

**MR CHAN'S FURTHER MECHANICS 1
ELASTIC STRINGS AND SPRINGS (HOOKES LAW
AND ELASTIC POTENTIAL ENERGY)
QUESTIONS BY TOPIC PACK**



MR CHAN'S FURTHER MECHANICS 1
ELASTIC STRINGS AND SPRINGS (HOOKES LAW AND ELASTIC POTENTIAL ENERGY)
QUESTIONS BY TOPIC PACK

- Questions taken from either Edexcel M3 or IAL M3
- IAL M3 tends to be slightly harder.





6. A light elastic string has natural length a and modulus of elasticity $\frac{1}{2}mg$. A particle P of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A . The particle is released from rest at A and falls vertically. When P has fallen a distance $a + x$, where $x > 0$, the speed of P is v .

(a) Show that

$$v^2 = 2g(a + x) - \frac{3gx^2}{2a}.$$

(4)

(b) Find the greatest speed attained by P as it falls.

(4)

After release, P next comes to instantaneous rest at a point D .

(c) Find the magnitude of the acceleration of P at D .

(6)

(Total 14 marks)

▪ <https://youtu.be/-wdIjz0PbWs>

13Fm Further Mechanics 1 - EPE & Work Energy - A Level Further Maths - M3 Jan 10 Q7

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13Fm - Further Mechanics 1 - A Level Further Maths - M3 Jan 10 Q7 - EPE & Work Energy
Also Practice Paper 5 Q6



5.

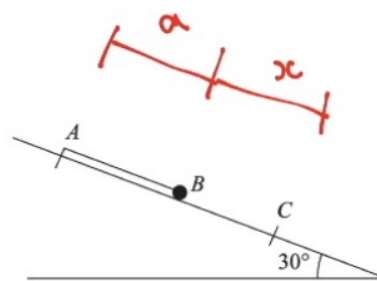


Figure 2

One end A of a light elastic string, of natural length a and modulus of elasticity $6mg$, is fixed at a point on a smooth plane inclined at 30° to the horizontal. A small ball B of mass m is attached to the other end of the string. Initially B is held at rest with the string lying along a line of greatest slope of the plane, with B below A and $AB = a$. The ball is released and comes to instantaneous rest at a point C on the plane, as shown in Figure 2. Find

- (a) the length AC , (5)
- (b) the greatest speed attained by B as it moves from its initial position to C . (7)

Energy

Pen Style



Uniform

Variable



Rectangle Oval Triangle



Star Polygon Sides

Stroke

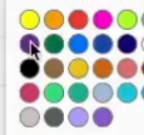
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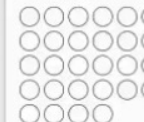
Shape Fill

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13Fm (Further Mechanics) - Elastic Potential Energy on inclined planes M3

25 views • 20 Mar 2021



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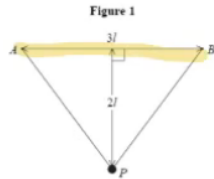
EDIT VIDEO

Two questions on Elastic Potential Energy on inclined planes in old specification M3, M3 June 2014 Q4 will be a nice follow-up question.

<https://drive.google.com/file/d/1YpFv...>

Questions

Q1.



A light elastic string, of natural length $3l$ and modulus of elasticity λ , has its ends attached to two points A and B , where $AB = 3l$ and AB is horizontal. A particle P of mass m is attached to the mid-point of the string. Given that P rests in equilibrium at a distance $2l$ below AB , as shown in Figure 1.

(a) show that $\lambda = \frac{15mg}{16}$.

(9)

The particle is pulled vertically downwards from its equilibrium position until the total length of the elastic string is $7.8l$. The particle is released from rest.

(b) Show that P comes to instantaneous rest on the line AB .

(6)

(Total 15 marks)



1



2



3



4



▪ <https://youtu.be/OD21E2NjnMc>

13Fm - Elastic Strings and Springs (M3 questions by topic pack)

19 views • 17 Apr 2021

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13Fm - Elastic Strings and Springs (M3 questions by topic pack)

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- [8. jan 2002 \(Q2\) M3](#)
- [10. jan 2002 \(Q3\) M3](#)
- [12. June 2002 m3](#)
- [14. Jan 2003 m3 \(Q1\)](#)
- [16. Jan 2003 m3 \(Q6\)](#)
- [18. June 2003 m3](#)
- [20. Jan 2004 m3](#)
- [23. June 2004 M3](#)
- [25. Jan 2005 m3](#)
- [27. June 2005 m3 \(Q1\)](#)
- [29. June 2005 m3 \(Q3\)](#)
- [32. Jan 2006 m3](#)
- [34. Jan 2006 m3](#)
- [37. June 2006 m3](#)
- [40. Jan 2007 m3](#)
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- [45. Jan 2008 m3 \(Q1\)](#)
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- [50. June 2008 m3 \(Q1\)](#)
- [52. JAN 2009 M3 \(Q2\)](#)
- [54. JAN 2009 M3 \(Q5\)](#)
- [58. June 2009 m3](#)
- [60. JAN 2010 M3 \(Q4\)](#)
- [62. Jan 2010 m3 \(q7\)](#)
- [66. JUNE 2010 M3](#)
- [68. JAN 2011 M3](#)
- [71. JUNE 2011 M3](#)
- [75. JAN 2012 M3 Q1](#)
- [77. JUNE 2012 M3](#)
- [80. JAN 2013 M3](#)
- [83. JUN 2013 ® M3](#)
- [85. JUNE 2013 M3](#)
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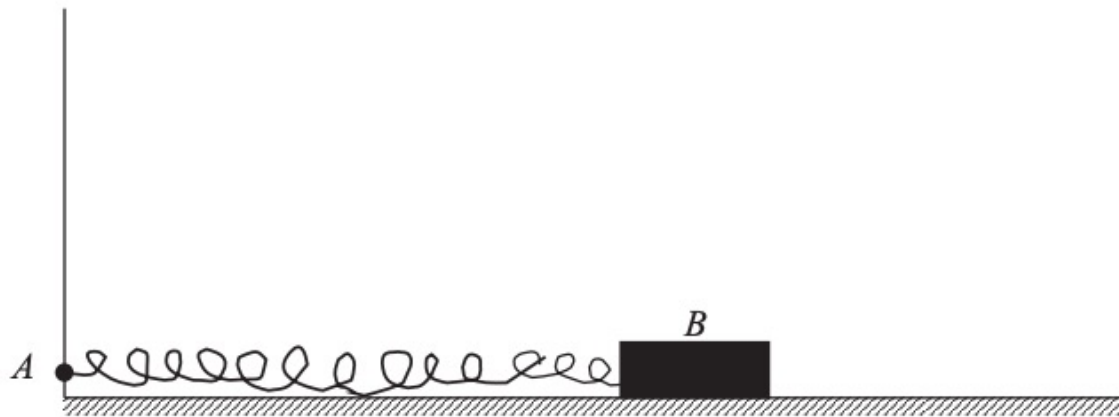
EDEXCEL IAL M3

- 98. Jan 2016 ial m3
- 102. June 2016 ial m3
- 106. Jan 2017 ial m3
- 111. June 2017 ial m3
- 115. June 2019 ial m3
- 117. Jan 2020 ial m3
- 121. Oct 2020 ial m3
- 125. Jan 2015 ial m3
- 129. June 2018 Ial m3
- 131. Jan 2019 ial m3
- 138. Jan 2018 m3 ial



3.

Figure 2



A light horizontal spring, of natural length 0.25 m and modulus of elasticity 52 N, is fastened at one end to a point A . The other end of the spring is fastened to a small wooden block B of mass 1.5 kg which is on a horizontal table, as shown in Fig. 2. The block is modelled as a particle.

The table is initially assumed to be smooth. The block is released from rest when it is a distance 0.3 m from A . By using the principle of the conservation of energy,

(a) find, to 3 significant figures, the speed of B when it is a distance 0.25 m from A .

(5)

It is now assumed that the table is rough and the coefficient of friction between B and the table is 0.6.

(b) Find, to 3 significant figures, the minimum distance from A at which B can rest in equilibrium.

(5)

JUNE 2001 M3



3. (a)	$\frac{1}{2} \times 1.5 v^2 = \frac{52 \times 0.05^2}{2 \times 0.25}$ $v = \underline{0.589 \text{ ms}^{-1}} \quad (3\text{SF})$	<p style="text-align: right;">(+)</p> <p>→ M A A </p> <p> M A </p> <p style="text-align: right;">(5)</p>
(b)	$F = 0.6 \times 1.5g$ $\frac{52x}{0.25} \quad \text{or} \quad \frac{52x}{25}$ $T = F \Rightarrow x = 0.0424 \text{ m} \quad \text{or} \quad 4.24 \text{ cm}$ $\text{Min distance} = \underline{0.208 \text{ m}} \quad \text{or} \quad \underline{20.8 \text{ cm}}$	<p>M </p> <p>B </p> <p>M A </p> <p>A V</p> <p style="text-align: right;">(5)</p> <p style="text-align: right;">(0)</p>



2. One end of a light elastic string, of natural length 2 m and modulus of elasticity 19.6 N, is attached to a fixed point A . A small ball B of mass 0.5 kg is attached to the other end of the string. The ball is released from rest at A and first comes to instantaneous rest at the point C , vertically below A .

(a) Find the distance AC .

(6)

(b) Find the instantaneous acceleration of B at C .

(3)

JAN 2002 (Q2) M3



2. (a)

$$\text{PE Loss} = 0.5g(2+x); \quad \text{EPE}_{\text{at C}} = \frac{19.6x^2}{4}$$

$$0.5g(2+x) = \frac{19.6x^2}{4}$$

$$k(x^2 - 2 - 2) = 0$$

solving

$$\underline{AC = 4x}$$

B | ; B |

M |

A |

M |

A | ✓

(6)

(b)

$$T_c = \frac{19.6 \times 2}{2} = 19.6$$

$$19.6 - 0.5g = 0.5a$$

$$\underline{a = 29.4 \text{ ms}^{-2}}$$

B | ✓

M |

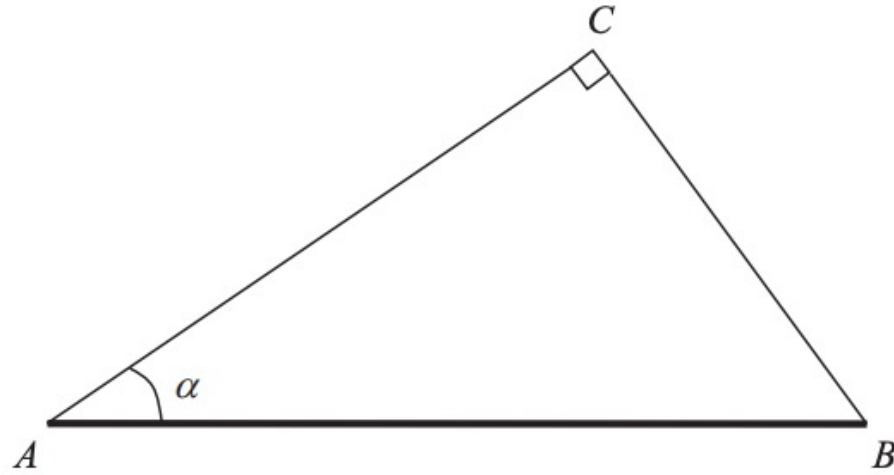
A |

(3)
(9)



3.

Figure 1



A rod AB , of mass $2m$ and length $2a$, is suspended from a fixed point C by two light strings AC and BC . The rod rests horizontally in equilibrium with AC making an angle α with the rod, where $\tan \alpha = \frac{3}{4}$, and with AC perpendicular to BC , as shown in Fig. 1.

(a) Give a reason why the rod cannot be uniform. (1)

(b) Show that the tension in BC is $\frac{8}{5}mg$ and find the tension in AC . (5)

The string BC is elastic, with natural length a and modulus of elasticity kmg , where k is constant.

(c) Find the value of k . (4)

JAN 2002 (Q3) M3



3.(a)	Line of action of weight must pass through c which is not above centre of rod (or equivalent)	BI (1)
(b)	<p><u>Method A:</u></p> $R(\text{along } AC) : T_1 = 2mg \sin \alpha = \frac{6mg}{5}$ $R(\text{along } BC) : T_2 = 2mg \cos \alpha = \frac{8mg}{5}^*$ <p>[Equiv. to moments about A, B respectively]</p> <p>OR <u>Method B:</u> $R(\uparrow), T_1 \sin \alpha + T_2 \cos \alpha = 2mg$ $L(\rightarrow), T_1 \cos \alpha = T_2 \sin \alpha$ solving to find T_1 or T_2 $T_1 = \frac{6mg}{5} ; T_2 = \frac{8mg}{5}^*$</p>	<p>M1 M1 A1 M1 A1</p> <p>→ M1 → M1 M1 A1 ; A1 (3)</p>
(c)	$\frac{8mg}{5} = \frac{kg(BC-a)}{a}$ $BC = 2a \sin \alpha$ $\underline{k = 8}$	<p>M1 A1</p> <p>B1</p> <p>A1 (4)</p> <p>(10)</p>



4. A light elastic string AB of natural length 1.5 m has modulus of elasticity 20 N. The end A is fixed to a point on a smooth horizontal table. A small ball S of mass 0.2 kg is attached to the end B . Initially S is at rest on the table with $AB = 1.5$ m. The ball S is then projected horizontally directly away from A with a speed of 5 m s^{-1} . By modelling S as a particle,

(a) find the speed of S when $AS = 2$ m.

(5)

When the speed of S is 1.5 m s^{-1} , the string breaks.

(b) Find the tension in the string immediately before the string breaks.

(5)

JUNE 2002 M3

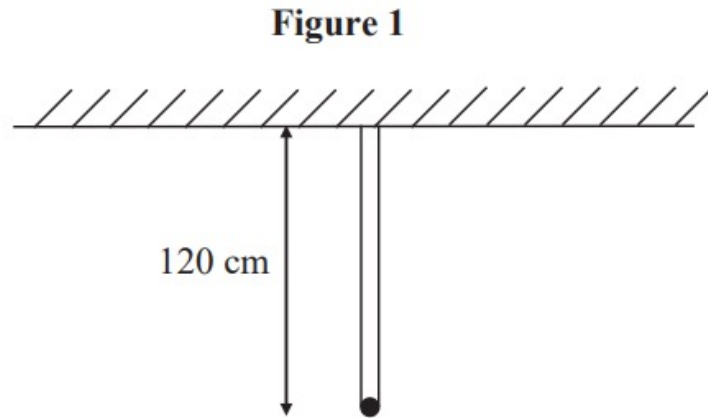


Question Number	Scheme	Marks
4.	<p>(a) $\frac{1}{2} \times 0.2 \times 5^2 - \frac{1}{2} \times 0.2 \times u^2 = \frac{1}{2} \times \frac{20(0.5)^2}{1.5}$</p> $u^2 = \frac{25}{3}$ $u = 2.89 \text{ ms}^{-1}$ <p>(b) $\frac{1}{2} \times 0.2 \times 5^2 - \frac{1}{2} \times 0.2 \times 1.5^2 = \frac{1}{2} \times \frac{20x^2}{1.5}$</p> $x^2 = 0.34125$ $T = \frac{20x}{1.5} = 7.8 \text{ N}$	<p>M1 A1 A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1 A1</p> <p>M1</p> <p>M1 A1 (5)</p> <p>(10 marks)</p>



JAN 2003 M3 (Q1)

1.

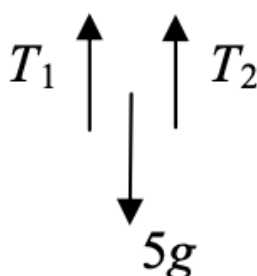


A particle of mass 5 kg is attached to one end of two light elastic strings. The other ends of the strings are attached to a hook on a beam. The particle hangs in equilibrium at a distance 120 cm below the hook with both strings vertical, as shown in Fig. 1. One string has natural length 100 cm and modulus of elasticity 175 N. The other string has natural length 90 cm and modulus of elasticity λ newtons.

Find the value of λ .

(5)



Question Number	Scheme	Marks
1.	<div style="display: flex; align-items: center; justify-content: center;"> <div style="text-align: center; margin-right: 20px;">  </div> <div style="margin-right: 20px;"> $T_1 = \frac{175 \times 0.2}{1}$ </div> <div> $\frac{175 \times 0.2}{1} + \frac{\lambda \times 0.3}{0.9} = 49$ $\Rightarrow \lambda = 42$ </div> </div>	<p>B1</p> <p>M1 A 1</p> <p>M1 A1 (5)</p> <p>(5 marks)</p>

6. A light elastic string has natural length 4 m and modulus of elasticity 58.8 N. A particle P of mass 0.5 kg is attached to one end of the string. The other end of the string is attached to a vertical point A . The particle is released from rest at A and falls vertically.

(a) Find the distance travelled by P before it immediately comes to instantaneous rest for the first time.

(7)

The particle is now held at a point 7 m vertically below A and released from rest.

(b) Find the speed of the particle when the string first becomes slack.

(5)

JAN 2003 M3 (Q6)



Question Number	Scheme	Marks
6. (a)	$\frac{1}{2} \times \frac{58.8}{4} x^2 = 0.5 \times 9.8 (x + 4)$ $3x^2 - 2x - 8 = 0$ $(3x + 4)(x - 2) = 0, \quad x = 2$ <p>Distance fallen = 6 m</p>	M1 A1 A1
(b)	$\frac{1}{2} \times 0.5v^2 = \frac{1}{2} \times \frac{58.8}{4} \times 3^2 - 0.5 \times 9.8 \times 3$ $v = 14.3 \text{ m s}^{-1}$	M1 A1
		M1 A1 (7)
		M1 A1 A1
		M1 A1 (5)
		(12 marks)



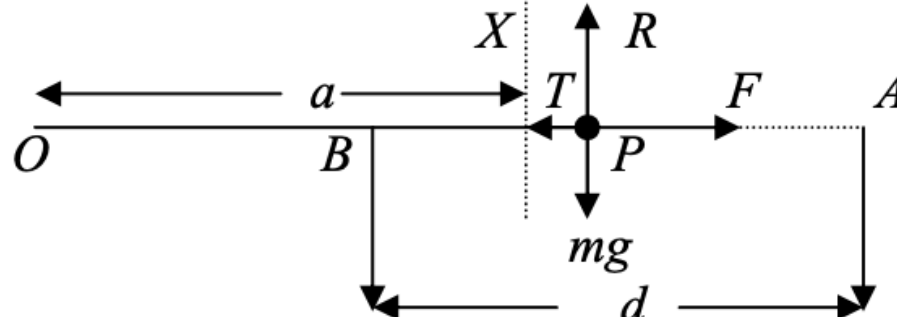
JUNE 2003 M3

1. A particle P of mass m is held at a point A on a rough horizontal plane. The coefficient of friction between P and the plane is $\frac{2}{3}$. The particle is attached to one end of a light elastic string, of natural length a and modulus of elasticity $4mg$. The other end of the string is attached to a fixed point O on the plane, where $OA = \frac{3}{2}a$. The particle P is released from rest and comes to rest at a point B , where $OB < a$.

