FM1 Ch 1 Momentum and Impulse Exam Questions that caused some difficulty!

Q1.

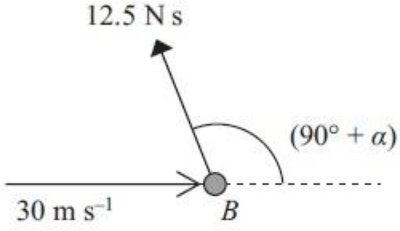

Figure 3

A small ball *B* of mass 0.25 kg is moving in a straight line with speed 30 m s^{-1} on a smooth horizontal plane when it is given an impulse. The impulse has magnitude 12.5 N s and is applied in a horizontal direction making an angle of (90° + *α*),
where $\tan \alpha = \frac{3}{4}$, with the initial direction of motion of the ball, as shown in Figure 3.

(i) Find the speed of *B* immediately after the impulse is applied.
(ii) Find the direction of motion of *B* immediately after the impulse is applied.

(6)
(Total 6 marks)

Q2.

Two particles *A* and *B* are moving on a smooth horizontal plane. The mass of *A* is *km*, where $2 < k < 3$, and the mass of *B* is *m*. The particles are moving along the same straight line, but in opposite directions, and they collide directly. Immediately before they collide the speed of *A* is 2*u* and the speed of *B* is 4*u*. As a result of the collision the speed of *A* is halved and its direction of motion is reversed.

(a) Find, in terms of *k* and *u*, the speed of *B* immediately after the collision.

(3)

(b) State whether the direction of motion of *B* changes as a result of the collision, explaining your answer.

(3)

Given that $k = \frac{7}{3}$,

(c) find, in terms of *m* and *u*, the magnitude of the impulse that *A* exerts on *B* in the collision.

(3)
(Total 9 marks)

Q3.

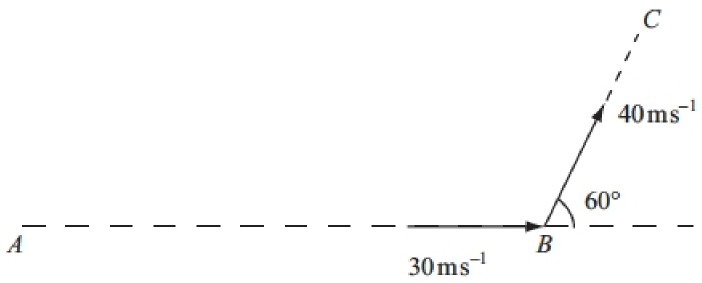

Figure 1

The points *A*, *B* and *C* lie in a horizontal plane. A batsman strikes a ball of mass 0.25 kg. Immediately before being struck, the ball is moving along the horizontal line *AB* with speed 30 m s^{-1}. Immediately after being struck, the ball moves along the horizontal line *BC* with speed 40 m s^{-1}. The line *BC* makes an angle of 60° with the original direction of motion *AB*, as shown in Figure 1.

Find, to 3 significant figures,

(i) the magnitude of the impulse given to the ball,
(ii) the size of the angle that the direction of this impulse makes with the original direction of motion *AB*.

(8)

(Total 8 marks)

Q4.

Two particles *A* and *B* are moving on a smooth horizontal plane. The mass of *A* is 2*m* and the mass of *B* is *m*. The particles are moving along the same straight line but in opposite directions and they collide directly. Immediately before they collide the speed of *A* is 2*u* and the speed of *B* is 3*u*. The magnitude of the impulse received by each particle in the collision is $\frac{7mu}{2}$

Find

(a) the speed of *A* immediately after the collision,

(3)

(b) the speed of *B* immediately after the collision.

(3)

(Total 6 marks)

Q5.

Two particles, *P* and *Q*, of masses *m* and 4*m* respectively are moving on a smooth horizontal plane when they collide directly. Immediately **before** the collision the particles are moving towards each other along the same straight line.

Immediately **after** the collision, the direction of motion of *P* is the same as the direction of motion of *Q*, the speed of *P* is $\frac{3u}{2}$ and the speed of *Q* is $\frac{u}{8}$. In the collision *Q* exerts an impulse of magnitude $\frac{7mu}{2}$ on *P*.

(a) Give a reason why the direction of motion of *P* is reversed by the collision.

(1)

(b) Find, in terms of *u*, the speed of *P* immediately before the collision.

(3)

(c) Find, in terms of *u*, the speed of *Q* immediately before the collision.

(3)

(Total for question = 7 marks)

Q6.

