for q. E.g. eliminating one of x, y or z
M1: Solves a suitable equation to obtain a value for q
A1: Correct value
(ii)
B1: Describes the correct geometrical configuration.
Must include the **two** ideas of **planes** and meeting in a **line** or forming a **sheaf** with no contradictory statements.

AQA 2018 AS CORE PURE

AS
FURTHER MATHEMATICS
Paper 1

Monday 14 May 2018 Afternoon Time allowed: 1 hour 30 minutes

2 Three matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{bmatrix} 5 & 2 & -3 \\ 0 & 7 & 6 \\ 4 & 1 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 0 \\ 3 & -5 \\ -2 & 6 \end{bmatrix} \quad \text{and } \mathbf{C} = \begin{bmatrix} 6 & 4 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

Which of the following **cannot** be calculated?

Circle your answer.

[1 mark]

AB **AC** **BC** $\mathbf{A}^2$

Q	Marking instructions	AO	Mark	Typical solution
2	Circles correct answer	1.1a	B1	**AC**
	Total		**1**	

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5 Describe fully the transformation given by the matrix $\begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

[3 marks]

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Q	Marking instructions	AO	Mark	Typical solution
5	Identifies correct values of sine **and** cosine.	1.1a	M1	$\cos\theta = -\frac{1}{2}$ and $\sin\theta = \frac{\sqrt{3}}{2}$
	Selects correct angle. Accept $\frac{2\pi}{3}$ or $-240°$ or $-\frac{4\pi}{3}$	1.1b	A1	$\theta = 120^\circ$
	Deduces the transformation giving a full description. FT their angle. Accept $\frac{2\pi}{3}$ or $-240°$ or $-\frac{4\pi}{3}$ Condone missing degree sign. Condone missing 'anticlockwise'. NMS scores 3/3	2.2a	A1F	Rotation about the z-axis through $120°$ anti-clockwise.
	Total		**3**	

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7 Find two invariant points under the transformation given by $\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$

[2 marks]

Q	Marking instructions	AO	Mark	Typical solution
7	Obtains two equations in x and y. May be seen as a single vector equation. At least one equation must be correct. Accept a pair of letters other than x and y. Ignore any subsequent incorrect working.	1.1a	M1	$\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$ $\begin{bmatrix} 2x+3y \\ x+4y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$ $2x + 3y = x$ and $x + 4y = y$
	Obtains any two correct invariant points, with no incorrect points. Condone correct points given as position vectors. NMS: Correct answer scores 2/2. NMS: One correct invariant point and only one incorrect point scores SC1.	1.1b	A1	$x = -3y$ $\therefore$ two invariant points are $(0,0)$ and $(-3,1)$
	Total		**2**	

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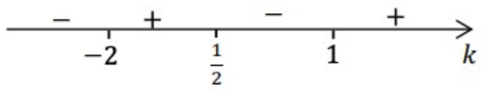
12 (a) Show that the matrix $\begin{bmatrix} 5-k & 2 \\ k^3+1 & k \end{bmatrix}$ is singular when $k = 1$.

[1 mark]

12 (b) Find the values of k for which the matrix $\begin{bmatrix} 5-k & 2 \\ k^3+1 & k \end{bmatrix}$ has a negative determinant.

Fully justify your answer.

[5 marks]

Q	Marking instructions	AO	Mark	Typical solution
12(a)	Substitutes $k = 1$ and correctly calculates the determinant, and concludes that the matrix is singular.	2.2a	B1	determinant $= 4 \times 1 - 2 \times 2 = 0$ $\therefore$ the matrix is singular **AG**
12(b)	Finds determinant in terms of k. Allow one error.	1.1a	M1	determinant $= k(5-k) - 2(k^3+1)$
	Obtains a correct inequality in k.	1.1b	A1	$5k - k^2 - 2k^3 - 2 < 0$
	Obtains three correct critical values.	1.1b	A1	$2k^3 + k^2 - 5k + 2 > 0$ $(k-1)(2k^2+3k-2) > 0$ $(k-1)(2k-1)(k+2) > 0$ 
	Deduces one correct region. FT their three real distinct critical values if given as $a < k < b$, $k > c$ (o.e.) where $a < b < c$	2.2a	A1F	$-2 < k < \frac{1}{2}$, $k > 1$
	Deduces the other correct region. FT their three real distinct critical values if given as $a < k < b$, $k > c$ (o.e.) where $a < b < c$ Condone the use of 'and'.	2.2a	A1F	
	Total		**6**	