Using the work-energy principle, or otherwise, calculate the distance AB .

(6)



Question Number	Scheme	Marks
1.	 <p>Attempt to relate Fd to EPE</p> $\frac{2}{3} mg d = \frac{4mg\left(\frac{a}{2}\right)^2}{2a}$ <p>Final answer: $d = \frac{3}{4} a$</p>	$R = mg$ B1 $F = \mu R = \mu mg$ B1 M1 M1 A1 ft A1 (6) (6 marks)



4. A particle P of mass m is attached to one end of a light elastic string of length a and modulus of elasticity $\frac{1}{2}mg$. The other end of the string is fixed at the point A which is at a height $2a$ above a smooth horizontal table. The particle is held on the table with the string making an angle β with the horizontal, where $\tan \beta = \frac{3}{4}$.

(a) Find the elastic energy stored in the string in this position.

(3)

The particle is now released. Assuming that P remains on the table,

(b) find the speed of P when the string is vertical.

(3)

By finding the vertical component of the tension in the string when P is on the table and AP makes an angle θ with the horizontal,

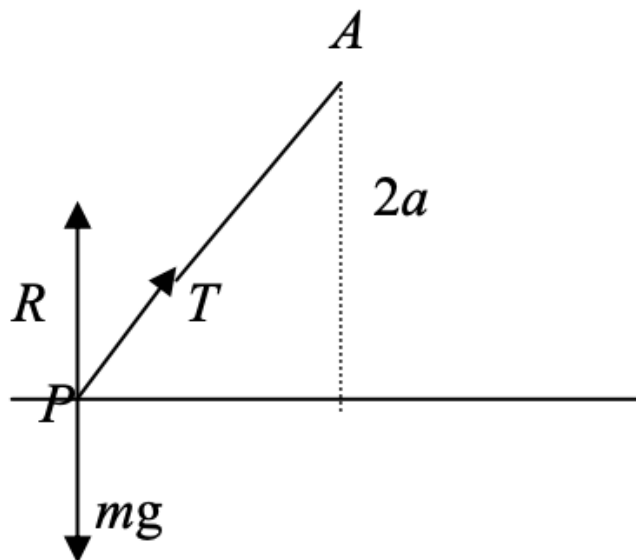
(c) show that the assumption that P remains in contact with the table is justified.

(5)

JAN 2004 M3



4.



(a) Length of string = $\frac{10}{3} a$

B1

$$\text{EPE} = \frac{\frac{1}{2} mg}{2a} (L - a)^2$$

M1

$$= \frac{49}{36} mga$$

A1 (3)

(b) Energy equation: $\frac{1}{2} mv^2 + \frac{\frac{1}{2} mg}{2a} a^2 = \left(\frac{49}{36} mga\right)_C$

M1A1★

$$v = \frac{2}{3} \sqrt{5ga} \text{ or equivalent}$$

A1 (3)



(c) When string at angle θ to horizontal, length of string = $\frac{2a}{\sin \theta}$

$$\begin{aligned}\Rightarrow \text{Vert. Comp. of } T, T_V, &= T \sin \theta = \frac{mg}{2a} \left(\frac{2a}{\sin \theta} - a \right) \sin \theta \\ &= \frac{mg}{2} (2 - \sin \theta)\end{aligned}$$

M1A1

(\Downarrow) $R + T_V = mg$ and find $R = \dots$

M1

$$R = mg - \frac{mg}{2} (2 - \sin \theta) = \frac{mg}{2} \sin \theta$$

A1

$\Rightarrow R > 0$ (as $\sin \theta > 0$), so stays on table

A1 (5)

[Alternative final 3 marks: As θ increases so T_V decreases M1

$$\text{Initial } T_V \text{ (string at } \beta \text{ to hor.)} = \frac{7}{10} mg \quad \text{A1}$$

$\Rightarrow T_V \leq \frac{7}{10} mg < mg$, so stays on table A1]

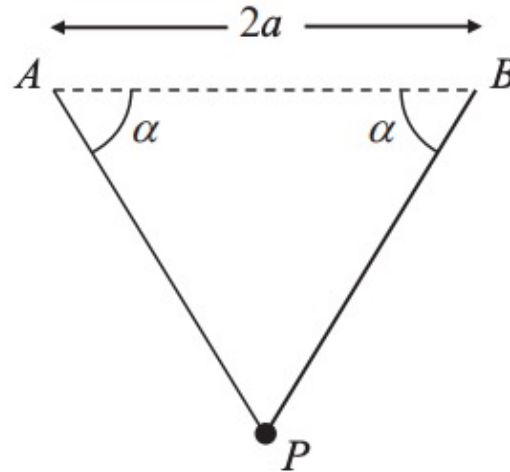
[11]



JUNE 2004 M3

2.

Figure 1



Two light elastic strings each have natural length a and modulus of elasticity λ . A particle P of mass m is attached to one end of each string. The other ends of the strings are attached to points A and B , where AB is horizontal and $AB = 2a$. The particle is held at the mid-point of AB and released from rest. It comes to rest for the first time in the subsequent motion when PA and PB make angles α with AB , where $\tan \alpha = \frac{4}{3}$, as shown in Fig. 1.

Find λ in terms of m and g .



<p>2.</p>	<p>Extn at bottom = $\frac{a}{\cos \alpha} - a = \frac{2a}{3}$ (0.67a or better)</p> <p>Energy: $mga \tan \alpha = \frac{2\lambda \left(\frac{2a}{3}\right)^2}{2a}$</p> <p>$3mg = \lambda$</p> <p>Second M0 if treated as equilibrium Third M1 for solving for λ</p>	<p>M1 A1</p> <p>M1 A1 A1 ft</p> <p>M1 A1</p> <p>(7 marks)</p>
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JAN 2005 M3

1. A particle P of mass 0.5 kg is attached to one end of a light inextensible string of length 1.5 m. The other end of the string is attached to a fixed point A . The particle is moving, with the string taut, in a horizontal circle with centre O vertically below A . The particle is moving with constant angular speed 2.7 rad s^{-1} . Find
- (a) the tension in the string, (4)
- (b) the angle, to the nearest degree, that AP makes with the downward vertical. (3)



EDEXCEL

190 High Holborn London WC1V 7BH

January 2005


Advanced Subsidiary/Advanced Level

General Certificate of Education

Subject: **Mechanics**

Paper: **M3**

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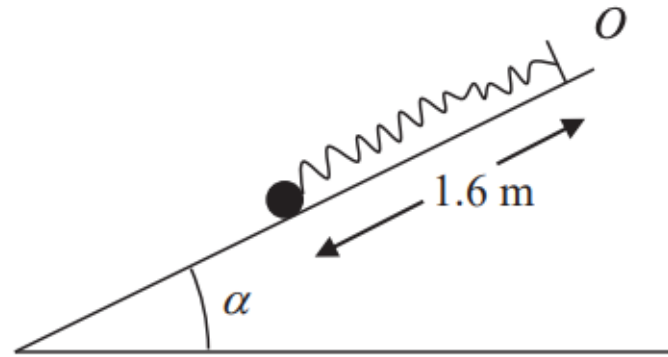
Question Number	Scheme	Marks
1.(a)	 $r = 1.5 \sin \theta$ $T \sin \theta = m r \omega^2$ $T \sin \theta = 0.5 \times 1.5 \sin \theta \times 2.7^2$ $T = \underline{5.4675 \text{ N}}$	B1 M1 A1 A1 (4)
(b)	$T \cos \theta = 0.5g$ $\cos \theta = \frac{0.5g}{5.4675}$ $\underline{\theta = 26^\circ \text{ (nearest degree)}}$	M1 A1 A1 (3) (7)



JUNE 2005 M3 (Q1)

1.

Figure 1

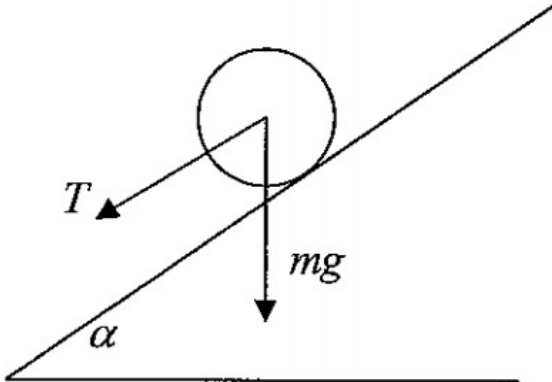


A particle of mass 0.8 kg is attached to one end of a light elastic spring, of natural length 2 m and modulus of elasticity 20 N. The other end of the spring is attached to a fixed point O on a smooth plane which is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$. The particle is held on the plane at a point which is 1.6 m down a line of greatest slope of the plane from O , as shown in Figure 1. The particle is then released from rest.

Find the initial acceleration of the particle.

(Total 6 marks)



Question Number	Scheme	Marks
1.	 <p>HL $T = \frac{20 \times 0.4}{2} (= 4)$ accept -4</p> <p>[$mg \sin \alpha + T = ma$</p> <p>$0.8g \times 0.6 + 4 = 0.8a$</p> <p>$a = 10.88 \approx 10.9 \text{ (ms}^{-2}\text{)}$ accept 11</p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>A1</p> <p>[6]</p>



JUNE 2005 M3 (Q3)

3. A light elastic string has natural length $2l$ and modulus of elasticity $4mg$. One end of the string is attached to a fixed point A and the other end to a fixed point B , where A and B lie on a smooth horizontal table, with $AB = 4l$. A particle P of mass m is attached to the mid-point of the string.

The particle is released from rest at the point of the line AB which is $\frac{5l}{3}$ from B . The speed of P at the mid-point of AB is V .

(a) Find V in terms of g and L .

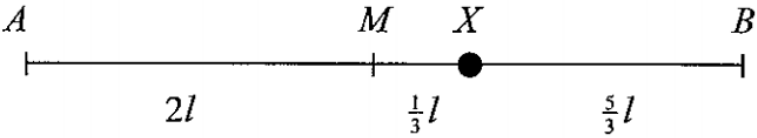
(7)

(b) Explain why V is the maximum speed of P .

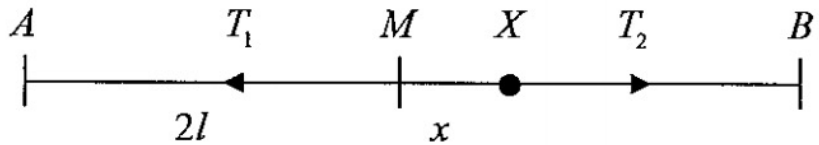
(2)

(Total 9 marks)



Question Number	Scheme	Marks
3.	<p>(a)</p>  <p>Elastic energy when P is at X: $E = \frac{4mg\left(\frac{2}{3}l\right)^2}{2l} + \frac{4mg\left(\frac{4}{3}l\right)^2}{2l} \left(= \frac{40mgl}{9} \right)$</p> $\frac{1}{2}mV^2 + 2 \times \frac{4mgl^2}{2l} = \frac{4mg\left(\frac{2}{3}l\right)^2}{2l} + \frac{4mg\left(\frac{4}{3}l\right)^2}{2l}$ $\frac{1}{2}V^2 + 4gl = \frac{8}{9}gl + \frac{32}{9}gl$ $V^2 = \frac{8gl}{9}$ $V = \left(\frac{8gl}{9}\right)^{\frac{1}{2}}$ <p>(b) The maximum speed occurs when $a = 0$ At M the particle is in equilibrium (the sum of the forces is zero) $\Rightarrow a = 0$.</p> <p><i>The alternative method using Newton's Second Law is considered on the next page.</i></p>	<p>M1 A1</p> <p>M1A1=A1ft</p> <p>M1</p> <p>A1 (7)</p> <p>B1</p> <p>B1 (2)</p> <p>[9]</p>

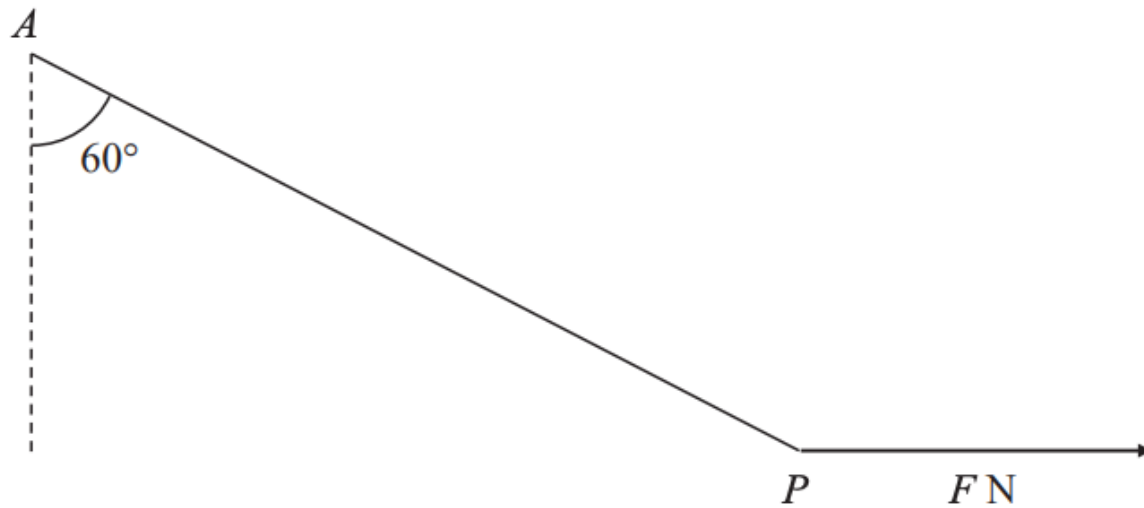


Question Number	Scheme	Marks
3.	<p><i>Alternative using Newton's second law.</i></p> <p>(a)</p>  <p>HL $T_1 = \frac{4mg(l+x)}{l}, T_2 = \frac{4mg(l-x)}{l}$</p> <p>N2L $m\ddot{x} = T_2 - T_1 = -\frac{8mg}{l}x$</p> <p>This is SHM, centre M</p> <p>$a = \frac{l}{3}, \omega^2 = \frac{8g}{l}$</p> <p>$v^2 = \omega^2(a^2 - x^2) \Rightarrow v^2 = \frac{8g}{l} \left(\frac{l^2}{9} - x^2 \right)$ Depends on showing SHM</p> <p>At $M, x = 0, V^2 = \frac{8gl}{9}, V = \left(\frac{8gl}{9} \right)^{\frac{1}{2}}$ or exact equivalents</p> <p>(b) The particle is performing SHM about the mid-point of AB. The maximum speed occurs at the centre of the oscillation (when $x = 0$)</p>	<p>M1 A1</p> <p>A1, A1ft</p> <p>M1</p> <p>M1, A1 (7)</p> <p>B1</p> <p>B1 (2)</p> <p>[9]</p>



1.

Figure 1



A particle P of mass 0.8 kg is attached to one end of a light inelastic string, of natural length 1.2 m and modulus of elasticity 24 N. The other end of the string is attached to a fixed point A . A horizontal force of magnitude F newtons is applied to P . The particle P is in equilibrium with the string making an angle 60° with the downward vertical, as shown in Figure 1.

Calculate

- (a) the value of F , (3)
- (b) the extension of the string, (3)
- (c) the elasticity stored in the string. (2)

JAN 2006 M3



Question Number	Scheme	Marks
1.	<p>(a) $\rightarrow F = T \sin 60^\circ \quad \uparrow T \cos 60^\circ = 0.8g$ both</p> <p>[or $Z F \cos 60^\circ = 0.8g \cos 30^\circ$]</p> <p>$F = 0.8g \tan 60^\circ \approx 14 \text{ (N)}$ accept 13.6</p> <p>(b) $T = \frac{0.8g}{\sin 30^\circ} (=15.68)$ allow in (a)</p> <p>HL $15.68 = \frac{24 \times x}{1.2} \Rightarrow x \approx 0.78 \text{ (cm)}$ accept 0.784</p> <p>(c) $E = \frac{24 \times x^2}{2 \times 1.2} \approx 6.1 \text{ (J)}$ accept 6.15</p>	<p>M1 (M2) M1 A1 (3)</p> <p>M1 M1 A1 (3)</p> <p>M1 A1ft (2)</p> <p>Total 8 marks</p>



6. One end of a light inextensible string of length l is attached to a fixed point A . The other end is attached to a particle P of mass m which is hanging freely at rest at point B . The particle P is projected horizontally from B with speed $\sqrt{3gl}$. When AP makes an angle θ with the downward vertical and the string remains taut, the tension in the string is T .