A ball of mass 0.2 kg is projected vertically downwards with speed *U* m s^{-1} from a point *A* which is 2.5 m above horizontal ground. The ball hits the ground. Immediately after hitting the ground, the ball rebounds vertically with a speed of 10 m s^{-1}. The ball receives an impulse of magnitude 7 N s in its impact with the ground. By modelling the ball as a particle and ignoring air resistance, find

(a) the value of *U*.

(6)

After hitting the ground, the ball moves vertically upwards and passes through a point *B* which is 1 m above the ground.

(b) Find the time between the instant when the ball hits the ground and the instant when the ball first passes through *B*.

(4)

(c) Sketch a velocity-time graph for the motion of the ball from when it was projected from *A* to when it first passes through *B*. (You need not make any further calculations to draw this sketch.)

(3)

(Total for question = 13 marks)

Mark Scheme

Q1.

Question Number	Scheme	Marks	Notes
			NB In a Q with parts labelled (i) & (ii) marks are awarded when seen – they do not belong to a particular part of the Q.
	$12.5\sin\alpha = \frac{1}{4}(v_1 - -30)$ or $-12.5\sin\alpha = \frac{1}{4}(v_1 - 30)$ $(v_1 = 0)$	M1 A1	Impulse = change in momentum parallel to the initial direction. . Correct equation
	$12.5\cos\alpha = \frac{1}{4}(v_2 - 0)$ $(v_2 = 40)$	M1 A1	Impulse = change in momentum perpendicular to the initial direction. Condone sin/cos confusion. Correct equation. NB could be in the form: $\begin{pmatrix}-12.5\sin\alpha \\ 12.5\cos\alpha\end{pmatrix} = 0.25v - 0.25\begin{pmatrix}30 \\ 0\end{pmatrix}$
	speed is 40 m s^{-1};	A1	cwo. Correct magnitude of speed after impulse. NB Must be speed, not velocity.
	perpendicular to original direction	A1	cwo. Correct direction (relative to the line given on the diagram – e.g. accept "vertically", "North", j direction, "up").
		6	
OR	Using a vector triangle: $(\frac{1}{4}v)^2 = 7.5^2 + 12.5^2 - 2x7.5x12.5\cos(90° - \alpha)$	M1	Use cosine rule to find $\frac{1}{4}v$. Terms must be of correct form, but accept unsimplified or slips e.g. their $\frac{1}{4}\times 30$
		A1	Correct equation
	$v = 40\text{m s}^{-1}$	A1	cao (penultimate mark on epen)
	$\frac{12.5}{\sin\theta} = \frac{7.5}{\sin\alpha}$	M1 A1	Use sine rule to find angle between initial and final directions. Correct equation in α and θ
	$\theta = 90°$	A1	cao. (final mark on epen)
		6	

Q2.

(a)	$2u \rightarrow \quad \leftarrow 4u$ $km2u - 4mu = -kmu + mv$	M1 A1
	$km \quad m$ $u(3k-4) = v$	A1 (3)
	$u \leftarrow \quad \rightarrow v$	
(b)	$k > 2 \Rightarrow v > 0 \Rightarrow$ dir^n of motion reversed	M1A1A1 cso (3)
(c)	For B, $m(u(3k-4) - -4u)$ $= 7mu$	M1 A1 f.t. A1 (3) [9]

Q3.

Question Number	Scheme	Marks
	(i) $I\uparrow = 0.25\times 40\sin 60 = 5\sqrt{3}$ (8.66) one component	
	$I\leftarrow = 0.25(-20+30) = 2.5$ both	M1
	$\lvert I\rvert = \sqrt{75+6.25} = 9.01$ (Ns)	A1
		M1 A1 (4)
	(ii) $\frac{\sin\theta}{40} = \frac{\sin 60^\circ}{\sqrt{1300}}$	
	$\theta = 106^\circ$ (3 s.f.)	M1 A1
	or $\tan\theta = \pm\frac{5\sqrt{3}}{2.5}$ oee $\theta = 106^\circ$	M1 A1 (4)
		[8]
	Alternative to 4(i)	
	Use of $\mathbf{I} = m(\mathbf{v}-\mathbf{u})$	M1
	$30^2 + 40^2 - 2 \text{ x } 30 \text{ x } 40\cos 60^\circ$ (= 1300)	M1 A1
	$I = 0.25\sqrt{1300} = 9.01$ N s (3 s.f.)	A1
	2nd Alternative to 4(i)	
	$\mathbf{u} = 30\mathbf{i}$, $\mathbf{v} = 40\cos 60\mathbf{i} + 40\sin 60\mathbf{j} = 20\mathbf{i} + 20\sqrt{3}\mathbf{j}$	
	$\mathrm{I} = \frac{1}{4}(-10\mathbf{i} + 20\sqrt{3}\mathbf{j}) = -2.5\mathbf{i} + 5\sqrt{3}\mathbf{j}$	M1
		A1 etc

Q4.