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16 Two matrices **A** and **B** satisfy the equation

$$\mathbf{AB} = \boldsymbol{I} + 2\mathbf{A}$$

where $\boldsymbol{I}$ is the identity matrix and $\mathbf{B} = \begin{bmatrix} 3 & -2 \\ -4 & 8 \end{bmatrix}$

Find **A**.

[3 marks]

Q	Marking instructions	AO	Mark	Typical solution
16	Uses factorisation or pre-multiplication to isolate $\boldsymbol{A}$	3.1a	M1	$\boldsymbol{AB} - 2\boldsymbol{A} = \boldsymbol{I}$ $\boldsymbol{A}(\boldsymbol{B} - 2\boldsymbol{I}) = \boldsymbol{I}$
	Deduces $\boldsymbol{A}$ in terms of $\boldsymbol{B}$ and $\boldsymbol{I}$. Could be implied by sight of $\begin{bmatrix} 1 & -2 \\ -4 & 6 \end{bmatrix}$ with attempt to invert.	2.2a	A1	$\boldsymbol{A} = (\boldsymbol{B} - 2\boldsymbol{I})^{-1}$
	Obtains correct matrix $\boldsymbol{A}$.	1.1b	A1	$\boldsymbol{A} = \begin{bmatrix} 1 & -2 \\ -4 & 6 \end{bmatrix}^{-1}$ $\boldsymbol{A} = \frac{1}{-2}\begin{bmatrix} 6 & 2 \\ 4 & 1 \end{bmatrix}$ $\boldsymbol{A} = \begin{bmatrix} -3 & -1 \\ -2 & -0.5 \end{bmatrix}$

ALT 16	Sets up four equations with at least three correct.	3.1a	M1	Let $\boldsymbol{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -4 & 8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 2\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $3a - 4b = 1 + 2a$ and $3c - 4d = 0 + 2c$ and $-2a + 8b = 0 + 2b$ and $-2c + 8d = 1 + 2d$
	Deduces at least two correct elements of $\boldsymbol{A}$. Note: Two correct elements from just two correct equations can score M1A1.	2.2a	A1	$a = 4b + 1$ and $c = 4d$ $6b = 2a$ and $6d = 2c + 1$ $\therefore\ 3b = 4b + 1$ and $6d = 2(4d) + 1$ $-1 = b$ and $-1 = 2d \Rightarrow d = -\frac{1}{2}$ $\therefore\ a = 4 \times -1 + 1$ and $c = 4 \times -\frac{1}{2}$ $a = -3$ and $c = -2$
	Obtains correct matrix $\boldsymbol{A}$	1.1b	A1	$\therefore\ \boldsymbol{A} = \begin{bmatrix} -3 & -1 \\ -2 & -0.5 \end{bmatrix}$
	Total		**3**	

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1 Which of the following matrices is an identity matrix?

Circle your answer.

[1 mark]

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

2 Which of the following expressions is the determinant of the matrix $\begin{bmatrix} a & 2 \\ b & 5 \end{bmatrix}$?

Circle your answer.

[1 mark]

$5a - 2b$ $2a - 5b$ $5b - 2a$ $2b - 5a$

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.2	B1	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	$5a - 2b$
	Total		**1**	

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Paper 1

Monday 13 May 2019 Afternoon Time allowed: 1 hour 30 minutes

12 The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$$

12 (a) Prove by induction that, for all integers $n \geq 1$,

$$\mathbf{A}^n = \begin{bmatrix} 1 & 3^n - 1 \\ 0 & 3^n \end{bmatrix}$$

[4 marks]

12 (b) Find all invariant lines under the transformation matrix **A**.