(a) Show that $T = mg(1 + 3 \cos \theta)$. (6)

(b) Find the speed of P at the instant when the string becomes slack. (3)

(c) Find the maximum height above the level of B reached by P . (5)

JAN 2006 M3



Question Number	Scheme	Marks
6.	<p>(a)</p> <p>Energy $\frac{1}{2}m(u^2 - v^2) = mgl(1 - \cos \theta)$ $[v^2 = gl + 2gl \cos \theta]$</p> <p>N2L $T - mg \cos \theta = \frac{mv^2}{l}$ $= \frac{mg \lambda (1 + 2 \cos \theta)}{\lambda}$ $T = mg(1 + 3 \cos \theta) *$</p>	<p>M1 A1</p> <p>M1 A1</p> <p>M1</p> <p>cs0 A1</p> <p>(6)</p>



(b)

$$T = 0 \Rightarrow \cos \theta = -\frac{1}{3}$$

$$v^2 = gl - \frac{2}{3}gl \Rightarrow v = \left(\frac{gl}{3}\right)^{\frac{1}{2}}$$

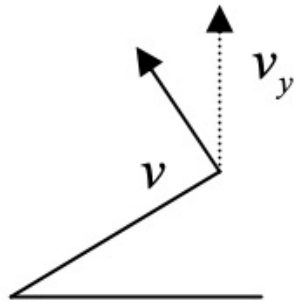
B1

M1 A1

(3)

(c)

$$\uparrow v_y = \left(\frac{gl}{3}\right)^{\frac{1}{2}} \sin \theta \left[= \left(\frac{gl}{3}\right)^{\frac{1}{2}} \cdot \frac{2\sqrt{2}}{3} \right]$$



$$v^2 = u^2 - 2gh \Rightarrow 2gh = \frac{gl}{3} \cdot \frac{8}{9} \Rightarrow h = \frac{4l}{27}$$

$$H = l(1 - \cos \theta) + \frac{4l}{27} = \frac{40l}{27}$$

M1

M1 A1

M1 A1

(5)

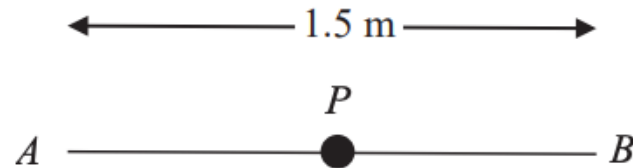
Total 14 marks



JUNE 2006 M3

5. Two light elastic strings each have natural length 0.75 m and modulus of elasticity 49 N. A particle P of mass 2 kg is attached to one end of each string. The other ends of the strings are attached to fixed points A and B , where AB is horizontal and $AB = 1.5$ m.

Figure 2



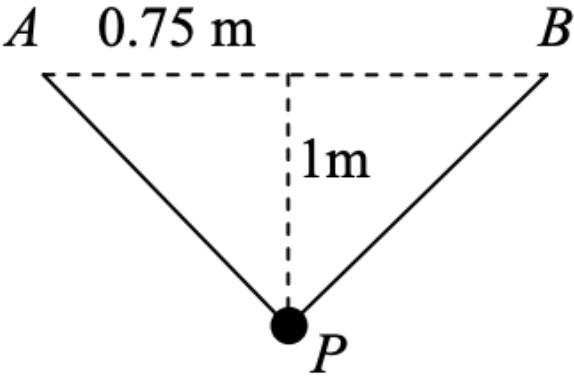
The particle is held at the mid-point of AB . The particle is released from rest, as shown in Figure 2.

- (a) Find the speed of P when it has fallen a distance of 1 m. (6)

Given instead that P hangs in equilibrium vertically below the mid-point of AB , with $\angle APB = 2\alpha$,

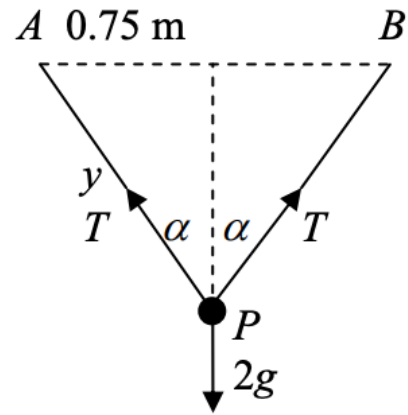
- (b) show that $\tan \alpha + 5 \sin \alpha = 5$. (6)



Question Number	Scheme	Marks
5.	<p>(a)</p>  <p style="text-align: center;">$AP = \sqrt{(0.75^2 + 1^2)} = 1.25$</p> <p>Conservation of energy</p> $\frac{1}{2} \times 2 \times v^2 + 2 \times \frac{49 \times 0.5^2}{2 \times 0.75} = 2g \times 1 \quad -1$ <p>for each incorrect term</p> <p style="text-align: center;">Leading to $v \approx 1.8 \text{ (ms}^{-1}\text{)}$</p> <p>accept 1.81</p>	<p>M1 A1</p> <p>M1 A2 (1, 0)</p> <p>A1 (6)</p>



(b)



$$R(\uparrow) \quad 2T \cos \alpha = 2g \quad \text{M1 A1}$$

$$y = \frac{0.75}{\sin \alpha}$$

Hooke's Law

$$T = \frac{49}{0.75} \left(\frac{0.75}{\sin \alpha} - 0.75 \right) \quad \text{M1 A1}$$

$$= 49 \left(\frac{1}{\sin \alpha} - 1 \right)$$

$$\frac{9.8}{\cos \alpha} = 49 \left(\frac{1}{\sin \alpha} - 1 \right) \quad \text{M1}$$

Eliminating T

$$\tan \alpha = 5(1 - \sin \alpha)$$

$$5 = \tan \alpha + 5 \sin \alpha \quad *$$

CSO

A1 (6)

[12]



JAN 2007 M3

3. A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $3.6mg$. The other end of the string is fixed at a point O on a rough horizontal table. The particle is projected along the surface of the table from O with speed $\sqrt{2ag}$. At its furthest point from O , the particle is at the point A , where $OA = \frac{4}{3}a$.
- (a) Find, in terms of m , g and a , the elastic energy stored in the string when P is at A . (3)
- (b) Using the work-energy principle, or otherwise, find the coefficient of friction between P and the table. (6)



3.

$$\begin{aligned} \text{(a)} \quad \text{E.P.E.} &= \frac{1}{2} \frac{3.6mg}{a} x^2 = \frac{1}{2} \frac{3.6mg}{a} \left(\frac{a}{3}\right)^2 \\ &= \underline{0.2 mga} \end{aligned}$$

$$\text{(b)} \quad \text{Friction} = \mu mg \Rightarrow \text{work done by friction} = \mu mg \left(\frac{4a}{3}\right)$$

$$\text{Work-energy: } \frac{1}{2} m \cdot 2ga = \mu mgd + 0.2 mga \quad (\text{3 relevant terms})$$

$$\text{Solving to find } \mu: \quad \underline{\mu = 0.6}$$

(b) 1st M1: allow for attempt to find work done by frictional force (i.e. not just finding friction).

2nd M1: “relevant” terms, i.e. energy or work terms!

A1 f.t. on their work done by friction

M1 A1

A1

(3)

M1 A1

M1 A1√

↓

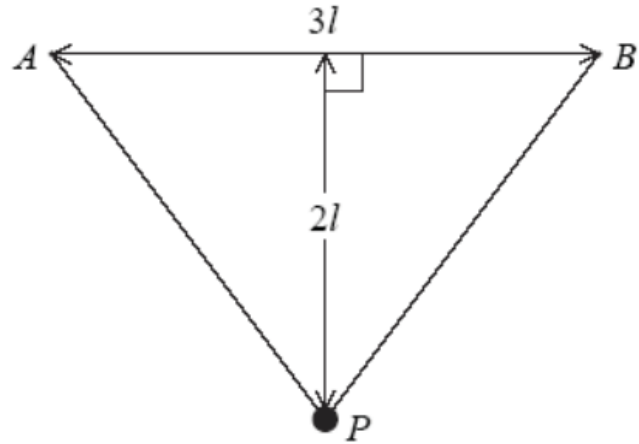
M1 A1

(6)



7.

Figure 1



A light elastic string, of natural length $3l$ and modulus of elasticity λ , has its ends attached to two points A and B , where $AB = 3l$ and AB is horizontal. A particle P of mass m is attached to the mid-point of the string. Given that P rests in equilibrium at a distance $2l$ below AB , as shown in Figure 1,

(a) show that $\lambda = \frac{15mg}{16}$.

(9)

The particle is pulled vertically downwards from its equilibrium position until the total length of the elastic string is $7.8l$. The particle is released from rest.

(b) Show that P comes to instantaneous rest on the line AB .

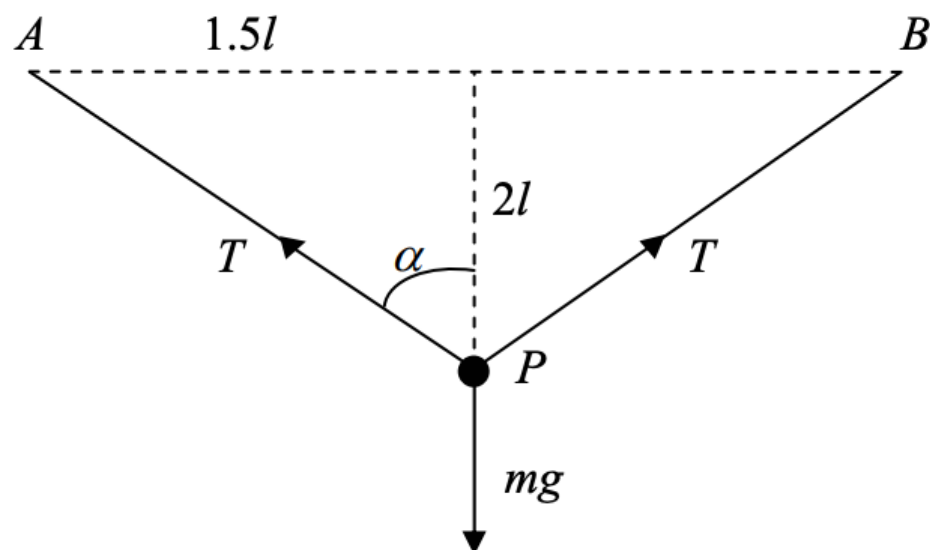
(6)

M3 JUNE 2007 (Q7)



7.

(a)



$$AP = \sqrt{(1.5l)^2 + (2l)^2} = 2.5l$$

$$\cos \alpha = \frac{4}{5}$$

Hooke's Law $T = \frac{\lambda(2.5l - 1.5l)}{1.5l} \left(= \frac{2\lambda}{3} \right)$

\uparrow $2T \cos \alpha = mg \quad \left(T = \frac{5mg}{8} \right)$

$$2 \times \frac{2\lambda}{3} \times \frac{4}{5} = mg \quad \left(\frac{2\lambda}{3} = \frac{5mg}{8} \right)$$

$$\lambda = \frac{15mg}{16} *$$

M1 A1

B1

M1 A1

M1 A1

M1

cso

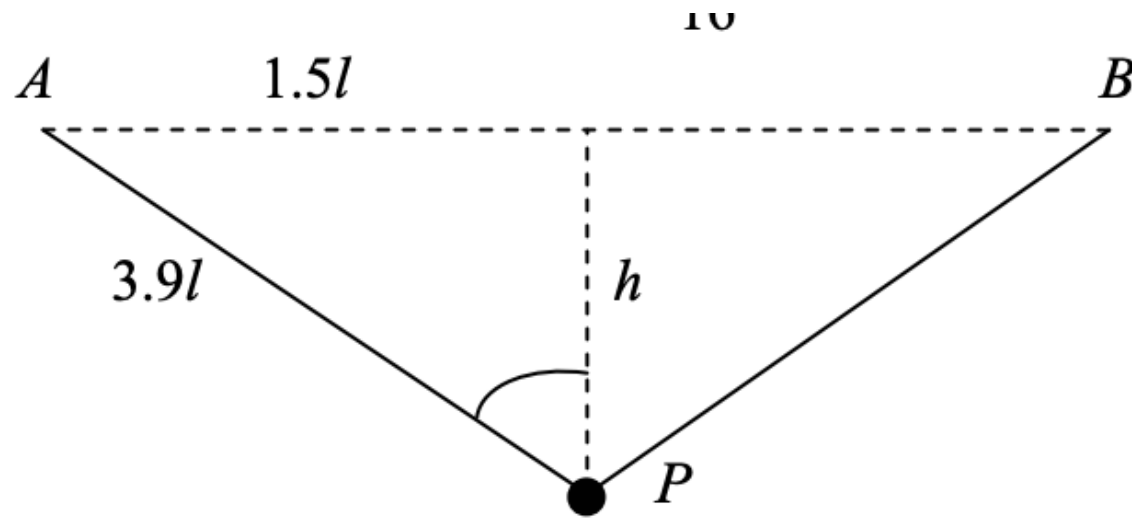
A1

(9)

[Back to Edexcel M3](#)



(b)



$$h = \sqrt{((3.9l)^2 - (1.5l)^2)} = 3.6l$$

Energy

$$\frac{1}{2}mv^2 + mg \times h = 2 \times \frac{15mg}{16} \times \frac{(2.4l)^2}{2 \times 1.5l}$$

Leading to

$$v = 0 \quad *$$

ft their h

cso

M1 A1

M1 A1ft = A1

A1

(6)
[15]



JAN 2008 M3 (Q1)

1. A light elastic string of natural length 0.4 m has one end A attached to a fixed point. The other end of the string is attached to a particle P of mass 2 kg. When P hangs in equilibrium vertically below A , the length of the string is 0.56 m.

(a) Find the modulus of elasticity of the string.

(3)

A horizontal force is applied to P so that it is held in equilibrium with the string making an angle θ with the downward vertical. The length of the string is now 0.72 m.

(b) Find the angle θ .

(3)



January 2008
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks
1.(a)	$T \text{ or } \frac{\lambda \times e}{l} = mg \quad (\text{even } T=m \text{ is M1, A0, A0 sp case})$ $\frac{\lambda \times 0.16}{0.4} = 2g$	M1
(b)	$\Rightarrow \lambda = \underline{49 \text{ N}} \quad \text{or } 5g$	A1 (3)
	<div style="border: 1px solid black; padding: 5px; width: fit-content;"> <p>Special case $T \sin \theta = mg$ giving $\theta = 30$ is M1 A0 A0 unless there is evidence that they think θ is with horizontal – then M1 A1 A0</p> </div> $R(\uparrow) \quad T \cos \theta = mg \text{ or } \cos \theta = \frac{mg}{T}$	M1
	$49 \cdot \frac{0.32}{0.4} \cdot \cos \theta = 19.6 \text{ or } 4g \cdot \cos \theta = 2g \text{ or } 2mg \cdot \cos \theta = mg \quad (\text{ft on their } \lambda)$	A1ft
	$\Rightarrow \cos \theta = \frac{1}{2} \Rightarrow \theta = 60^\circ \quad (\text{or } \frac{\pi}{3} \text{ radians})$	A1 (3)
		6



JAN 2008 M3 (Q4)

4. A particle P of mass m lies on a smooth plane inclined at an angle 30° to the horizontal. The particle is attached to one end of a light elastic string, of natural length a and modulus of elasticity $2mg$. The other end of the string is attached to a fixed point O on the plane. The particle P is in equilibrium at the point A on the plane and the extension of the string is $\frac{1}{4}a$. The particle P is now projected from A down a line of greatest slope of the plane with speed V . It comes to instantaneous rest after moving a distance $\frac{1}{2}a$.

By using the principle of conservation of energy,

(a) find V in terms of a and g ,

(6)

(b) find, in terms of a and g , the speed of P when the string first becomes slack.

(4)



4. (a)

Energy equation with at least three terms, including K.E term

$$\frac{1}{2}mV^2 + ..$$

$$+ .. \frac{1}{2} \cdot \frac{2mg}{a} \cdot \frac{a^2}{16}, + mg \cdot \frac{1}{2} a \cdot \sin 30, = \frac{1}{2} \cdot \frac{2mg}{a} \cdot \frac{9a^2}{16}$$

$$\Rightarrow V = \sqrt{\frac{ga}{2}}$$

M1

A1, A1, A1

dM1 A1 (6)

(b)

Using point where velocity is zero and point where string becomes slack:

$$\frac{1}{2}mw^2 =$$

$$\frac{1}{2} \cdot \frac{2mg}{a} \cdot \frac{9a^2}{16}, - mg \cdot \frac{3a}{4} \cdot \sin 30$$

$$\Rightarrow w = \sqrt{\frac{3ag}{8}}$$

M1

A1, A1

A1 (4)

Alternative (using point of projection and point where string becomes slack):

$$\frac{1}{2}mw^2 - \frac{1}{2}mV_1^2, = \frac{mga}{16} - \frac{mga}{8}$$

$$\text{So } w = \sqrt{\frac{3ag}{8}}$$

M1, A1 A1

A1

10

[Back to Edexcel M3](#)

In part (a)

DM1 requires EE, PE and KE to have been included in the energy equation.

If sign errors lead to $V^2 = -\frac{ga}{2}$, the last two marks are M0 A0

In parts (a) and (b) A marks need to have the correct signs

In part (b) for M1 need **one** KE term in energy equation of at least **3 terms** with distance $\frac{3a}{4}$ to indicate first method, and **two** KE terms in energy equation of at least **4 terms** with

distance $\frac{a}{4}$ to indicate second method.

SHM approach in part (b). (**Condone this method only if SHM is proved**)

Using $v^2 = \omega^2(a^2 - x^2)$ with $\omega^2 = \frac{2g}{a}$ and $x = \pm \frac{a}{4}$.

Using ' a ' = $\frac{a}{2}$ to give $w = \sqrt{\frac{3ag}{8}}$.

M1 A1 A1

A1



1.

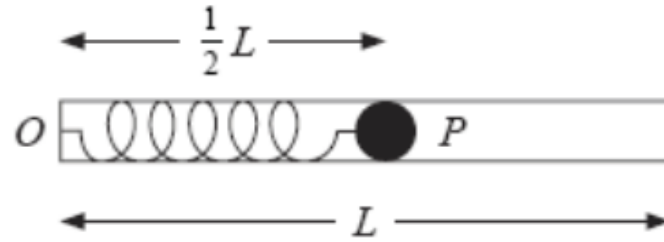


Figure 1

A light elastic spring, of natural length L and modulus of elasticity λ , has a particle P of mass m attached to one end. The other end of the spring is fixed to a point O on the closed end of a fixed smooth hollow tube of length L .

The tube is placed horizontally and P is held inside the tube with $OP = \frac{1}{2}L$, as shown in Figure 1. The particle P is released and passes through the open end of the tube with speed $\sqrt{2gL}$.

(a) Show that $\lambda = 8mg$. (4)

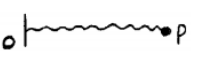

The tube is now fixed vertically and P is held inside the tube with $OP = \frac{1}{2}L$ and P above O . The particle P is released and passes through the open top of the tube with speed u .

