Year 13 Further Mathematics Mock Set#02b
Core Pure Paper 2

- Advised to print in "A3-booklets", this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

This exam paper has 8 questions, for a total of 75 marks.

Question	Marks	Score
1	13	
2	9	
3	6	
4	4	
5	9	
6	11	
7	13	
8	10	
Total:	75	

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1.

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 1 & 1 \end{pmatrix}$$

Given that the transformation in the x-y plane represented by $\mathbf{A}^{-1}$ transforms a triangle of area 30 cm^2 into a triangle of area d cm^2.

(a) Find the value of d.

(3)

(b) Prove by mathematical induction that, for all positive integers n,

$$\mathbf{A}^n = \begin{pmatrix} 2^n & 0 \\ 2^n - 1 & 1 \end{pmatrix}$$

(5)

Given that the line $y = 2x$ is invariant under the transformation in the x-y plane represented by $\mathbf{A}^n\mathbf{B}$, where

$$\mathbf{B} = \begin{pmatrix} 1 & 0 \\ 33 & 0 \end{pmatrix}$$

(c) Find the value of n

(5)

Question 1 continued

Question 1 continued

Question 1 continued

(Total for Question 1 is 13 marks)

2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The complex numbers u and v are defined by $u = -4 + 2i$ and $v = 3 + i$

(a) Find $\frac{u}{v}$ in the form $x + iy$, where x and y are real.

(3)

(b) Hence express $\frac{u}{v}$ in the form $re^{i\theta}$, where r and θ are real and exact.

(2)

In an Argand diagram, with origin O, the points A, B and C represent the complex numbers u, v and $2u + v$ respectively.

(c) State fully the geometrical relationship between OA and BC.

(2)

(d) Prove that $\angle AOB = \frac{3}{4}\pi$

(2)

Question 2 continued

Question 2 continued

Question 2 continued

(Total for Question 2 is 9 marks)

3.

$$\begin{aligned} 2x \quad -3y \quad +4z &= 1 \\ 3x \quad - \; y \qquad\quad &= 2 \\ x \quad +2y \quad +kz &= 1 \end{aligned}$$

where k is a constant.

(a) Find the value k for which the system of equations does not have a unique solution.
(2)

(b) Hence, solve the system of equations when it does not have a unique solution.
(4)

Question 3 continued

Question 3 continued

Question 3 continued

(Total for Question 3 is 6 marks)

4. The roots of the cubic equation

$$2x^3 + x^2 - 7 = 0$$

are α, β, and γ.

Using the substitution $y = 1 + \frac{1}{x}$, or otherwise, find the cubic equation whose roots are $1 + \frac{1}{\alpha}$, $1 + \frac{1}{\beta}$ and $1 + \frac{1}{\gamma}$.

Give your answer in the form $ay^3 + by^2 + cy + d = 0$, where a, b, c and d are constants to be found.

(4)

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 4 marks)

5. Given that

$$x = \operatorname{arsinh} t \quad y = \arccos t \qquad \{-1 < t < 1\}$$

(a) By differentiating $\cos y$ with respect to t, show that $\dfrac{\mathrm{d}y}{\mathrm{d}t} = -\dfrac{1}{\sqrt{1-t^2}}$.

(4)

(b) Find $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ in terms of t, simplifying your answer.

(5)

Question 5 continued

Question 5 continued

Question 5 continued

(Total for Question 5 is 9 marks)

6. With respect to a fixed origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = -\mathbf{i} + \mathbf{j} + 2\mathbf{k} \qquad \overrightarrow{OB} = -2\mathbf{i} - \mathbf{j} \qquad \overrightarrow{OC} = 2\mathbf{i} + 2\mathbf{k}$$

(a) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$

(5)

(b) Find the perpendicular distance from O to the plane ABC.

(2)

(c) Find the acute angle between the planes OAB and ABC.

(4)

Question 6 continued

Question 6 continued

Question 6 continued

(Total for Question 6 is 11 marks)

7.

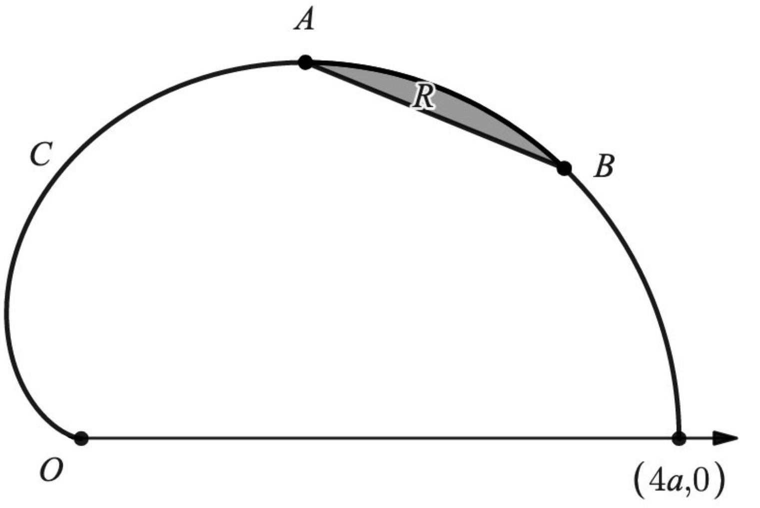

Figure 1: https://www.desmos.com/calculator/j9eg2tqsn8

The curve C, shown shaded in Figure 1, has polar equation

$$r = 2a(1 + \cos\theta) \qquad \{0 \leq \theta \leq \pi\}$$

where a is a positive constant.

The tangent to C at the point A is parallel to the initial line.

(a) Determine the polar coordinates of A.

(6)

The point B on the curve has polar coordinates $\left(a\left(2+\sqrt{3}\right), \frac{\pi}{6}\right)$

The finite region R, shown shaded in Figure 1, is bounded by the curve C and the line AB.

(b) Use calculus to determine the exact area of the shaded region R.
Give your answer in the form

$$\frac{a^2}{4}\left(d\pi - e + f\sqrt{3}\right)$$

where d, e and f are integers.

(7)

Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 13 marks)

8. Given that

$$f(x) = \text{artanh}\left(\frac{1-x}{3+x}\right) \qquad \{x > -1\}$$

(a) Show that $f''(x) = \dfrac{1}{2(x+1)^2}$.

(6)

(b) Hence find the Maclaurin series for $f(x)$ up to and including the term in x^2.

(4)

Question 8 continued

Question 8 continued

Question 8 continued

(Total for Question 8 is 10 marks)

Total for paper is 75 marks