Fully justify your answer.

[6 marks]

12 (c) Find a line of invariant points under the transformation matrix **A**.

[2 marks]

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Q	Marking instructions	AO	Marks	Typical solution
12(a)	Demonstrates the rule is correct for $n = 1$ and states that it is true for $n = 1$ (may appear at any stage).	1.1b	B1	Try $n = 1$: $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}^1 = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ and $\begin{bmatrix}1 & 3^1 - 1\\0 & 3^1\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ $\therefore$ true for $n = 1$ Assume true for $n = k$ $\therefore\ \mathbf{A}^k = \begin{bmatrix}1 & 3^k - 1\\0 & 3^k\end{bmatrix}$ $\Rightarrow\ \mathbf{A}^k \times \mathbf{A} = \begin{bmatrix}1 & 3^k - 1\\0 & 3^k\end{bmatrix} \times \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ $\Rightarrow\ \mathbf{A}^{k+1} = \begin{bmatrix}1 & 2 + 3(3^k - 1)\\0 & 3^k \times 3\end{bmatrix}$ $= \begin{bmatrix}1 & 3^{k+1} - 1\\0 & 3^{k+1}\end{bmatrix}$ $\therefore$ it is also true for $n = k + 1$ True for $n = 1$, and true for $n = k \Rightarrow$ true for $n = k + 1$ Then, by induction, it is true for all integers $n \geq 1$
	Multiplies $\begin{bmatrix}1 & 3^k - 1\\0 & 3^k\end{bmatrix}$ and $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ Accept any letter in place of k (condone n).	2.4	M1	
	Obtains $\begin{bmatrix}1 & 3^{k+1} - 1\\0 & 3^{k+1}\end{bmatrix}$ from multiplying $\begin{bmatrix}1 & 3^k - 1\\0 & 3^k\end{bmatrix}$ and $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ Must include an intermediate step for the top right element.	2.2a	A1	
	Completes a rigorous argument and explains how their argument proves the required result. e.g. states "assume that the rule is true for $n = k$" (or equivalent) and "also true for $n = k + 1$" (or equivalent) and "for all n" and includes the base case with a conclusion. Do not accept the use of n in place of k. NMS scores 0/4	2.1	R1	

Q	Marking instructions	AO	Marks	Typical solution
12(b)	Sets up two equations in (x, y) and its image (x', y')	3.1a	M1	$\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}x'\\y'\end{bmatrix}$ $x' = x + 2y$ and $y' = 3y$
	Correctly substitutes $y = mx + c$ and $y' = mx' + c$	1.1b	A1	$x' = x + 2(mx + c)$ and $mx' + c = 3(mx + c)$
	Eliminates one variable to leave an equation in m, c and just one other variable.	1.1a	M1	$m(x + 2mx + 2c) + c \equiv 3mx + 3c$
	Compares coefficients to produce two correct equations in m and c	1.1b	A1	$m + 2m^2 = 3m$ and $2mc + c = 3c$
	Gives $y = 0$ or $y = x + c$ as invariant lines. Condone other incorrect invariant lines.	1.1b	B1	$m(m-1) = 0$ and $c(m-1) = 0$ $m = 0$ or $m = 1$ and $c = 0$ or $m = 1$
	Gives $y = 0$ and $y = x + c$ as invariant lines, with no incorrect invariant lines. NMS can score 2/6	2.2a	B1	$y = 0x + 0$ or $y = 1x + c$ Invariant lines are $y = 0$ and $y = x + c$
Q12b: Alternative mark scheme for students who assume that all invariant lines pass through the origin – max 3 marks				
Q12b ALT	Sets up two equations in (x, y) and its image (x', y')	3.1a	M1	$\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}x'\\y'\end{bmatrix}$ $x' = x + 2y$ and $y' = 3y$
	Eliminates three variables to leave an equation in m and just one other variable.	1.1a	M1	$x' = x + 2(mx)$ and $mx' = 3(mx)$ $m(x + 2mx) \equiv 3mx$
	Gives $y = 0$ and $y = x$ as invariant lines, with no other incorrect invariant lines. NMS can score 1/3	2.2a	B1	$m + 2m^2 = 3m$ $m(m-1) = 0$ $m = 0$ or $m = 1$ $y = 0x$ or $y = 1x$ Invariant lines are $y = 0$ and $y = x$