(b) Find u . (5)

JUNE 2008 M3 (Q1)



**June 2008
6679 Mechanics M3
Mark Scheme**

Question Number	Scheme	Marks
Q1(a)	 $\text{EPE stored} = \frac{1}{2} \frac{\lambda}{L} \left(\frac{1}{2} L \right)^2 \quad \left(= \frac{\lambda L}{8} \right)$ $\text{KE gained} = \frac{1}{2} m 2gL \quad (= mgL)$ $\text{EPE} = \text{KE} \Rightarrow \frac{\lambda L}{8} = mgL \quad \text{i.e. } \lambda = 8mg^*$	<p>B1</p> <p>B1</p> <p>M1A1cso</p> <p style="text-align: right;">(4)</p>
(b)	 $\text{EPE} = \text{GPE} + \text{KE}$ $\frac{1}{2} \frac{8mg}{L} \left(\frac{1}{2} L \right)^2 = \frac{8mgL}{8} = mg \frac{L}{2} + \frac{1}{2} mu^2$ $\frac{mgL}{2} = \frac{m}{2} u^2 \quad \therefore u = \sqrt{gL}$	<p>M1</p> <p>A1A1</p> <p>M1A1 (5)</p> <p>9 Marks</p>



2.

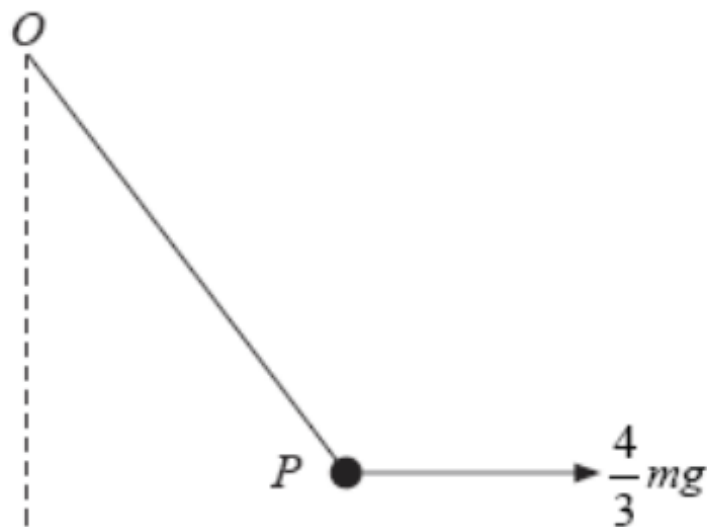


Figure 1

A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $3mg$. The other end of the string is attached to a fixed point O .

The particle P is held in equilibrium by a horizontal force of magnitude $\frac{4}{3}mg$ applied to P .

This force acts in the vertical plane containing the string, as shown in Figure 1. Find

(a) the tension in the string,

(5)

(b) the elastic energy stored in the string.

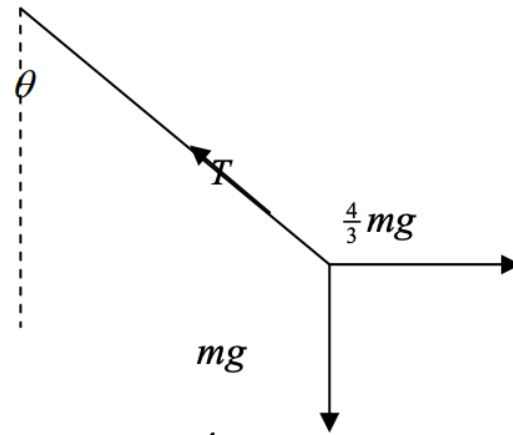
(4)

**JAN 2009 M3
(Q2)**

[Back to Edexcel M3](#)



2



(a) (\leftarrow) $T \sin \theta = \frac{4}{3} mg$

 (\uparrow) $T \cos \theta = mg$

$$T^2 = \left(\frac{4}{3} mg\right)^2 + (mg)^2$$

Leading to $T = \frac{5}{3} mg$

M1 A1

A1

M1

A1 (5)

(b) HL $T = \frac{\lambda x}{a} \Rightarrow \frac{5}{3} mg = \frac{3mge}{a}$ ft their T

$$e = \frac{5}{9} a$$

$$E = \frac{\lambda x^2}{2a} = \frac{3mg}{2a} \times \left(\frac{5}{9} a\right)^2 = \frac{25}{54} mga$$

M1 A1ft

M1 A1 (4)

[9]

[Back to Edexcel M3](#)



5.

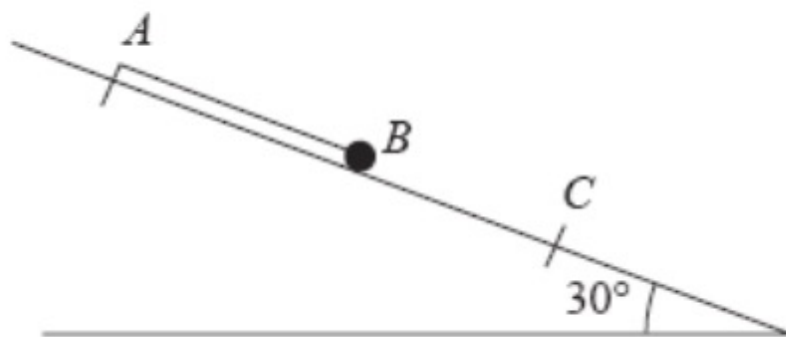


Figure 2

One end A of a light elastic string, of natural length a and modulus of elasticity $6mg$, is fixed at a point on a smooth plane inclined at 30° to the horizontal. A small ball B of mass m is attached to the other end of the string. Initially B is held at rest with the string lying along a line of greatest slope of the plane, with B below A and $AB = a$. The ball is released and comes to instantaneous rest at a point C on the plane, as shown in Figure 2.

Find

(a) the length AC , (5)

(b) the greatest speed attained by B as it moves from its initial position to C . (7)

JAN 2009
M3 (Q5)

[Back to Edexcel M3](#)



Question Number	Scheme	Marks
5	<p>(a) Let x be the distance from the initial position of B to C GPE lost = EPE gained</p> $mgx \sin 30^\circ = \frac{6mgx^2}{2a}$ <p>Leading to $x = \frac{a}{6}$</p> $AC = \frac{7a}{6}$ <p>(b) The greatest speed is attained when the acceleration of B is zero, that is where the forces on B are equal.</p> $(\nearrow) \quad T = mg \sin 30^\circ = \frac{6mge}{a}$ $e = \frac{a}{12}$ <p>CE $\frac{1}{2}mv^2 + \frac{6mg}{2a} \left(\frac{a}{12}\right)^2 = mg \frac{a}{12} \sin 30^\circ$</p> <p>Leading to $v = \sqrt{\left(\frac{ga}{24}\right)} = \frac{\sqrt{6ga}}{12}$</p> <p><i>Alternative approaches to (b) are considered on the next page.</i></p>	<p>M1 A1=A1</p> <p>M1</p> <p>A1 (5)</p> <p>M1</p> <p>A1</p> <p>M1 A1=A1</p> <p>M1 A1 (7)</p> <p>[12]</p>

[Back to Edexcel M3](#)



Question Number	Scheme	Marks
5	<p><i>Alternative approach to (b) using calculus with energy.</i></p> <p>Let distance moved by B be x</p> <p>CE $\frac{1}{2}mv^2 + \frac{6mg}{2a}x^2 = mgx \sin 30^\circ$</p> $v^2 = gx - \frac{6g}{a}x^2$ <p>For maximum v $\frac{d}{dx}(v^2) = 2v \frac{dv}{dx} = g - \frac{12g}{a}x = 0$</p> $x = \frac{a}{12}$ $v^2 = g\left(\frac{a}{12}\right) - \frac{6g}{a}\left(\frac{a}{12}\right)^2 = \frac{ga}{24}$ $v = \sqrt{\left(\frac{ga}{24}\right)}$	<p>M1 A1=A1</p> <p>M1 A1</p> <p>M1</p> <p>A1 (7)</p>



Alternative approach to (b) using calculus with Newton's second law.

As before, the centre of the oscillation is when extension is $\frac{a}{12}$

$$\text{N2L} \quad mg \sin 30^\circ - T = m\ddot{x}$$

$$\frac{1}{2}mg - \frac{6mg \left(\frac{a}{12} + x \right)}{a} = m\ddot{x}$$

$$\ddot{x} = -\frac{6g}{a}x \Rightarrow \omega^2 = \frac{6g}{a}$$

$$v_{\max} = \omega a = \sqrt{\left(\frac{6g}{a} \right)} \times \frac{a}{12} = \sqrt{\left(\frac{ga}{24} \right)}$$

M1 A1

M1 A1

A1

M1 A1 (7)

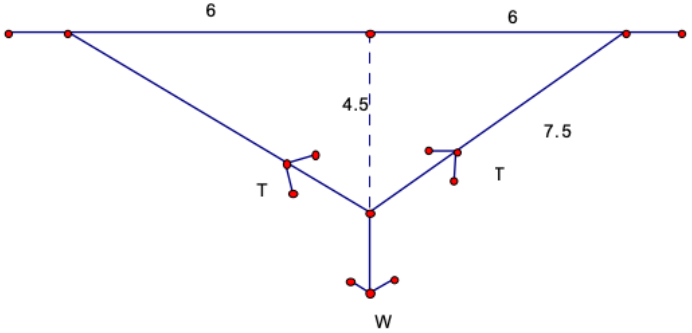


1. A light elastic string has natural length 8 m and modulus of elasticity 80 N. The ends of the string are attached to fixed points P and Q which are on the same horizontal level and 12 m apart. A particle is attached to the mid-point of the string and hangs in equilibrium at a point 4.5 m below PQ .
- (a) Calculate the weight of the particle. (6)
- (b) Calculate the elastic energy in the string when the particle is in this position. (3)
-

JUNE 2009 M3



**June 2009
6679 Mechanics M3
Mark Scheme**

Question Number	Scheme	Marks
Q1 (a)	 <p>Resolving vertically: $2T \cos \theta = W$</p> <p>Hooke's Law: $T = \frac{80 \times 3.5}{4}$ $W = 84\text{N}$</p>	M1A2,1,0 M1A1 A1
(b)	<p>EPE = $2 \times \frac{80 \times 3.5^2}{2 \times 4}$, = 245 (or awrt 245)</p> <p>(alternative $\frac{80 \times 7^2}{16} = 245$)</p>	M1A1ft,A1 [9]



4.

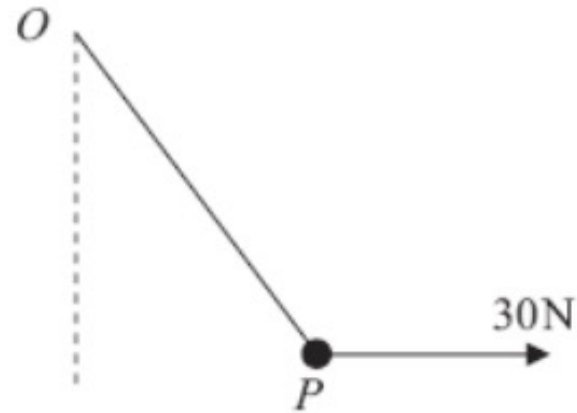


Figure 3

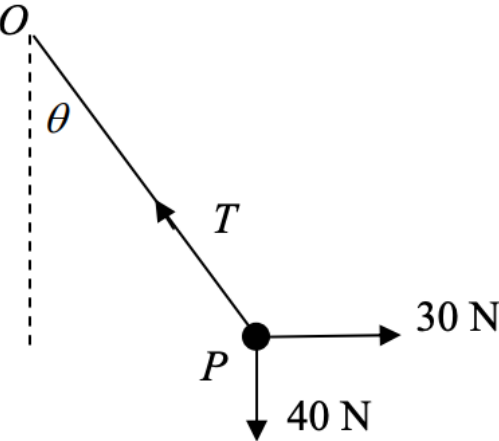
JAN 2010 M3
(Q4)

A particle P of weight 40 N is attached to one end of a light elastic string of natural length 0.5 m . The other end of the string is attached to a fixed point O . A horizontal force of magnitude 30 N is applied to P , as shown in Figure 3. The particle P is in equilibrium and the elastic energy stored in the string is 10 J .

Calculate the length OP .

(10)



Question Number	Scheme	Marks
Q4.	<div style="text-align: center;">  </div> <p style="margin-left: 40px;"> $\uparrow \quad T \cos \theta = 40$ $\rightarrow \quad T \sin \theta = 30$ leading to $T = 50$ </p> <p style="margin-left: 100px;"> $E = \frac{\lambda x^2}{2a} = 10$ </p> <p style="margin-left: 40px;"> HL $T = \frac{\lambda x}{a} = 50$ </p> <p style="margin-left: 40px;"> leading to $x = 0.4$ </p> <p style="margin-left: 40px;"> $OP = 0.5 + 0.4 = 0.9 \text{ (m)}$ </p>	<p>M1 A1 A1 M1 A1</p> <p>B1</p> <p>M1</p> <p>M1 A1</p> <p>A1ft (10) [10]</p>

[Back to Edexcel M3](#)



JAN 2010 M3 (Q7)

7. A light elastic string has natural length a and modulus of elasticity $\frac{3}{2}mg$. A particle P of mass m is attached to one end of the string. The other end of the string is attached to a fixed point A . The particle is released from rest at A and falls vertically. When P has fallen a distance $a + x$, where $x > 0$, the speed of P is v .

(a) Show that

$$v^2 = 2g(a + x) - \frac{3gx^2}{2a}. \quad (4)$$

(b) Find the greatest speed attained by P as it falls. (4)

After release, P next comes to instantaneous rest at a point D .

(c) Find the magnitude of the acceleration of P at D . (6)



Q7.

(a)

$$\frac{1}{2}mv^2 + \frac{3mgx^2}{4a} = mg(a+x)$$

$$\text{leading to } v^2 = 2g(a+x) - \frac{3gx^2}{2a} *$$

cso

M1 A2 (1, 0)

A1 (4)

(b) Greatest speed is when the acceleration is zero

$$T = \frac{\lambda x}{a} = \frac{3mgx}{2a} = mg \Rightarrow x = \frac{2a}{3}$$

$$v^2 = 2g\left(a + \frac{2a}{3}\right) - \frac{3g}{2a} \times \left(\frac{2a}{3}\right)^2 \left(= \frac{8ag}{3}\right)$$

$$v = \frac{2}{3}\sqrt{(6ag)} \quad \text{accept exact equivalents} \quad \text{A1} \quad (4)$$

M1 A1

M1

$$(c) v=0 \Rightarrow 2g(a+x) - \frac{3gx^2}{2a} = 0$$

$$3x^2 - 4ax - 4a^2 = (x-2a)(3x+2a) = 0$$

$$x = 2a$$

M1

M1 A1

At D,

$$m\ddot{x} = mg - \frac{\lambda \times 2a}{a}$$

ft their 2a

M1 A1ft

$$|\ddot{x}| = 2g$$

A1 (6)

[14]

[Back to Edexcel M3](#)

Alternative to (b)

$$v^2 = 2g(a+x) - \frac{3gx^2}{2a}$$

Differentiating with respect to x

$$2v \frac{dv}{dx} = 2g - \frac{3gx}{a}$$

$$\frac{dv}{dx} = 0 \Rightarrow x = \frac{2a}{3}$$

$$v^2 = 2g\left(a + \frac{2a}{3}\right) - \frac{3g}{2a} \times \left(\frac{2a}{3}\right)^2 \left(= \frac{8ag}{3}\right)$$

$$v = \frac{2}{3} \sqrt{(6ag)}$$

accept exact equivalents

M1 A1

M1

A1

(4)



Q7.

Alternative approach using SHM for (b) and (c)

If SHM is used mark (b) and (c) together placing the marks in the grid as shown.

Establishment of equilibrium position

$$T = \frac{\lambda x}{a} = \frac{3mge}{2a} = mg \Rightarrow e = \frac{2a}{3}$$

bM1 bA1

N2L, using y for displacement from equilibrium position

$$m\ddot{y} = mg - \frac{\frac{3}{2}mg(y+e)}{a} = -\frac{3g}{2a}y$$

bM1 bA1

$$\omega^2 = \frac{3g}{2a}$$

Speed at end of free fall $u^2 = 2ga$

cM1

Using A for amplitude and $v^2 = \omega^2(a^2 - x^2)$

$$u^2 = 2ga \text{ when } y = -\frac{2}{3}a \Rightarrow 2ga = \frac{3g}{2a} \left(A^2 - \frac{4a^2}{9} \right)$$

cM1

$$A = \frac{4a}{3}$$

cA1

$$\text{Maximum speed } A\omega = \frac{4a}{3} \times \sqrt{\left(\frac{3g}{2a}\right)} = \frac{2}{3}\sqrt{(6ag)}$$

cM1 cA1

$$\text{Maximum acceleration } A\omega^2 = \frac{4a}{3} \times \frac{3g}{2a} = 2g$$

cA1

[Back to Edexcel M3](#)



3.

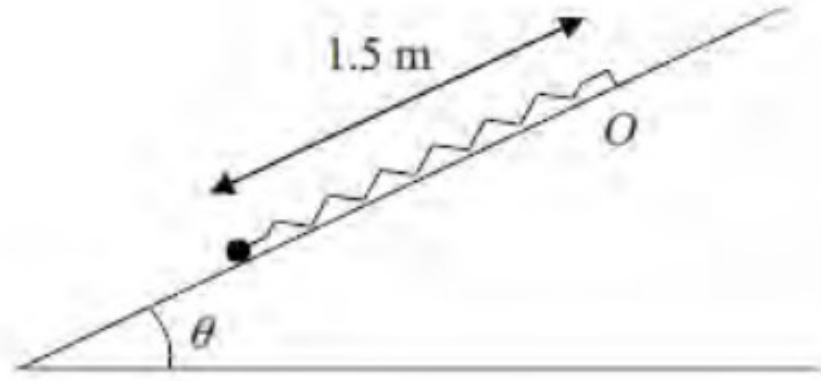


Figure 2

A particle of mass 0.5 kg is attached to one end of a light elastic spring of natural length 0.9 m and modulus of elasticity λ newtons. The other end of the spring is attached to a fixed point O on a rough plane which is inclined at an angle θ to the horizontal, where $\sin \theta = \frac{3}{5}$. The coefficient of friction between the particle and the plane is 0.15. The particle is held on the plane at a point which is 1.5 m down the line of greatest slope from O , as shown in Figure 2. The particle is released from rest and first comes to rest again after moving 0.7 m up the plane.

Find the value of λ .

