Year 13Fm Pre-Mock Set #03c
Further Pure FP1
14^{th} March 2022

- Advised to print in "A3-booklets", this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

This exam paper has 8 questions, for a total of 75 marks.

Question	Marks	Score
1	6	
2	8	
3	9	
4	9	
5	9	
6	9	
7	8	
8	17	
Total:	75	

Andrew Chan
Last updated: 28th February 2023

1. An ellipse has equation

$$\frac{x^2}{25} + \frac{y^2}{4} = 1$$

The point P lies on the ellipse and has coordinates $(5\cos\theta, 2\sin\theta)$, $\{0 < \theta < \frac{\pi}{2}\}$

The line L is a normal to the ellipse at the point P.
An equation of L is

$$5x\sin\theta - 2y\cos\theta = 21\sin\theta\cos\theta$$

Given that the line L crosses the y-axis at the point Q and that M is the midpoint of PQ.

Find the exact area of triangle OPM, where O is the origin.

Give your answer as a multiple of $\sin 2\theta$.

(6)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 6 marks)

2. The plane Π has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ -2 \\ -4 \end{pmatrix} + s \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} + t \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$$

The point A has coordinates $(4, -3, c)$, where c is a constant.

(a) Find, in terms of c, an equation of the line that passes through the point A and is perpendicular to the plane Π.
Give your answer in the form $(\mathbf{r} - \mathbf{a}) \times \mathbf{b} = 0$

(3)

The point A' is a reflection of the point A in the plane Π

(b) Find, in terms of c, the coordinates of the point A'

(5)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3. Given that

$$y = \cot x$$

(a) Show that

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 2\cot x + 2\cot^3 x$$

(3)

(b) Hence show that

$$\frac{\mathrm{d}^3y}{\mathrm{d}x^3} = p\cot^4 x + q\cot^2 x + r$$

where p, q and r are integers to be found.

(3)

(c) Find the Taylor series expansion of $\cot x$ in ascending powers of $\left(x - \frac{\pi}{3}\right)$, up to and including the term in $\left(x - \frac{\pi}{3}\right)^3$.

(3)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 9 marks)

4. Given that

$$\frac{7-2x}{|5x-3|+1} < 4$$

(a) show that

$$|5x-3| > \frac{3-2x}{4}$$

(2)

(b) On the same diagram, sketch

(i) $y = |5x-3|$

(ii) $y = \frac{3-2x}{4}$

Show the coordinates of the points where the graphs meet the coordinate axes.

(3)

(c) Use algebra to determine the exact values of x for which

$$\frac{7-2x}{|5x-3|+1} < 4$$

Give your answer in set notation.

(4)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 9 marks)

5. With respect to a fixed origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = 18\mathbf{i} - 14\mathbf{j} - 2\mathbf{k} \qquad \overrightarrow{OB} = -7\mathbf{i} - 5\mathbf{j} + 3\mathbf{k} \qquad \overrightarrow{OC} = -2\mathbf{i} - 9\mathbf{j} - 6\mathbf{k}$$

The points O, A, B and C form the vertices of a tetrahedron.

(a) Show that the area of the triangular face ABC of the tetrahedron is 108 to 3 significant figures.

(3)

(b) Find the volume of the tetrahedron.

(4)

An oak wood block is made in the shape of the tetrahedron, with centimetres taken for the units.

The density of oak is 0.85 g/cm^3

(c) Determine the mass of the block, giving your answer in kg.

(2)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. On a particular day, the depth of water at Brighton Pier is modelled by the equation

$$D = 2\sin\left(\frac{x}{3}\right) + 3\cos\left(\frac{x}{3}\right) + 6 \qquad \{0 \leq x \leq 7\pi\} \quad \text{(I)}$$

where the depth of water is D metres at time x hours after midnight on that day.

(a) Write down the depth of water at midnight, according to the model.

(1)

Using the substitution $t = \tan\left(\frac{x}{6}\right)$

(b) show that the equation (I) can be re-written as

$$D = \frac{3t^2 + 4t + 9}{1 + t^2}$$

(3)

(c) Hence determine, according to the model, the time after midnight when the depth of water is 5 metres for the first time.
Give your answer to the nearest minute.

(5)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 9 marks)

7. The curve H has equation

$$xy = a^2 \qquad \{x > 0\}$$

where a is a positive constant.

The line with equation $y = kx$, where k is a positive constant, intersects H at the point P.

(a) Use calculus to determine, in terms of a and k, an equation for the tangent to H at P.

(4)

The tangent to H at P meets the x-axis at the point A and meets the y-axis at the point B.

(b) Determine the coordinates of A and the coordinates of B.
Give your answers in terms of a and k.

(2)

(c) Hence show that the area of triangle AOB, where O is the origin, is independent of k.

(2)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 8 marks)

8. A community is concerned about the rising level of pollutant in its local pond and applies a chemical treatment to stop the increase of pollutant.

The concentration, x parts per million (ppm), of the pollutant in the pond water t days after the chemical treatment was applied, is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{3+\cosh t}{3x^2\cosh t} - \frac{1}{3}x\tanh t \qquad \text{(I)}$$

When the chemical treatment was applied the concentration of pollutant was 3 ppm.

(a) Use the iteration formula

$$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_n \approx \frac{(y_{n+1}-y_n)}{h}$$

once to estimate the concentration of the pollutant in the pond water 6 hours after the chemical treatment was applied.

(4)

(b) Show that the transformation $u = x^3$ transforms the differential equation (I) into the differential equation

$$\frac{\mathrm{d}u}{\mathrm{d}t} + u\tanh t = 1 + \frac{3}{\cosh t} \qquad \text{(II)}$$

(3)

(c) Determine the general solution of equation (II)

(4)

(d) Hence find an equation for the concentration of pollutant in the pond water t days after the chemical treatment was applied.

(3)

(e) Find the percentage error of the estimate found in part (a) compared to the value predicted by the model, stating if it is an overestimate or an underestimate.

(3)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 17 marks)

Total for paper is 75 marks