Q	Marking instructions	AO	Marks	Typical solution
12(c)	Sets up at least one correct equation in x and y Accept alternative variables for this mark.	1.1a	M1	$x = x + 2y$ and $y = 3y$
	Gives $y = 0$ as the only line of invariant points. NMS can score 2/2	1.1b	A1	$y = 0$
	Total		**12**	

A-level
FURTHER MATHEMATICS
Paper 1

Monday 3 June 2019 Morning Time allowed: 2 hours

AQA

7 Three non-singular square matrices, **A**, **B** and **R** are such that

$$\mathbf{AR} = \mathbf{B}$$

The matrix **R** represents a rotation about the z-axis through an angle θ and

$$\mathbf{B} = \begin{bmatrix} -\cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

7 (a) Show that **A** is independent of the value of θ.

[3 marks]

7 (b) Give a full description of the single transformation represented by the matrix **A**.
[1 mark]

Q	Marking Instructions	AO	Marks	Typical solution
7(a)	Finds the correct matrix for $\mathbf{R}^{-1}$ PI by correct $\mathbf{A}$	AO2.2a	B1	$\mathbf{R}^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{A} = \mathbf{BR}^{-1}$ $\mathbf{A} = \begin{bmatrix} -\cos\theta & \sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\mathbf{A}$ is independent of θ .
	Appropriate method to find $\mathbf{A}$, such as post multiplying $\mathbf{B}$ by $\mathbf{R}^{-1}$ PI by correct $\mathbf{A}$	AO1.1a	M1	
	Completes a rigorous argument to show the required result, including finding the correct matrix for $\mathbf{A}$. Must include conclusion that $\mathbf{A}$ is independent of θ	AO2.1	R1	
7(b)	States fully correct (single) geometrical description. Eg Reflection in y/z plane.	AO3.2a	E1	Reflection in $x = 0$ plane.
	Total		**4**	

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A-level
FURTHER MATHEMATICS
Paper 1

Monday 3 June 2019 | Morning | Time allowed: 2 hours

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12 Three planes have equations

$$\begin{aligned} 4x - 5y + z &= 8 \\ 3x + 2y - kz &= 6 \\ (k-2)x + ky - 8z &= 6 \end{aligned}$$

where k is a real constant.

The planes do **not** meet at a unique point.

12 (a) Find the possible values of k.

[3 marks]

12 (b) For each value of k found in part **(a)**, identify the configuration of the given planes.

Fully justify your answer, stating in each case whether or not the equations of the planes form a consistent system.

[5 marks]

Q	Marking Instructions	AO	Marks	Typical solution
12(a)	Recognises the need to set the determinant = 0	AO3.1a	M1	$9k^2 - 9k - 180 = 0$ $k = 5$ and $k = -4$
	Obtains and solves a three-term quadratic equation in k	AO1.1a	M1	
	Obtains the correct values of k	AO1.1b	A1	
12(b)	Selects an appropriate method and substitutes their first value of k	AO3.1a	M1	For $k = 5$ $\begin{bmatrix} 4 & -5 & 1 & 8 \\ 0 & 23 & -23 & 0 \\ 0 & 35 & -35 & 0 \end{bmatrix}$ Consistent Line of intersection (sheaf) For $k = -4$ $3x + 2y + 4z = 6$ $-6x - 4y - 8z = 6$ Inconsistent Two planes parallel and distinct with third plane crossing both
	For $k = 5$ (k must be correct): Deduces that equations are consistent – must have sufficient working to justify comment.	AO2.2a	M1	
	Gives correct geometrical description with full working.	AO3.2a	A1	
	For $k = -4$ (k must be correct): Deduces that equations are inconsistent by comparing eqs 2 & 3 – must have comment.	AO2.2a	B1	
	Gives correct geometrical description.	AO3.2a	B1	
	Total		**8**	