(9)

**JUNE
2010 M3**

[Back to Edexcel M3](#)



Question Number	Scheme	Marks
Q3	<div data-bbox="420 185 1172 642" data-label="Diagram"> </div> <div data-bbox="369 664 993 763" data-label="Equation-Block"> $\text{EPE lost} = \frac{\lambda \times 0.6^2}{2 \times 0.9} - \frac{\lambda \times 0.1^2}{2 \times 0.9} \left(= \frac{7}{36} \lambda \right)$ </div> <div data-bbox="356 778 700 835" data-label="Equation-Block"> $R(\uparrow) \quad R = mg \cos \theta$ </div> <div data-bbox="522 849 802 935" data-label="Equation-Block"> $= 0.5g \times \frac{4}{5} = 0.4g$ </div> <div data-bbox="484 942 828 985" data-label="Equation-Block"> $F = \mu R = 0.15 \times 0.4g$ </div> <div data-bbox="331 992 1197 1035" data-label="Text"> <p>P.E. gained = E.P.E. lost - work done against friction</p> </div> <div data-bbox="293 1049 1133 1142" data-label="Equation-Block"> $0.5g \times 0.7 \sin \theta = \frac{\lambda \times 0.6^2}{2 \times 0.9} - \frac{\lambda \times 0.1^2}{2 \times 0.9} - 0.15 \times 0.4g \times 0.7$ </div> <div data-bbox="382 1149 1159 1235" data-label="Equation-Block"> $0.1944\lambda = 0.5 \times 9.8 \times 0.7 \times \frac{3}{5} + 0.15 \times 0.4 \times 9.8 \times 0.7$ </div> <div data-bbox="484 1242 700 1285" data-label="Equation-Block"> $\lambda = 12.70 \dots$ </div> <div data-bbox="484 1292 789 1335" data-label="Equation-Block"> $\lambda = 13 \text{ N} \quad \text{or } 12.7$ </div>	<div data-bbox="1490 692 1592 735" data-label="Text">M1 A1</div> <div data-bbox="1490 785 1541 828" data-label="Text">M1</div> <div data-bbox="1490 942 1592 985" data-label="Text">M1 A1</div> <div data-bbox="1490 1071 1643 1113" data-label="Text">M1 A1 A1</div> <div data-bbox="1490 1292 1541 1335" data-label="Text">A1</div> <div data-bbox="1668 1335 1732 1378" data-label="Text">[9]</div>



6.

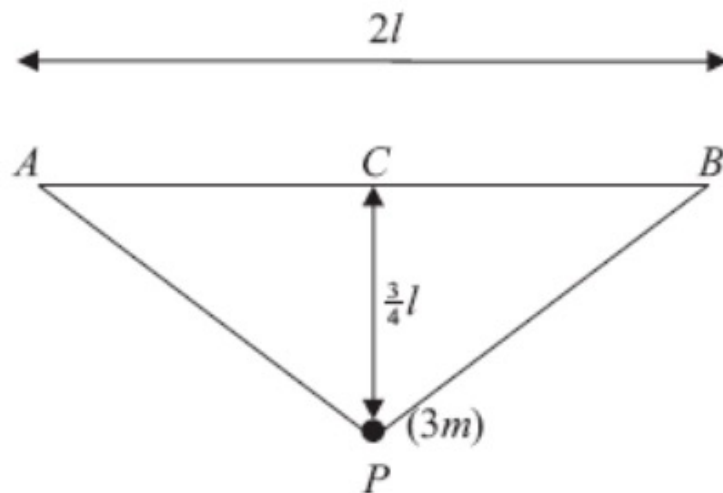


Figure 4

A small ball of mass $3m$ is attached to the ends of two light elastic strings AP and BP , each of natural length l and modulus of elasticity kmg . The ends A and B of the strings are attached to fixed points on the same horizontal level, with $AB = 2l$. The mid-point of AB is C . The ball hangs in equilibrium at a distance $\frac{3}{4}l$ vertically below C as shown in Figure 4.

(a) Show that $k = 10$.

(7)

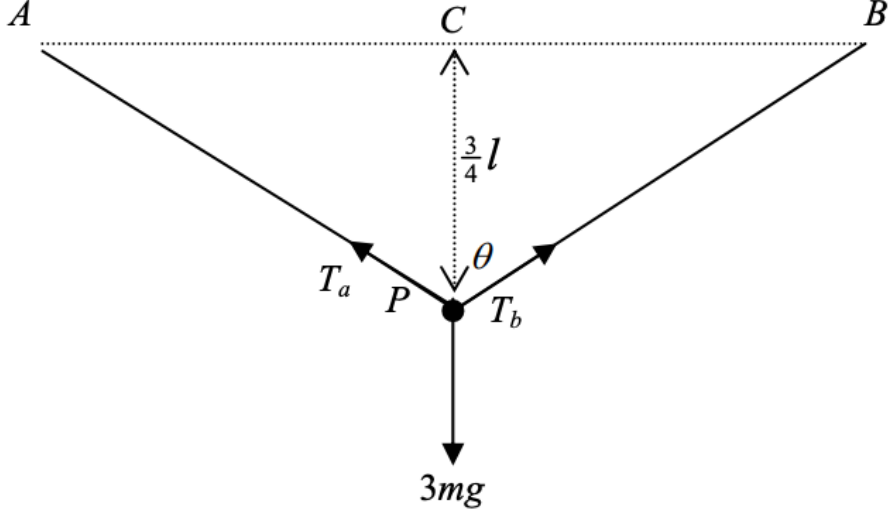
The ball is now pulled vertically downwards until it is at a distance $\frac{12}{5}l$ below C . The ball is released from rest.

(b) Find the speed of the ball as it reaches C .

(6)

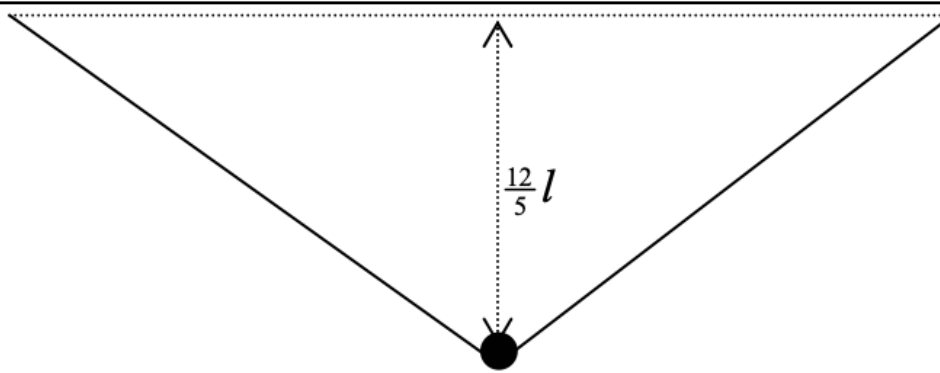
JAN 2011
M3



Question Number	Scheme	Marks
<p>6. (a)</p>	 <p>length $AP = \text{length } BP = \frac{5}{4}l$</p> $T_a = T_b = \frac{kmg \left(\frac{1}{4}l\right)}{l} = \frac{1}{4}kmg \quad (\text{or } T = \dots)$ <p>R(\uparrow) $T_a \cos \theta + T_b \cos \theta = 3mg \quad (\text{or } 2T \cos \theta = 3mg)$</p> $\frac{1}{4}kmg \times \frac{3}{5} + \frac{1}{4}kmg \times \frac{3}{5} = 3mg \quad \left(\text{or } \frac{1}{2}kmg \times \frac{3}{5} = 3mg \right)$ $\frac{3}{10}kmg = 3mg$ <p>$k = 10$ *</p>	<p>B1</p> <p>M1A1</p> <p>M1A1</p> <p>A1</p> <p>A1</p> <p>(7)</p>



(b)



$$\text{initial extn} = \frac{13}{5}l - l = \frac{8}{5}l$$

$$\text{E.P.E. lost} = 2 \times \frac{\lambda x^2}{2l} = 2 \times \frac{10mg}{2l} \left(\frac{8l}{5}\right)^2 = \frac{128mgl}{5}$$

$$\text{P.E. gained} = 3mg \times \frac{12l}{5} = \frac{36mgl}{5}$$

$$\frac{1}{2} \times 3mv^2 + \frac{36mgl}{5} = \frac{128mgl}{5}$$

$$v^2 = \frac{256gl}{15} - \frac{72gl}{15}$$

$$v = \sqrt{\left(\frac{184}{15}gl\right)}$$

B1

M1A1

M1A1

A1

(6)
[13]



5. A particle P of mass m is attached to one end of a light elastic string of natural length l and modulus of elasticity $3mg$. The other end of the string is attached to a fixed point O on a rough horizontal table. The particle lies at rest at the point A on the table, where $OA = \frac{7}{6}l$.

The coefficient of friction between P and the table is μ .

- (a) Show that $\mu \geq \frac{1}{2}$. (4)

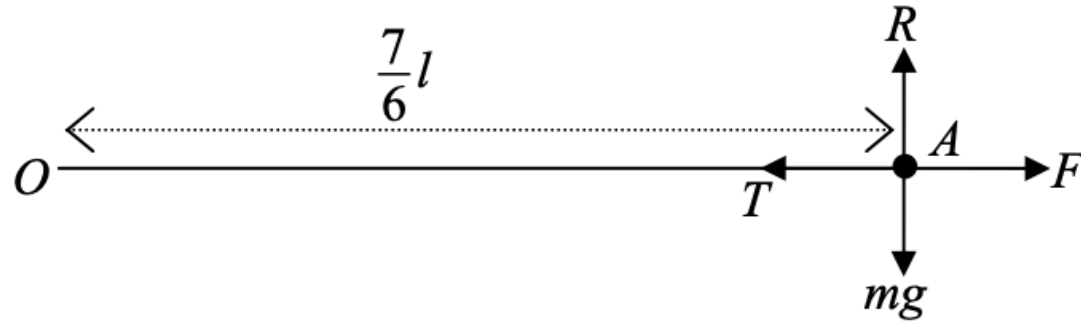
The particle is now moved along the table to the point B , where $OB = \frac{3}{2}l$, and released from rest. Given that $\mu = \frac{1}{2}$, find

- (b) the speed of P at the instant when the string becomes slack, (5)
- (c) the total distance moved by P before it comes to rest again. (3)
-



5.

(a)



$$T = \frac{3mg}{l} \left(\frac{1}{6}l \right) = \frac{1}{2}mg$$

$$R(\uparrow) \quad R = mg \quad R(\rightarrow) \quad F = T = \frac{1}{2}mg$$

$$F \leq \mu R$$

$$\frac{1}{2}mg \leq \mu mg$$

$$\mu \geq \frac{1}{2} \quad *$$

B1

M1

M1

A1

(4)



<p>(b)</p>	$\text{E.P.E. lost} = \frac{1}{2} \times \frac{3mg}{l} \left(\frac{l}{2}\right)^2 = \frac{3mgl}{8}$ $\text{Work done by friction} = \frac{1}{2} mg \left(\frac{l}{2}\right)$ $\frac{3mgl}{8} = \frac{1}{2} mv^2 + \frac{1}{2} mg \left(\frac{l}{2}\right)$ $v^2 = \frac{gl}{4}$ $v = \frac{1}{2} \sqrt{gl}$	<p>B1</p> <p>B1</p> <p>M1 A1ft</p> <p>A1</p> <p>(5)</p>
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Question Number	Scheme	Marks
(c)	$\frac{3mgl}{8} = \frac{1}{2}mgx$ $x = \frac{3l}{4}$	<p>M1 A1 ft</p> <p>A1</p> <p>(3)</p> <p>12</p>



JAN 2012

M3 Q1

1. A particle of mass 0.8 kg is attached to one end of a light elastic string of natural length 0.6 m . The other end of the string is attached to a fixed point A . The particle is released from rest at A and comes to instantaneous rest 1.1 m below A .

Find the modulus of elasticity of the string.

(4)



January 2012
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks
1.	$\text{EPE} = \frac{\lambda \times 0.5^2}{1.2}$ $\text{GPE lost} = \text{EPE gained}$ $0.8 \times 9.8 \times 1.1 = \frac{\lambda \times 0.5^2}{1.2}$ $\lambda = 41.4 \text{ N or } 41 \text{ N}$	B1 M1 (used) A1ft A1 4



JUNE 2012 M3

[Back to Edexcel M3](#)

7. A particle B of mass 0.5 kg is attached to one end of a light elastic string of natural length 0.75 m and modulus of elasticity 24.5 N. The other end of the string is attached to a fixed point A . The particle is hanging in equilibrium at the point E , vertically below A .

(a) Show that $AE = 0.9$ m.

(3)

The particle is held at A and released from rest. The particle first comes to instantaneous rest at the point C .

(b) Find the distance AC .

(5)

(c) Show that while the string is taut, B is moving with simple harmonic motion with centre E .

(4)

(d) Calculate the maximum speed of B .

(2)



Question Number	Scheme	Marks
7(a)	Use of $T = \frac{\lambda x}{a} = mg$ $T = \frac{24.5x}{0.75} = 0.5g$ $x = \frac{0.75 \times 0.5g}{24.5} = 0.15, \quad AE = 0.75 + 0.15 = 0.9 \text{ (m)} \quad (**)$	M1 A1 A1 (3)
(b)	Using $\text{gain in EPE} = \text{loss in GPE}$ $\frac{\lambda x^2}{2a} = \frac{24.5x^2}{1.5} = \dots$ $\dots = 0.5g(0.75 + x)$ Form quadratic in x and attempt to solve for x : $24.5x^2 = 5.5125 + 7.35x, \quad 24.5x^2 - 7.35x - 5.5125 = 0,$ $x = \frac{7.35 \pm \sqrt{7.35^2 + 4 \times 24.5 \times 5.5125}}{49}$ (or $40x^2 - 12x - 9 = 0, \quad x = \frac{12 \pm \sqrt{144 + 3600}}{80}$) $x = 0.647\dots \text{ (m)} \quad AC \approx 1.4 \text{ (m)}$	M1 A1 A1 DM1 A1 (5)



(c)	<p>Using $F = ma$ and displacement x from E:</p> $0.5g - \frac{24.5(x + 0.15)}{0.75} = 0.5a$ $a = -\frac{196}{3}x, \text{ so SHM}$	<p>M1 A2,1,0</p>
(d)	<p>Max speed = their a x their ω</p> $= (0.647 - 0.15) \times \sqrt{\frac{196}{3}}$ $\approx 4.0 \text{ ms}^{-1} \quad (4.02)$	<p>M1 (4)</p> <p>A1 (2)</p> <p>14</p>



JAN 2013 M3

7. A particle P of mass 1.5 kg is attached to the mid-point of a light elastic string of natural length 0.30 m and modulus of elasticity λ newtons. The ends of the string are attached to two fixed points A and B , where AB is horizontal and $AB = 0.48$ m. Initially P is held at rest at the mid-point, M , of the line AB and the tension in the string is 240 N.

(a) Show that $\lambda = 400$.

(3)

The particle is now held at rest at the point C , where C is 0.07 m vertically below M . The particle is released from rest at C .

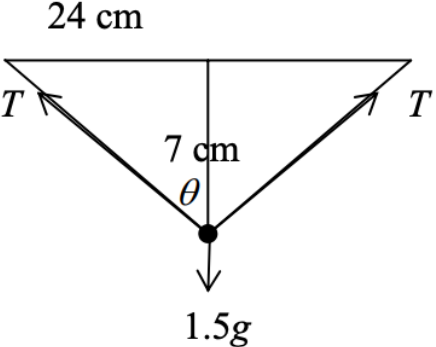
(b) Find the magnitude of the initial acceleration of P .

(6)

(c) Find the speed of P as it passes through M .

(6)



Question Number	Scheme	Marks
7 (a)	$T = \frac{\lambda x}{l} \Rightarrow 240 = \frac{\lambda \times 18}{30}$ $\lambda = 400$	M1A1 A1
7 (b)	<div style="text-align: center;">  </div> <p>Extension = 10 cm or 20 cm (used in (b) or (c))</p> $T = \frac{400 \times 10}{15} = \left(\frac{800}{3} \right)$ $R(\uparrow) \quad 2T \cos \theta - 1.5g = (\pm)1.5a$ $\frac{1600}{3} \times \frac{7}{25} - 1.5 \times 9.8 = (\pm)1.5a$ $a = 89.75\dots \quad a = 90 \text{ m s}^{-2} \text{ or } 89.8 \text{ (positive)}$	B1 M1A1ft M1A1 A1



(c)	$\text{E.P.E.} = \frac{1}{2} \times 400 \times \frac{0.2^2}{0.3}$ $1.5g \times 0.07 + \frac{1}{2} \times 1.5v^2 = 200 \times \frac{0.2^2}{0.3} - \frac{200 \times 0.18^2}{0.3}$ $v^2 = \frac{1}{0.75} \left(200 \times \frac{0.2^2}{0.3} - \frac{200 \times 0.18^2}{0.3} - 1.5g \times 0.07 \right)$ $v = 2.32\dots = 2.3 \text{ m s}^{-1}$	<p>B1ft (any correct EPE)</p> <p>M1A1A1</p> <p>M1dep</p> <p>A1</p>
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JUN 2013 ® M3

3. A particle P of mass 0.5 kg is attached to one end of a light elastic spring, of natural length 2 m and modulus of elasticity 20 N. The other end of the spring is attached to a fixed point A . The particle P is held at rest at the point B , which is 1 m vertically below A , and then released.

(a) Find the acceleration of P immediately after it is released from rest.

(4)

The particle comes to instantaneous rest for the first time at the point C .

(b) Find the distance BC .

(6)



Question Number	Scheme	Marks
3. (a)	Weight + thrust = mass x accn. $0.5 \times g + \frac{20 \times 1}{2} = 0.5a$ $a = g + 20 = 29.8 \approx 30 \text{ (m s}^{-2}\text{)}$	M1 B1(thrust) A1ft A1 (4)
(b)	Change in GPE = $mg(x+1)$ $\text{EPE at B} = \frac{20 \times 1^2}{2 \times 2} \quad \text{or} \quad \text{EPE at C} = \frac{20 \times x^2}{2 \times 2}$ $\text{Conservation of energy: } \frac{20 \times 1^2}{2 \times 2} + mgh = \frac{20 \times x^2}{2 \times 2} \quad h = x + 1$ $5 + 0.5g(x + 1) = 5x^2$ $5x^2 - 0.5gx - (5 + .5g) = 0$ $x = \frac{0.5g + \sqrt{(0.5g)^2 + 20(5 + 0.5g)}}{10} = 1.98$ Distance $BC = 1 + 1.98 = 2.98 \text{ (m)}$	B1 B1 M1A1 M1dep A1 (6) [10]



JUNE 2013 M3

4. A particle P of mass 2 kg is attached to one end of a light elastic string of natural length 1.2 m. The other end of the string is attached to a fixed point O on a rough horizontal plane. The coefficient of friction between P and the plane is $\frac{2}{5}$. The particle is held at rest at a point B on the plane, where $OB = 1.5$ m. When P is at B , the tension in the string is 20 N. The particle is released from rest.