Question Number	Scheme	Marks	
(a)			
	For A: $-\frac{7mu}{2} = 2m(v_A - 2u)$	M1 A1	
	$v_A = \frac{u}{4}$	A1	(3)
(b)	$\frac{7mu}{2} = m(v_B - -3u)$	M1 A1	
	For B: $v_B = \frac{u}{2}$	A1	(3)
	OR CLM:	OR	
	$4mu - 3mu = 2m\frac{u}{4} + mv_B$	M1 A1	
	$v_B = \frac{u}{2}$	A1	(3)
			[6]

Q5.

Question Number	Scheme	Marks
(a)	Otherwise one particle would have to move through the other oe	B1 (1)
(b)	For P: $m\left(\frac{3u}{2}-(-v_P)\right)=\frac{7mu}{2}$	M1 A1
	speed of $P = 2u$	A1 (3)
(c)	For $Q: 4m(\frac{u}{8}-v_Q)=-\frac{7mu}{2}$	M1 A1
	$v_Q = u$	A1 (3)
		OR
	OR $4mv_Q - m2u = 4m\frac{u}{8}+m\frac{3u}{2}$	M1 A1
	$v_Q = u$	A1 (3)
		(7)
	Notes	
(a)	B1 for a clear statement e.g if they are moving in the same direction, the one at the front must be going faster (than the one behind) or: the <u>original</u> momentum of P is less than the impulse exerted on P oe	
	N.B. For (b) and (c), they may find v_Q first and then use it to find v_P	
(b)	M1 for impulse-momentum principle applied to P ; condone sign errors but must be using m for mass and subtracting momenta M0 if g included	
	First A1 for a correct equation. They may have v_P instead of $-v_P$	
	Second A1 for $2u$; must be **positive**	
(c)	M1 for impulse-momentum principle applied to Q ; condone sign errors but must be using $4m$ for mass and subtracting momenta M0 if g included	
	First A1 for a correct equation. They may have $(-v_Q)$ instead of v_Q	
	OR	
	M1 for conservation of momentum, with correct no. of terms, condone sign errors, using their v_P or their v_Q. M0 if equation has more than one unknown	
	First A1 for a <u>correct</u> equation. They may have $(-v_Q)$ instead of v_Q	
	Second A1 for u ; must be **positive**	

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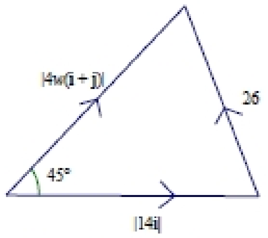
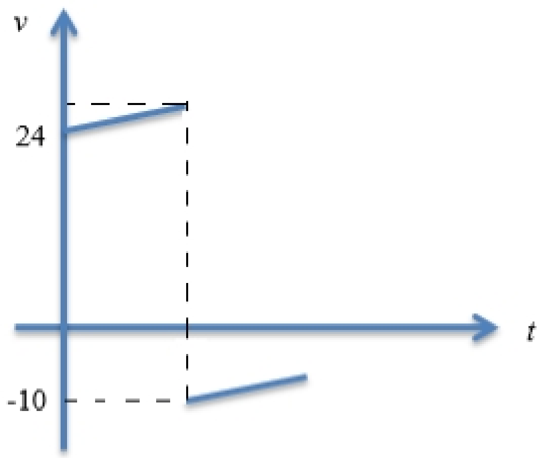
Q5 continued

Q	Scheme	Marks	Notes
	Use of $\mathbf{I} = m\mathbf{v} - m\mathbf{u}$: $\mathbf{I} = 4w(\mathbf{i}+\mathbf{j}) - 4\times 3.5\mathbf{i}$	M1	Correct use of formula but condone subtraction in wrong order. Ignore 26 if seen
	$= (4w-14)\mathbf{i} + 4w\mathbf{j}$	A1	$\mathbf{I}$ in terms of w. Any equivalent form Ignore 26 if seen. Accept + / -
	Magnitude of vector: $26^2 = (4w-14)^2 + (4w)^2$ $(32w^2 - 112w - 480 = 0)$	M1	Correct use of Pythagoras to find magnitude Must square both sides.
	$(2w+5)(w-6) = 0$	M1	Solve a 3 term quadratic in w
	$w = 6$	A1	Positive root only
			Can score fill marks from $\mathbf{I} = m\mathbf{u} - m\mathbf{v}$
		[5]	
alt	Impulse momentum triangle: 	M1	
	Angle and lengths/vectors	A1	
	$26^2 = 14^2 + 16w^2 \times 2 - 2\times 14\times 4w\sqrt{2}\times\frac{1}{\sqrt{2}}$ $(32w^2 - 112w - 480 = 0)$	M1	Use of cosine rule:
	$(2w+5)(w-6) = 0$	M1	Solve a 3 term quadratic in w
	$w = 6$	A1	Positive root only:
		[5]	
		(5)	

Q6.