(a) Find the speed of P when $OP = 1.2$ m.

(7)

The particle comes to rest at the point C .

(b) Find the distance BC .

(2)



Question Number	Scheme	Marks
<p style="text-align: center;">4</p> <p>(a)</p>	$T = \frac{\lambda x}{l}$ $20 = \frac{\lambda \times 0.3}{1.2}$ $\lambda = 80 \text{ N}$ $\text{Initial EPE} = \frac{\lambda x^2}{2l} = \frac{80 \times 0.3^2}{2.4} (= 3 \text{ J})$ $\frac{80 \times 0.3^2}{2.4} - 0.4 \times 2g \times 0.3 = \frac{1}{2} \times 2v^2$ $v^2 = 0.648$ $v = 0.80 \text{ or } 0.805 \text{ m s}^{-1}$	<p style="text-align: center;">M1A1</p> <p style="text-align: center;">A1</p> <p style="text-align: center;">B1</p> <p style="text-align: center;">M1A1ft</p> <p style="text-align: center;">A1 (7)</p>



Notes for Question 4

(a)

M1 for attempting Hooke's Law, formula must be correct, either explicitly or by correct substitution.

A1 for $20 = \frac{\lambda \times 0.3}{1.2}$

A1 for obtaining $\lambda = 80$

B1 for the initial EPE $\frac{\lambda \times 0.3^2}{2.4}$ (= 3 J) their value for λ allowed. May only be seen in the equation.

M1 for a work-energy equation with one EPE term, one KE term and work done against friction (Award if second EPE/KE terms included **provided** these become 0). The EPE must be dimensionally correct, but need not be fully correct (eg denominator 1.2 instead of 2.4)

A1ft for a completely correct equation follow through their EPE

A1 cao for $v = 0.80$ or 0.805 **must be 2 or 3 sf**

NB: This is damped harmonic motion (due to friction) so all SHM attempts lose the last 4 marks.



(b)

Comes to rest $0.4 \times 2g \times y = 3$

$$y = \frac{3}{0.4 \times 2 \times 9.8} = 0.38 \text{ or } 0.383 \text{ m}$$

Alternatives:

Energy from string going slack to rest:

$$\frac{1}{2} \times 2 \times 0.648 = 0.4 \times 2g \times x$$

$$x = 0.8265\dots$$

$$y = 0.3 + 0.08265\dots = 0.38 \text{ or } 0.383$$

NL2 to obtain the accel when string is slack $\left(-\frac{2g}{5}\right)$ and $v^2 = u^2 + 2as$

$$0 = 0.648 + 2 \times \left(-\frac{2g}{5}\right)s$$

$$BC = \frac{0.648 \times 5}{4g} + 0.3 = 0.38 \text{ or } 0.383$$

M1

A1 (2)

[9]

M1 Complete
method

A1

M1A1

[Back to Edexcel M3](#)



(b)

M1 for any **complete** method leading to a value for either BC . If the distance travelled after the string becomes slack is found the work must be completed by adding 0.3 Their EPE found in (a) used in energy methods.

MS method is energy from B to C ie work done against friction = loss of EPE.

OR Energy from point where the string becomes slack to C ie work done against friction = loss of KE and completed for the required distance

OR NL2 to obtain the acceleration $\left(-\frac{2g}{5}\right)$ while the string is slack **and** $v^2 = u^2 + 2as$ to find the distance and completed for the required distance

A1cso for $BC = 0.38$ or 0.383 (m) **must be 2 or 3 sf**



JAN 2014 M3

2. A particle P of mass m is attached to one end of a light elastic spring, of natural length l and modulus of elasticity $2mg$. The other end of the spring is attached to a fixed point A on a rough horizontal plane. The particle is held at rest on the plane at a point B , where $AB = \frac{1}{2}l$, and released from rest. The coefficient of friction between P and the plane is $\frac{1}{4}$.

Find the distance of P from B when P first comes to rest.

(9)



2	$\frac{2mg}{2l} \left(\left(\frac{1}{2}l \right)^2 - x^2 \right) = \frac{1}{4} mg \left(\frac{1}{2}l + x \right)$ $8x^2 + 2lx - l^2 = 0$ $(4x - l)(2x + l) = 0$ $x = \frac{1}{4}l \text{ or } -\frac{1}{2}l$ $\text{distance} = \frac{1}{2}l + \frac{1}{4}l = \frac{3}{4}l$	M1A1;M1 A 1 M1 A1 M1dep A1 A1	9
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Notes

M1 for the difference of 2 elastic energy terms, not nec in a complete energy equation.

A1 for a correct difference

M1 for a work energy equation, loss of EPE = work done against friction(not dep on previous mark)

A1 for a fully correct equation

M1dep for re-arranging to a three term quadratic, dependent on the second M mark, or use the difference of 2 squares to get a linear equation

A1 for a correct 3 term quadratic, terms in any order

M1dep for solving the resulting quadratic, usual rules. Dependent on all second and third M marks

A1 for $x = \frac{1}{4}l$ $x = -\frac{1}{2}l$ need not be shown

A1cao and cso distance = $\frac{3}{4}l$



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3. One end A of a light elastic string AB , of modulus of elasticity mg and natural length a , is fixed to a point on a rough plane inclined at an angle θ to the horizontal. The other end B of the string is attached to a particle of mass m which is held at rest on the plane. The string AB lies along a line of greatest slope of the plane, with B lower than A and $AB = a$. The coefficient of friction between the particle and the plane is μ , where $\mu < \tan \theta$. The particle is released from rest.

(a) Show that when the particle comes to rest it has moved a distance $2a (\sin \theta - \mu \cos \theta)$ down the plane. (6)

(b) Given that there is no further motion, show that $\mu \geq \frac{1}{3} \tan \theta$.

(5)



Question Number	Scheme	Marks
3 (a)	$R = mg \cos \theta$ WD against friction = $\mu x mg \cos \theta$ $\mu x mg \cos \theta = mgx \sin \theta - \frac{mgx^2}{2a}$ $x = 2a(\sin \theta - \mu \cos \theta) \quad **$	B1 B1 M1 A2 A1 (6)
(b)	$T = \frac{mg2a(\sin \theta - \mu \cos \theta)}{a} = 2mg(\sin \theta - \mu \cos \theta)$ No motion if $T \leq mg \sin \theta + \mu mg \cos \theta$ $2mg(\sin \theta - \mu \cos \theta) \leq mg \sin \theta + \mu mg \cos \theta$ $\frac{1}{3} \tan \theta \leq \mu \quad **$	B1 M1 A1 DM1 A1 (5) 11
Notes		
(a)	B1 correct equation perpendicular to the plane B1 correct expression for work done against friction M1 work-energy equation A2 fully correct; A1 one error; A1 correct expression for x no errors in the working	
(b)	B1 use Hooke's law to obtain a correct expression for T M1 using NL2 parallel to the plane to set up an inequality for situation where no motion A1 correct inequality DM1 solving to get an inequality for μ A1 correct inequality and no errors in the working If only error is use of $<$ instead of \leq , deduct final A mark only	



4.

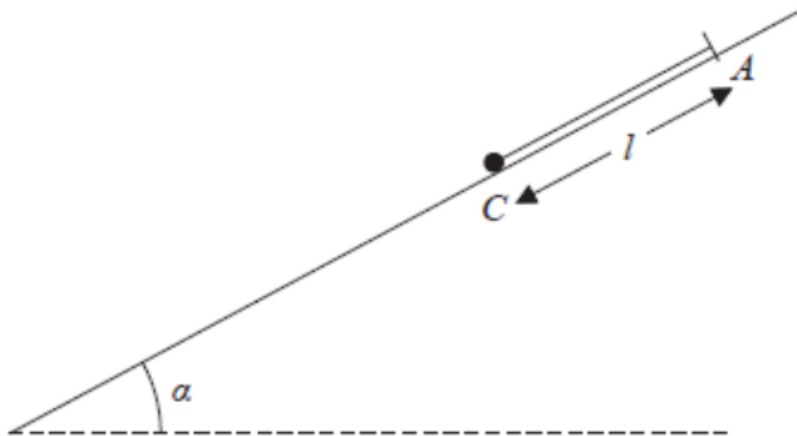


Figure 3

One end of a light elastic string, of natural length l and modulus of elasticity $3mg$, is fixed to a point A on a fixed plane inclined at an angle α to the horizontal, where $\sin \alpha = \frac{3}{5}$.

A small ball of mass $2m$ is attached to the free end of the string. The ball is held at a point C on the plane, where C is below A and $AC = l$ as shown in Figure 3. The string is parallel to a line of greatest slope of the plane. The ball is released from rest. In an initial model the plane is assumed to be smooth.

(a) Find the distance that the ball moves before first coming to instantaneous rest.

(5)

In a refined model the plane is assumed to be rough. The coefficient of friction between the ball and the plane is μ . The ball first comes to instantaneous rest after moving a distance $\frac{2}{5}l$.

(b) Find the value of μ .

(6)

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Question Number	Scheme	Marks
4	<p data-bbox="690 208 733 239">(a)</p> $\frac{3mgx^2}{2l} = 2mgx \sin \alpha$ $3x^2 = 4xl \times \frac{3}{5}$ $5x^2 = 4xl$ $x = \frac{4}{5}l$ <p data-bbox="690 762 733 793">(b)</p> $R = 2mg \cos \alpha \left(= \frac{8}{5}mg \right)$ $\frac{3mg}{2l} \times \frac{4}{25}l^2 = 2mg \times \frac{2}{5}l \times \frac{3}{5}, \quad \mu \frac{8}{5}mg \times \frac{2}{5}l$ $6 = 12 - 16\mu$ $16\mu = 6 \quad \mu = \frac{3}{8}$	<p data-bbox="2071 165 2257 282">M1A1 B1(A1 on e- pen)</p> <p data-bbox="2071 608 2290 639">DM1A1 (5)</p> <p data-bbox="2071 762 2112 793">B1</p> <p data-bbox="2071 888 2290 1005">M1A1ft, B1ft (A1 on e- pen)</p> <p data-bbox="2071 1179 2290 1210">DM1A1 (6)</p> <p data-bbox="2071 1250 2137 1282">[11]</p>



JAN 2016 IAL M3

Fixed points A and B are on a horizontal ceiling, where $AB = 4a$. A light elastic string has natural length $3a$ and modulus of elasticity λ . One end of the string is attached to A and the other end is attached to B . A particle P of mass m is attached to the midpoint of the string. The particle hangs freely in equilibrium at the point C , where C is at a distance $\frac{3}{2}a$ vertically below the ceiling.

(a) Show that $\lambda = \frac{5mg}{4}$

(5)

The point D is the midpoint of AB . The particle is now raised vertically upwards to D , and released from rest.

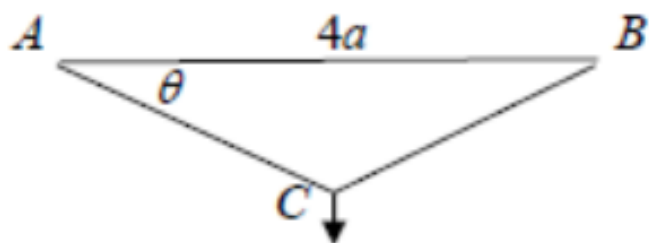
(b) Find the speed of P as it passes through C .

(5)

(Total for question = 10 marks)



(a)



$$R(\uparrow) \quad 2T \sin \theta = mg$$

$$\sin \theta = \frac{3}{5} \quad \text{or} \quad \cos \theta = \frac{4}{5}$$

$$T = \frac{5}{6} mg$$

$$T = \frac{\lambda \times 2a}{3a}$$

$$\lambda = \frac{3}{2} \times \frac{5}{6} mg = \frac{5}{4} mg *$$

M1A1

B1

M1

A1 cso (5)



<p>(b)</p>	<p>EPE at $D = \frac{\lambda a^2}{2 \times 3a}$ at $C = \frac{\lambda \times 4a^2}{2 \times 3a}$ (or equivalents with half strings)</p> $\frac{1}{2}mv^2 + \frac{4\lambda a^2}{6a} = \frac{\lambda a^2}{6a} + mg \times \frac{3}{2}a$ $v^2 = \frac{7}{4}ag$ $v = \frac{\sqrt{7ag}}{2}$	<p>B1 (either)</p> <p>M1A1</p> <p>dM1</p> <p>A1 (5) [10]</p>
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<p>(a)</p> <p>MI</p> <p>A1</p> <p>B1</p> <p>MI</p> <p>Alcso</p>	<p>Resolve vertically Must have $2T$.</p> <p>Correct equation</p> <p>A correct trig value for an angle - seen implicitly or used</p> <p>Hooke's law inc attempting the extension in terms of a</p> <p>Obtain the given value of λ from correct working</p>
<p>(b)</p> <p>B1</p> <p>MI</p> <p>A1</p> <p>MI</p> <p>A1</p>	<p>Obtain the correct EPE at either start or finish of the motion. May be those shown (full strings) or half of these (half strings) (May have already sub their λ)</p> <p>An energy equation with the correct number of terms. EPE terms to be of the form $k \frac{\lambda x^2}{l}$</p> <p>A fully correct equation</p> <p>Solve to $v^2 = \dots$</p> <p>A correct expression for v Any equivalent form</p>



This was one of the more successfully answered questions and there were many correct, or almost correct, solutions.

It was again surprising that many candidates failed to draw a diagram, meaning that their equations then concerned undefined forces and unspecified angles. Although the majority knew how to tackle part (a), the given answer (although needed here) generated a number of probably manipulated but error-free solutions. Many of these failed to show any calculations of lengths or angles or the general statement $2T \cos \theta = mg$ but simply wrote a numerical equation straight away $\left(2 \times \frac{3}{5} \times \frac{2\lambda}{3} = mg\right)$ – an impressive feat without a diagram. As before, they should be encouraged to show every step of their reasoning and calculations. There were a number of errors in Hooke's Law due to confusion between original lengths, extended lengths, whole and half strings but somehow almost all ended up claiming to have 'proved' the result.

The correct theory for (b) was well known and there were again many successful solutions. The most common error was to overlook the initial EPE, losing nearly all the marks, but a small minority left out both, settling for $PE = KE$. There were a number of errors in the EPE formula $\left(\frac{\lambda x}{l} \text{ or } \frac{\lambda x}{2l}\right)$ and errors in simplifying correct EPEs. Again, some could have rescued lost marks by quoting the formula first. A few lost the last mark of an otherwise correct solution by applying the $\sqrt{\quad}$ incompletely to their answer. A very few tried to solve (b) using SHM but without a clear idea of what x or \ddot{x} were measuring.



JUNE 2016 IAL M3

A particle P of mass m is attached to one end of a light elastic string, of natural length l and modulus of elasticity $4mg$. The other end of the string is attached to a fixed point O on a rough horizontal plane. The coefficient of friction between P and the plane is $\frac{2}{5}$.

The particle is held at a point A on the plane, where $OA = \frac{5}{4}l$, and released from rest. The particle comes to rest at the point B .

(a) Show that $OB < l$

(4)

(b) Find the distance OB .

(3)

(Total for question = 7 marks)



Question Number	Scheme	Marks
(a)	$\text{EPE at } A = \frac{4mg\left(\frac{1}{4}l\right)^2}{2l} \quad \left(= \frac{mgl}{8} \right)$ $\text{Work done against friction from } A \text{ to natural length} = \frac{2}{5}mg\left(\frac{1}{4}l\right)$ $\frac{mgl}{8} > \frac{mgl}{10}$ <p>$\therefore P$ is still moving when string goes slack, ie $OB < l$</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1 cso (4)</p>
(b)	$\frac{2}{5}mgx = \frac{mgl}{8}$ $x = \frac{5}{16}l$ $OB = \frac{5}{4}l - \frac{5}{16}l = \frac{15}{16}l$	<p>M1</p> <p>A1</p> <p>A1ft (3)</p> <p>[7]</p>



- (a)B1 Correct EPE at start
 B1 Correct work done against friction from release at A to string becoming slack.
 M1 Either showing the inequality above or using an equation to show P has KE at natural length. Comparing the 2 energy terms even if incorrect scores M1 EPE to be dimensionally correct
 Alcso Drawing the required conclusion, with evidence, from correct working.
- NB The two B marks can be awarded in (b) - Award B1 for work done against friction from release to coming to rest again.
- (b)M1 Work-energy from start to B
 A1 Correct distance moved
 Alft Subtract their distance moved from $\frac{5}{4}l$
- ALT for (b): Work from natural length to B :**
 M1 Find KE at natural length (may have been done in (a)) and then find further distance moved by any valid method.
 A1 Correct distance moved from natural length $= \frac{1}{16}l$
 Alft Subtract their distance moved from l .



[Question](#)[Mark Scheme](#)[Examiner's Report](#)[Resources](#)

Most candidates did not realise that having the particle moving on a rough table significantly complicated the situation as friction might cause the particle to stop moving before the string became slack. Hence the significance of part (a) was not appreciated and OB was calculated first and then stated to be less than l . The best solutions either used the inequality approach shown in the mark scheme or showed that the particle still had kinetic energy at the instant when the string became slack. The most common error was to assume that there was still some EPE after the string had gone slack and an extra EPE term was included in the energy equation. A more worrying error was to consider just forces and not consider energy at all.



JAN 2017 IAL M3

A particle P of mass $4m$ is attached to one end of a light elastic string of natural length l and modulus of elasticity $3mg$. The other end of the string is attached to a fixed point O on a rough horizontal table. The particle lies at rest at the point A on the table, where $OA = \frac{4}{3}l$. The coefficient of friction between P and the table is μ .

(a) Show that $\mu \geq \frac{1}{4}$

(4)

The particle is now moved along the table to the point B , where $OB = 2l$, and released from rest.

Given that $\mu = \frac{2}{5}$

(b) show that P comes to rest before the string becomes slack.

(5)

(Total for question = 9 marks)

[Back to Edexcel IAL M3](#)



Question Number	Scheme	Marks
(a)	$F \leq \mu 4mg$ $F = T = \frac{3mg}{l} \times \frac{1}{3}, \leq 4\mu mg$ $\mu \geq \frac{1}{4} \quad *$	B1 M1,A1ft A1cso (4)

(a)B1	Correct inequality or equation provided clear this is maximum friction.	
M1	Resolve horizontally including use of HL (correct formula) and an attempt at the extension (ie not l) Mass can be m or $4m$	
A1ft	Correct inequality follow through their max friction	
A1	Obtain GIVEN inequality with no errors seen.	
	If = used throughout and inequality only at final stage, B1M1 only unless they convince you.	
NB	If it is not clear that max friction is being used, only M mark is available.	



(b)

(Assume string returns to its natural length)

$$\text{EPE} = \frac{3mgl^2}{2l}$$

$$\text{Work done against friction} = \frac{2}{5} \times 4mg \times l$$

$$\frac{1}{2} \times 4mv^2 = \frac{3mgl^2}{2l} - \frac{2}{5} \times 4mg \times l$$

$$v^2 \leq 0 \Rightarrow \text{string does not become slack.}$$

B1

B1

M1A1ft

A1 cso (5) [9]

(Alternative: Assume extension is x when particle comes to rest)

$$\text{EPE lost} = \frac{3mgl^2}{2l} - \frac{3mgx^2}{2l}$$

$$\text{Work done against friction} = \frac{2}{5} \times 4mg \times (l - x)$$

$$\frac{3mgl^2}{2l} - \frac{3mgx^2}{2l} = \frac{8}{5} mg(l - x)$$

$$x = \frac{1}{15}l, \quad (x = l) \text{ pos ext at } x = \frac{1}{15}l \quad \therefore \text{at rest before string becomes slack}$$

B1(either)

B1

M1A1ft

A1cso



(b)B1	Correct EPE when extension is l
B1	Correct WD against friction
M1	Attempt energy equation with KE, EPE and WD terms. EPE term to be of the form $k \frac{\lambda x^2}{l}$, mass m or $4m$.
Alft	Correct equation, follow through their EPE and WD terms. All signs to be correct.
Alcso	Solving for v^2 or stating $v^2 \leq 0$ (from a fully correct equation) and giving the conclusion. No errors anywhere in the working.
ALT	For last 3 marks: Show WD to nat length $>$ initial EPE (M1A1) Correct conclusion (A1)
ALT B1	Either of the 2 required EPE terms correct
B1	Correct WD against friction
M1	Attempt energy equation with a difference of EPE terms and a WD term. EPE terms to be of the form $k \frac{\lambda x^2}{l}$
Alft	Correct equation, follow through their EPE and WD terms. All signs to be correct.
Alcso	Solve for x and state conclusion. No errors seen



Only a very few scored all four marks for (a). The majority did not give an adequate reason for the inequality and earned only one mark for a question that they probably considered simple. Inequalities in "Show" questions should always be given careful consideration so that the real reason for the inequality is identified. An ideal solution here needed three stages: tension = $\lambda x/l$, tension = friction and friction, μR . Statements such as $F \dots T$, supported by vague statements such as "Because it does not move" do not justify the inequality and the often seen statement that $F = \mu R$ is not true unless it is made very clear that this is the maximum possible friction.

Although the underlying principles were well known, many attempts for (b) were not well executed. Most students managed to score two marks by finding correct expressions for the initial EPE and the work done against friction but these were most often then used in equations which contained no other energy terms. Valid equations involved assuming either a final EPE at the point where the particle stopped or some KE when the string reached its natural length. Alternatively, a very neat solution showed by inequality that the initial EPE was insufficient to provide the necessary work for the particle to reach the point where the string became slack.



JUNE 2017 IAL M3

A light elastic string has natural length 0.4 m and modulus of elasticity 49 N. A particle P of mass 0.3 kg is attached to one end of the string. The other end of the string is attached to a fixed point A on a ceiling. The particle is released from rest at A and falls vertically. The particle first comes to instantaneous rest at the point B .

(a) Find the distance AB .

(6)

The particle is now held at the point 0.6 m vertically below A and released from rest.

(b) Find the speed of P immediately before it hits the ceiling.

(5)

(Total for question = 11 marks)



Question Number	Scheme	Marks
(a)	$0.3g(x+0.4) = \frac{49x^2}{2 \times 0.4} \quad \text{OR} \quad 0.3gy = \frac{49(y-0.4)^2}{2 \times 0.4}$ $5x^2 - 0.24x - 0.096 = 0$ $x = \frac{0.24 \pm \sqrt{0.24^2 + 20 \times 0.096}}{10}$ $x = 0.1646\dots (\text{neg not needed}) \quad y = 0.5646\dots (0.24 \text{ need not be shown})$ $AB = 0.56 \text{ or } 0.565 \text{ m} \quad AB = y = 0.56 \text{ or } 0.565$	M1A1A1 dM1 dM1 A1 (6)
(b)	$\frac{49 \times 0.2^2}{0.8} = \frac{1}{2} \times 0.3v^2 + 0.3g \times 0.6$ $v^2 = \frac{2}{0.3} \left(\frac{49 \times 0.2^2}{0.8} - 0.3 \times 9.8 \times 0.6 \right)$ $v = 2.1 \text{ or } 2.14 \text{ m s}^{-1}$	M1A1A1 dM1A1 (5) [11]



- (a)
- M1** Use an energy equation with x as the extension at B or y as the distance fallen. There must be a PE term and an EPE term. EPE term to be of the form $k \frac{\lambda x^2}{l}$
- A1A1** Deduct one mark per incorrect term.
- dM1** Simplify to a 3 term quadratic, terms in any order. Depends on first M mark
- M1** Solve their quadratic by formula or completing the square. Allow calculator solution **only** if $x = 0.1646$ or the final answer is correct. Depends on both previous M marks.
- A1** Correct length of AB . Must be 2 or 3 sf.
- ALT:** Find v at natural length by SUVAT and then use energy. No marks until the energy equation seen, then mark as above. (A1A1 deduct one per error.)
- (b)**M1** Forming an energy equation from release to A . Must have 3 terms, an initial EPE, a PE and a KE term. EPE term to be of the form $k \frac{\lambda x^2}{l}$ and extension $\neq 0.165$
- A1** Any two terms correct.
- A1** Completely correct equation.
- dM1** Solve their equation to $v^2 = (4.5733\dots)$ or $v = \dots$ Depends on the M mark above (in (b))
- A1** Speed = 2.1 or 2.14 m s⁻¹
- NB** Use of $g = 9.81$ produces the same 3 sf answers. Exceptionally allow this.
SHM solutions must first **prove** SHM and find the centre (equilibrium position). Send to review



Part (a) tended to be all or nothing. If students realised that they needed to use energy, they generally formed a correct quadratic, and solved for the correct answer. The most popular approach was the main mark scheme method, with virtually all students remembering to add 0.4 at the end. Those who used just x in both the PE and EPE terms could not obtain a 3 term quadratic and lost the last three marks. Of those who did obtain a quadratic, some used the formula and showed their working and others just wrote down a calculator answer. When this answer was wrong, they lost the last two marks. The answer had to be to 2 or 3 significant figures because a numerical value of g had been used but some students had not read the rubric. Very few used SUVAT to find the speed when the string became taut, but they generally got to the correct answer. A large number of students unfortunately found the distance at equilibrium, rather than rest, scoring no marks.

Most students recovered in (b), with the majority knowing to use energy and many getting the correct answer. Here it was more common to consider two stages, using energy to find the speed as the string became slack and then using SUVAT to complete.

In both parts of the question there were examples of the EPE formula not being used correctly – missing the 2 in the denominator or using the total length of the string instead of the extension. Another error was to put the GPE on the wrong side of the energy equation.

However, many completely correct solutions were seen.



JUNE 2019 IAL M3

One end of a light elastic spring, of natural length l and modulus of elasticity $2mg$, is attached to a particle of mass m . The other end of the spring is attached to a fixed point A on a rough horizontal plane. The particle is held at a point B on the plane, where $AB = 1.25l$, and released from rest. The particle first comes to instantaneous rest at the point C on the plane, where $AC = 0.9l$.

Find the coefficient of friction between the particle and the plane.

(Total for question = 6 marks)



Question Number	Scheme	Marks
	$\text{EPE } \frac{2mg(0.25l)^2}{2l} \text{ or } \frac{2mg(0.1l)^2}{2l}$	B1 either
	$\text{WD against friction: } 0.35l\mu mg$	B1
	$\frac{2mg(0.25l)^2}{2l} - \frac{2mg(0.1l)^2}{2l} = 0.35l\mu mg$	M1A1
	$\mu = 0.15 \left(\text{or } \frac{3}{20} \right)$	M1A1 (6)
		[6]

B1	A correct expression for EPE, either at the start or end of the motion.
B1	Correct expression for Work Done against friction
M1	Attempt at an energy equation with a difference between 2 EPE terms and a WD. EPE must be of form $\frac{\lambda x^2}{kl}$, $k = 2$ or 1
A1	Correct equation
M1	Solving to find μ . Independent, but must have 3 terms, but condone EPE added.
A1	$\mu = 0.15 \left(\text{or } \frac{3}{20} \right)$ o.e. (g cancels)



JAN 2020 IAL M3

A light elastic string has modulus of elasticity $2mg$ and natural length l . One end of the string is fixed to a point A on a rough plane inclined to the horizontal at angle α , where $\sin \alpha = \frac{3}{5}$. A particle, P , of mass m is attached to the other end of the string.

Initially P is held at rest on the plane at the point B , where B is above A and $AB = \frac{1}{2}l$. The string lies along a line of greatest slope of the plane.

The particle P is released from rest and moves down the plane along the line of greatest slope. The coefficient of friction between P and the plane is μ , where $\mu < \tan \alpha$.

Given that P comes to instantaneous rest at the point C , where $AC = l + e$,

(a) show that

$$\mu = \frac{9l^2 + 6le - 10e^2}{4l(3l + 2e)} \quad (6)$$

Given that $e = l$

(b) find the magnitude of the instantaneous change in the acceleration of P at C .

(5)

(Total for question = 11 marks)



Question Number	Scheme	Marks
(a)	$mg \sin \alpha \times \left(\frac{3l}{2} + e \right) = \mu mg \cos \alpha \times \left(\frac{3l}{2} + e \right) + \frac{1}{2} \times \frac{2mg}{l} e^2$ $\frac{3}{5} \left(\frac{3l}{2} + e \right) = \frac{4\mu}{5} \left(\frac{3l}{2} + e \right) + \frac{e^2}{l}$ $\mu = \frac{9l^2 + 6le - 10e^2}{4l(3l + 2e)} \quad *$	<p>M1B1B1A1</p> <p>dM1A1cso (6)</p>

(a) M1	Attempt a work-energy equation with a GPE term, a single EPE term and the work done against friction. . (Allow $EPE = k \frac{\lambda x^2}{l}$)
B1	Correct EPE at C. (Ignore any extra EPE terms for this mark)
B1	Correct GPE
A1ft	Correct equation. Follow through their EPE and GPE terms providing they are of the correct form
dM1	At least one line of correct working to rearrange towards $\mu =$. They do not need to reach $\mu =$ for this mark.
A1cso*	Given result obtained with no errors seen and at least one line of correct rearrangement. Must be exactly as printed on paper.



<p>(b)</p>	$e = l \Rightarrow \mu = \frac{1}{4} \text{ or } 0.25$ $F = \frac{1}{5}mg$ <p>Change in acceleration is due to change of direction of F</p> $F_1 = 2mg - mg \sin \alpha + F_r \left(= \frac{8}{5}mg \right) \text{ and } F_2 = 2mg - mg \sin \alpha - F_r \left(= \frac{6}{5}mg \right)$ $\text{Mag of change in accel} = \frac{F_1 - F_2}{m} = \frac{2g}{5} = 3.92 \text{ or } 3.9 \text{ (m s}^{-2}\text{)}$	<p>B1</p> <p>B1ft</p> <p>M1</p> <p>M1A1 (5)</p> <p>[11]</p>
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<p>(b)</p> <p>B1</p> <p>B1ft</p> <p>M1</p> <p>M1</p> <p>A1</p>	<p>Correct numerical value for μ seen anywhere in (b). This might be implied by later working.</p> <p>Correct value for F, seen anywhere in (b). Follow through their μ but must be dimensionally correct. μ</p> <p>Attempt 2 equations of motion to find resultant force. (Use of $Change = 2F$) would imply this mark.</p> <p>Subtract and divide by m to obtain the mag of the change in the acceleration.</p> <p>Must be $\frac{2g}{5}$, or 3.9 or 3.92 (m s⁻²)</p>
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Whilst almost all candidates attempted to form an energy equation with the correct terms, it was very common for a mistake to appear, usually in the distances used in both the work done and GPE terms (often $0.5l + e$, rather than $1.5l + e$). The EPE terms was usually found correctly and trig mistakes were rare. The algebra involved in reaching the given result was fairly complex and the clarity of working was variable, with many candidates often making fairly large jumps towards the given solution. Inevitably, candidates that made a mistake in forming their original equation managed to arrive at the given result. It should be remembered that no credit will be given for this, so if your equation is not yielding the required result, you should go back to the first line and look for the error.

In part (b) most candidates were able to pick up the first mark and most did find a correct friction, often embedded in their equations. However, the majority did not realise the need to form two equations and those that did often omitted a term, losing the final 3 marks.



OCT 2020
IAL M3

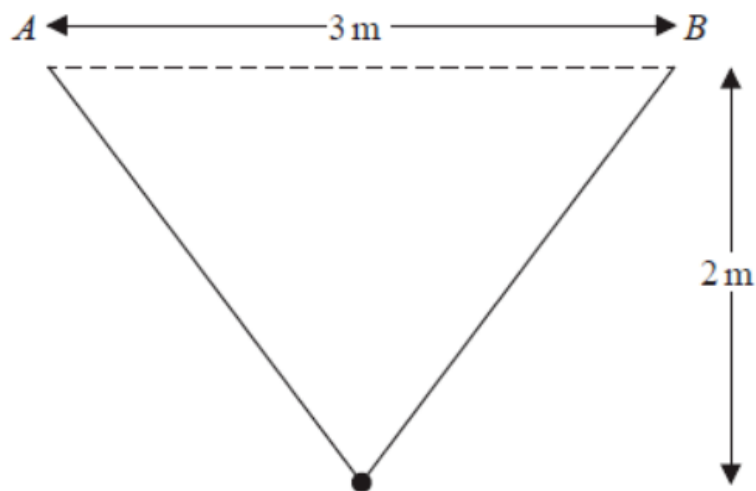


Figure 2

A smooth bead of weight 12 N is threaded onto a light elastic string of natural length 3 m . The points A and B are on a horizontal ceiling, with $AB = 3\text{ m}$. One end of the string is attached to A and the other end of the string is attached to B .

The bead hangs freely in equilibrium, 2 m below the ceiling, as shown in Figure 2.

(a) Find the tension in the string.

(4)

(b) Show that the modulus of elasticity of the string is 11.25 N .

(2)

The bead is now pulled down to a point vertically below its equilibrium position and released from rest.

(c) Find the elastic energy stored in the string at the instant when the bead is moving at its maximum speed.

(2)

(Total for question = 8 marks)



Question Number	Scheme	Marks
(a)	$\cos \theta = \frac{4}{5}$	B1
	$2T \cos \theta = 12$	M1A1
	$2T \left(\frac{4}{5} \right) = 12 \Rightarrow T = \frac{60}{8} = 7.5 \text{ (N)}$	A1
		(4)
(b)	$\text{Ext} = 2(2.5) - 3 = 2 \text{ m}$	
	$7.5 = \frac{\lambda \times 2}{3} \Rightarrow \lambda = 11.25 \text{ (N) } *$	M1A1*
		(2)
(c)	$\text{EPE} = \frac{11.25 \times 2^2}{2 \times 3} = 7.5 \text{ (J)}$	M1A1
		(2)
		[8]



(a)

B1 $\cos \theta = \frac{4}{5}$ seen or implied.

M1 Resolving vertically, with 2 equal tensions (implied) and weight.

A1 Correct equation.

A1 $T = \frac{15}{2} = 7.5 \text{ N}$ accept any equivalent fraction, since g not used.

(b)

M1 Use of Hooke's Law with their tension to form equation in λ . Must be using natural length of 3 (or 1.5 for half string), but condone incorrect extension for M mark.

A1* 11.25 or any equivalent fraction.

(c)

M1 Attempt at EPE in equilibrium position. Must have same extension as (b). Condone missing half in EPE formula. If using half strings, then they must include the EPE of both strings. Must be using natural length of 3 (or 1.5 for half string). Allow if an embedded term in an energy equation.

A1 7.5 J. Accept any equivalent. Must be a clear answer (not embedded).



Question

Mark Scheme

Examiner's Report

Resources

This was very straight forward and was answered well on the whole, but part c) did cause problems for some.

Part a). The vast majority answered this fully correctly. Correct trigonometry was used with a correct vertical resolution. Occasional errors involved the use of only one component of the tension used or candidates doubling the tension found to give an answer of 15N.

Part b) No problems at all. Some used the whole string while other used the half string in equal measure.

Part c). There were many fully correct answers, but a significant number left this blank. It was evident that many candidates were unclear on when the bead was moving at its maximum speed. Some thought this was when it became slack and therefore $EPE=0$. Others involved EPE in an energy equation with kinetic energy in some attempt to find the speed.



JAN 2015 IAL M3

A light elastic string has natural length 5 m and modulus of elasticity 20 N. The ends of the string are attached to two fixed points A and B , which are 6 m apart on a horizontal ceiling. A particle P is attached to the midpoint of the string and hangs in equilibrium at a point which is 4 m below AB .

(a) Calculate the weight of P .

(6)

The particle is now raised to the midpoint of AB and released from rest.

(b) Calculate the speed of P when it has fallen 4 m.