Question Number	Scheme	Marks
(a)	$V^2 = U^2 + 2g \times 2.5$	M1A1
	Eliminate V *and* solve for U	A1 (DM1)
	$7 = 0.2(10 - -V)$	M1A1
	$U = 24$	A1 (6)
(b)	$1 = 10t - 4.9t^2$ **OR** e.g. $v^2 = 10^2 - 2\times 9.8 \times 1$ and $v = 10 - 9.8t$	
	$1 = 10t - 4.9t^2$ to give $\sqrt{80.4} = 10 - 9.8t$	M1 A1
	$t = \dfrac{10 \pm \sqrt{100 - 19.6}}{9.8}$ so $t = \dfrac{10 - \sqrt{10^2 - 2\times 9.8 \times 1}}{9.8}$	**DM1**
	$t = 0.11$ s or 0.105 s	A1 (4)
(c)		B1ft1st line B1 2nd line B1 ,-10 (3)
		(13)

	Notes	
	(a) First M1 for complete method, using *suvat*, to find equation in U and V only First A1 for a correct equation Second A1 – **treat as third DM1**, dependent on the other two M's, for eliminating V and solving for U Second M1 for using Impulse = Change in Momentum of ball (must have 0.2 in both terms and be using 10 as one of the velocities) (M0 if *clearly* adding momenta or if g is included) but condone sign errors. Third A1 for a correct equation, 7 and 10 must have the same sign but equation may have V instead of $-V$ Fourth A1 for $U = 24$ (must appear here) **N.B.** If they use U instead of V in the impulse-momentum equation, can score max M1A0/6 for part (a). **N.B.** If they go from $V^2 = U^2 + 49$ to $V = U + 7$, can score max 5/6	

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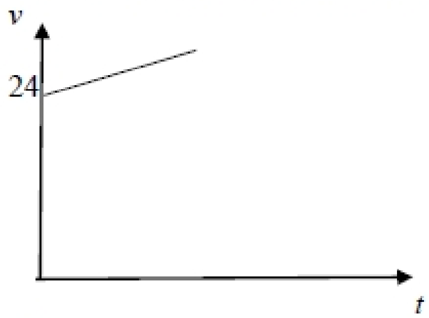
Q6 continued

	(b) First M1 for complete method, using one or more *suvat* formulae, to produce an equation in t only <u>using $s = 1$ or -1</u> First A1 for a correct equation in t only Second DM1, dependent on first M1, for solving their equation (this mark can be implied by a correct answer) Second A1 for either 0.105 (s) or 0.11 (s) (must be only ONE answer)	
	(c) First B1ft for a straight line, with positive gradient, starting at their U value (or just at U) on the positive v-axis. Second B1 for a parallel (approx.) line placed correctly <u>(B0 if a continuous vertical line is included)</u> i.e. starting at a point where the t coordinate is equal to the t coordinate of the point where the first line stopped, and the v coordinate is negative. Third B1 for second line, placed correctly, starting on $v = -10$ N.B. Whole graph could be reflected in the t-axis **SC: If second line is placed correctly but extends up to the t-axis, or beyond, lose second B1 but can score the third B1.**	

(b)	**ALTERNATIVE :** "the instant when the ball first passes through B" is taken to be when the ball is on the way down from A.	
	$s = vt - \frac{1}{2}at^2$ **OR** $v_B^{\ 2} = 24^2 + 2\times 9.8\times 1.5$ and $25 = v_B + 9.8t$	
	$1 = 25t - 4.9t^2$ to give $25 = \sqrt{605.4} + 9.8t$	M1 A1
	$t = \dfrac{25 \pm \sqrt{625 - 19.6}}{9.8}$ so $t = \dfrac{25 - \sqrt{625 - 19.6}}{9.8}$	**DM1**
	$t = 0.040$ (s) or 0.0403 (s) or 0.04 (s) (must only be ONE answer)	A1 (4)
(c)	**ALTERNATIVE :** again "when it first passes through B" is taken to be when the ball is on the way down from A.	
		B2 line B1ft 24 (3)
	(b) First M1 for complete method, using one or more *suvat* formulae, to produce an equation in t only <u>using $s = 1$ or -1</u> First A1 for a correct equation in t only Second DM1, dependent on first M1, for solving their equation (this mark can be implied by a correct answer) Second A1 $t = 0.040$ (s) or 0.0403 (s)	
	(c) B2 for a straight line, with positive gradient, starting on the positive v-axis. B1ft starting at their U value (or just at U)	