(5)

(Total for question = 11 marks)



Question Number	Scheme	Marks
(a)	<p>Length of string/half string = 10 m / 5 m (or extn = 5 m)</p> $T = \frac{\lambda x}{l} = \frac{20 \times 5}{5}, = 20$ $2T \cos \alpha = mg$ $2 \times 20 \times \frac{4}{5} = mg$ <p>Weight = 32 N (Accept $mg = 32$)</p>	<p>B1</p> <p>M1, A1</p> <p>M1</p> <p>A1ft</p> <p>A1 (6)</p>
(b)	<p>PE lost = "mg"$\times 4$</p> $\text{EPE gained} = \frac{20 \times 5^2}{2 \times 5} - \frac{20 \times 1^2}{2 \times 5}$ $\frac{1}{2}mv^2 = "mg" \times 4 - \left(\frac{20 \times 5^2}{2 \times 5} - \frac{20 \times 1^2}{2 \times 5} \right)$ $\frac{16}{g}v^2 = 32 \times 4 - \left(\frac{20 \times 5^2}{2 \times 5} - \frac{20 \times 1^2}{2 \times 5} \right)$ $v^2 = 5g$ <p>$v = 7, 7.0$ or 7.00</p>	<p>M1A1A1</p> <p>DM1</p> <p>A1 (5) [11]</p>



<p>(a)</p>	<p>B1 correct length of complete or half string or correct extension(need not be shown explicitly) M1 apply Hooke's law $x \neq 1$ A1 correct tension obtained M1 resolving vertically, both tensions resolved A1ft substitute their tension and $\cos \alpha = \frac{4}{5}$ A1 correct weight obtained (no ft)</p>	
<p>(b)</p>	<p>M1 energy equation with KE, PE and two EPE terms - all calculated with correct formulae A1A1 Deduct one A mark per error (if m is substituted early, ft their m) M1 Substitute their mass (not weight) A1 correct value for v 7, 7.0 or 7.00 only acceptable</p>	



Question

Mark Scheme

Examiner's Report

Resources

Part (a) was generally answered very well, with most correctly identifying the required lengths and most candidates finding the tension in the half string. When going on to resolve most made an acceptable attempt at resolving and it was generally only strange slips in copying across tensions that lead to trouble. Most actually identified their answer as weight, although some did only label it mg . The commonest problems encountered in this part of the question involved mixing up numbers for the string with those for the complete string. There was also some confusion about the natural length of the string; some candidates appeared to think this was 6 m whilst some were inconsistent and sometimes used 5 m and other times used 6 m.

Part (b) certainly caused more problems and for candidates with full marks in (a) it tended to be 0 or 5. Almost all realised that they had to use energy and gave a KE, GPE and EPE term, but a large number failed to include an initial EPE. Where all four terms were present, the correct answer was generally reached, unless they were carrying forward an incorrect weight from (a). Any candidate who made any sensible attempt (even if missing an EPE term) did go on to correctly use their mass from (a).



JUNE 2018 IAL M3

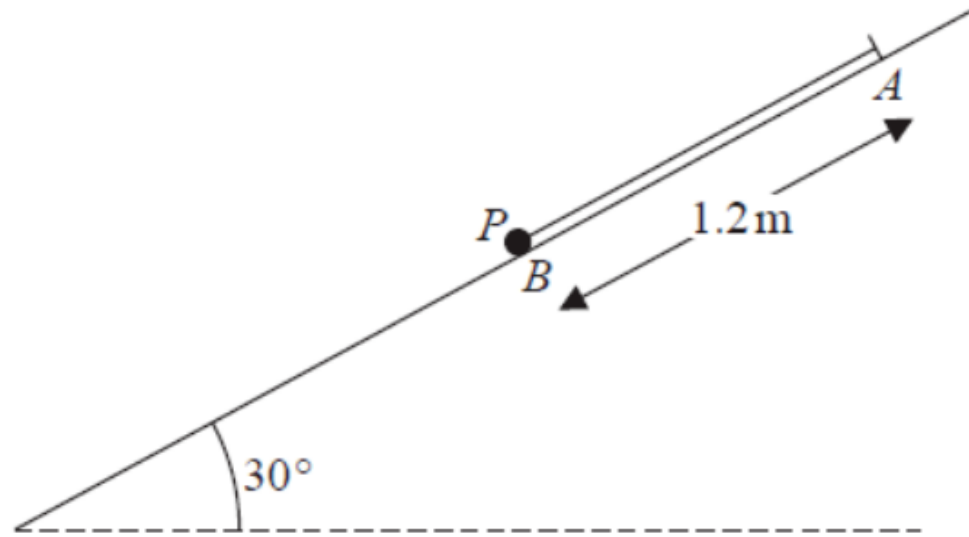


Figure 2

Figure 2 shows a light elastic string, of modulus of elasticity λ newtons and natural length 0.6 m . One end of the string is attached to a fixed point A on a rough plane which is inclined at 30° to the horizontal. The other end of the string is attached to a particle P of mass 0.5 kg . The string lies along a line of greatest slope of the plane. The particle is held at rest on the plane at the point B , where B is lower than A and $AB = 1.2\text{ m}$. The particle then receives an impulse of magnitude 1.5 N s in the direction parallel to the string, causing P to move up the plane towards A . The coefficient of friction between P and the plane is 0.7 . Given that P comes to rest at the instant when the string becomes slack, and the value of λ .

(Total for question = 8 marks)



Question Number	Scheme	Marks
	$0.5u = 1.5 \quad u = 3 \text{ m s}^{-1}$ <p>Work done against friction = $0.7 \times 0.5 \cos 30g \times 0.6$</p> $\text{Initial EPE} = \frac{\lambda \times 0.6^2}{2 \times 0.6} \left(= \frac{0.6\lambda}{2} = 0.3\lambda \right)$ $\frac{\lambda \times 0.6^2}{2 \times 0.6} + \frac{1}{2} \times 0.5 \times 9 = 0.7 \times 0.5 \cos 30g \times 0.6 + 0.5 \times g \times 0.6 \sin 30$ $\lambda = 3.340\dots = 3.3 \text{ or } 3.34$	<p>B1</p> <p>M1A1</p> <p>B1</p> <p>M1A1A1 Ft EPE and Work</p> <p>A1 [8]</p>
<p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>Alft</p> <p>Alft</p> <p>A1</p>	<p>Correct value for u, seen explicitly or used.</p> <p>Attempt the work done against friction. Weight must be resolved (sin/cos interchange accepted.) Distance moved to be 0.6 m. Mass can be 0.5 or m</p> <p>Correct work done. Mass can be 0.5 or m</p> <p>Allow both of the above marks if the work done against friction is embedded in some incorrect work eg including other forces to form a resultant force.</p> <p>Correct initial EPE Need not be simplified.</p> <p>The work done and the EPE may not be shown explicitly. Check the equation if necessary.</p> <p>Attempt a complete work-energy equation. Must have an EPE, a GPE, a KE and a (dimensionally correct) work against friction term. The final KE may be included provided it becomes 0 here or later. EPE term must be of the form $\frac{k\lambda x^2}{l}$ $k = \frac{1}{2}, 1$ or 2</p> <p>Deduct one per error. Follow through their EPE and work.</p> <p>Correct value of λ, 2 or 3 sf only.</p>	



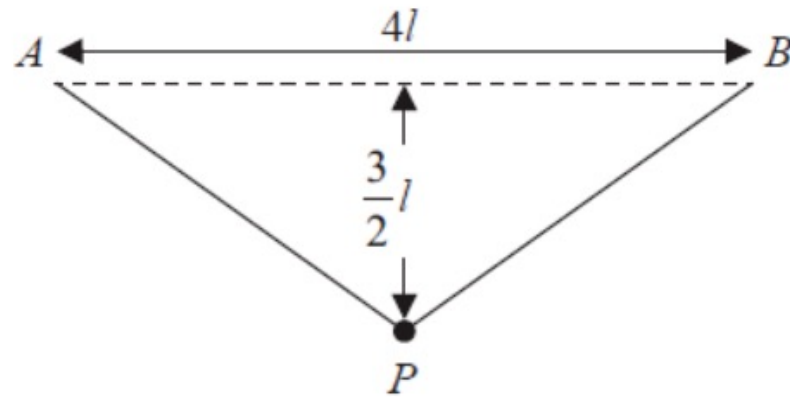


Figure 2

The ends of a light elastic string, of natural length $4l$ and modulus of elasticity λ , are attached to two fixed points A and B , where AB is horizontal and $AB = 4l$. A particle P of mass $2m$ is attached to the midpoint of the string. The particle hangs freely in equilibrium at a distance $\frac{3}{2}l$ vertically below the midpoint of AB , as shown in Figure 2.

(a) Show that $\lambda = \frac{20}{3}mg$.

(7)

The particle is pulled vertically downwards from its equilibrium position until the total length of the string is $6l$. The particle is then released from rest.

(b) Show that P comes to instantaneous rest before reaching the line AB .

(6)

(Total for question = 13 marks)

**JAN
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M3**



Question Number	Scheme	Marks
(a)	<div data-bbox="555 272 1161 558" data-label="Diagram"> </div> <p data-bbox="351 625 715 682">$R(\uparrow): 2T \cos \theta = 2mg$</p> <p data-bbox="351 743 988 829">$\cos \theta = \frac{3}{5}$ (or other correct trig function)</p> <p data-bbox="351 886 690 972">$T = \frac{\lambda \times l}{4l}$ or $\frac{\lambda \times 0.5l}{2l}$</p> <p data-bbox="351 1029 563 1115">$T = \frac{5mg}{3} = \frac{\lambda}{4}$</p> <p data-bbox="351 1172 580 1258">$\lambda = \frac{20}{3}mg$ *</p>	<p data-bbox="1541 639 1651 672">M1A1</p> <p data-bbox="1541 772 1600 805">B1</p> <p data-bbox="1541 915 1651 948">M1A1</p> <p data-bbox="1541 1182 1702 1215">M1A1cso</p> <p data-bbox="1702 1225 1760 1258">(7)</p>



<p>(b)</p>	<p>Dist below $AB = l\sqrt{3^2 - 2^2} = l\sqrt{5}$ (or $2.23l$)</p> <p>EPE at start: $= \frac{\lambda \times (2l)^2}{2 \times 4l} = \frac{20mg}{3} \times \frac{(2l)^2}{8l} \left(= \frac{10mgl}{3} \right)$</p> <p>GPE gained if P reaches $AB = 2mgl\sqrt{5} = 4.47\dots mgl$</p> <p>$\frac{10}{3} < 4.47\dots$</p> <p>$\therefore P$ cannot reach the line AB</p>	<p>B1</p> <p>M1A1</p> <p>B1</p> <p>M1</p> <p>Alcso (6)</p> <p>[13]</p>
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Question Number	Scheme	Marks
(a)M1 A1 B1 M1 A1 M1 A1cso	Resolve vertically. Must have 2 tensions, both resolved and $(2)m$ or $(2)mg$ not resolved Fully correct equation Correct sine, cosine or tangent, seen explicitly or used in an equation Use Hooke's law for the full string or half string with their attempt at the extension Fully correct equation Eliminate T between their 2 equations to obtain an expression for λ Correct given result obtained from correct working	
(b) B1 M1 A1 B1 M1 A1cso	Correct initial distance below the level of AB Calculate the initial EPE, formula to be of the form $\frac{\lambda x^2}{k \times \text{natural length}}$, $k = 2$ or 1 . Must use the full string or 2 x half strings Correct initial EPE Need not be simplified GPE gained if P reaches AB Compare the initial EPE with the GPE – using exact or decimal results Correct work and a conclusion (exact or decimals results used)	



Alternatives for last 3 marks:

ALT1

Assume P stops at distance x below AB

B1

GPE gained $2mg(l\sqrt{5} - x)$

M1

Attempt an energy equation with initial and final KE zero and show it has a positive, real root

$$\frac{10mgl}{3} - 2 \times \frac{20mg}{3} \times \frac{(\sqrt{4l^2 + x^2} - 2l)^2}{4l} = 2mg(l\sqrt{5} - x)$$

Final KE must be 0, 2 EPE terms needed

Alcso

Correct work and a conclusion

ALT 2

Assume final extension is x Similar work may be seen with final extension $2x$

B1

GPE gained $2mg \left(l\sqrt{5} - \sqrt{\left(2l + \frac{x}{2}\right)^2 - 4l^2} \right)$

M1

Attempt an energy equation with initial KE zero and show it has a positive, real root

$$\frac{10mgl}{3} - \frac{20mg}{3} \times \frac{x^2}{4l} = 2mg \left(l\sqrt{5} - \sqrt{\left(2l + \frac{x}{2}\right)^2 - 4l^2} \right)$$

Final KE must be 0, 2 EPE terms needed

Alcso

Correct work and a conclusion



ALT3

Assume P stops after rising a distance x

B1

GPE gained $2mgx$

M1

Attempt an energy equation with initial and final KE zero and show it has a positive, real root

$$\frac{10mgl}{3} - 2 \times \frac{20mg}{3} \times \frac{\left(\sqrt{4l^2 + (l\sqrt{5} - x)^2} - 2l \right)^2}{4l} = 2mgx$$

Final KE must be 0, 2 EPE terms needed

Alcso

Correct work and a conclusion

Alternative for last 2 marks:

M1

Attempt an energy equation including the KE at level of AB and solve for v^2

Alcso

$v^2 < 0$ so P cannot reach the level of AB (Equation must be correct)

Warning: in (b), use of HL with extension $2l$ can also lead to the “correct” result, but scores M0 as it is not an energy solution. (May possibly gain the B marks but this is unlikely.)



Part (a) was answered well by most candidates. When the answer is given it is advisable that candidates show clearly the separate equations they are using and clear working to make it clear how the answer was obtained. Marks were sometimes lost when candidates forced the result by fudging the lengths in their Hooke's Law equation.

Not many candidates got full marks for part (b). The most common fully correct approaches were to calculate the initial EPE and to show that this was less than the GPE required at AB or to set up an energy equation with KE being the difference of EPE and GPE and showing that v^2 would have to be negative. Errors in the initial EPE were often seen with some candidates confused over two strings or one string, mixing up the wrong extension with the wrong natural length. The most common response was to successfully calculate the initial EPE, but then to calculate the height reached incorrectly assuming that all of the initial EPE was

converted to GPE. This height of $\frac{5}{3}l$ was then compared with $l\sqrt{5}$.

Those who attempted an energy equation involving a final extension of x or distance moved up as h often struggled to arrive at the correct equation and usually did not or could not go on to solve it.



JAN 2018 M3 IAL

A particle of mass 0.9 kg is attached to one end of a light elastic string, of natural length 1.2 m and modulus of elasticity 29.4 N . The other end of the string is attached to a fixed point A on a ceiling.

The particle is held at A and then released from rest. The particle first comes to instantaneous rest at the point B .

Find the distance AB .

(Total for question = 5 marks)



Question Number	Scheme	Marks
	$\frac{29.4(y-1.2)^2}{2 \times 1.2} = 0.9 \times 9.8y$ $y^2 - 3.12y + 1.44 = 0$ $y = \frac{3.12 \pm \sqrt{3.12^2 - 4 \times 1.44}}{2}, \quad y = 2.556... = 2.6 \text{ or } 2.56 \text{ (m)}$	M1A1A1 DM1A1 (5) [5]
M1 A1 A1 DM1 Alcao	Attempt an energy equation with a GPE term and a single EPE term of the form $k \frac{\lambda x^2}{l}$ Either term correct Both terms correct Obtain a 3 term quadratic and attempt its solution. Formula to be correct (if shown). This mark can only be awarded for a calculator solution if final answer is correct. Correct distance AB . Second solution for quadratic need not be shown.	
ALT:	$\frac{29.4x^2}{2 \times 1.2} = 0.9 \times 9.8(x+1.2) \text{ etc (solves to 1.356...)}$ Second M mark here requires completion to distance AB	M1A1A1

If SUVAT (or energy) used to natural length, the first M mark is for the energy equation with an EPE term and second M mark is for solving and completing to required distance



Question

Mark Scheme

Examiner's Report

Resources

This question was also answered very well, with many students getting full marks. By far the most popular approach was the alternative on the mark scheme, possibly because this did not require the squaring of an awkward bracket. It's rare for students to fail to complete this method by adding on 1.2. The solving of the quadratic equation was very often completed with no supporting working and a few students lost both marks as a result. The only common mistake was to find the distance to the equilibrium position by using Hooke's law. As this was an elastic energy problem such an approach scored no marks. Any students who tried a more elaborate approach than the two given in the mark scheme tended not to get through a whole valid method.



A particle P of mass m is attached to one end of a light elastic spring, of natural length l and modulus of elasticity $2mg$. The other end of the spring is attached to a fixed point A on a rough horizontal plane. The particle is held at rest on the plane at a point B , where $AB = \frac{1}{2}l$, and released from rest. The coefficient of friction between P and the plane is $\frac{1}{4}$

Find the distance of P from B when P first comes to rest.

(9)

(Total for question = 9 marks)



	$\frac{2mg}{2l} \left(\left(\frac{1}{2}l \right)^2 - x^2 \right) = \frac{1}{4}mg \left(\frac{1}{2}l + x \right)$ $8x^2 + 2lx - l^2 = 0$ $(4x - l)(2x + l) = 0$ $x = \frac{1}{4}l \text{ or } -\frac{1}{2}l$ $\text{distance} = \frac{1}{2}l + \frac{1}{4}l = \frac{3}{4}l$	M1A1;M1 A 1 M1 A1 M1dep A1 A1
9		

Notes

- M1 for the difference of 2 elastic energy terms, not nec in a complete energy equation.
- A1 for a correct difference
- M1 for a work energy equation, loss of EPE = work done against friction(not dep on previous mark)
- A1 for a fully correct equation
- M1dep for re-arranging to a three term quadratic, dependent on the second M mark, or use the difference of 2 squares to get a linear equation
- A1 for a correct 3 term quadratic, terms in any order
- M1dep for solving the resulting quadratic, usual rules. Dependent on all second and third M marks
- A1 for $x = \frac{1}{4}l$ $x = -\frac{1}{2}l$ need not be shown
- A1cao and cso distance = $\frac{3}{4}l$



Question

Mark Scheme

Examiner's Report

Most recognised this as a work-energy question and found a correct expression for the work. Many of these, however, overlooked the fact that there would be a final as well as an initial EPE, allowing a very brief solution and only one available mark out of 9. Candidates should consider whether their solutions warrant the marks allocated. Algebraic errors which caused the reasoning not to result in a 3 term quadratic equation also proved very costly. Candidates should probably be reminded that evidence of a full solution to a quadratic equation should be shown. Some candidates used a 2 stage method, considering the motion to the natural length and then the extension and this method was usually completed successfully.

