Question 1:

(a) **[3 marks]** Express $\dfrac{5}{(x-1)(3x+2)}$ in partial fractions.

(b) **[3 marks]** Hence find

$$\int \frac{5}{(x-1)(3x+2)}\,\mathrm{d}x$$

(c) **[6 marks]** Find the particular solution of the differential equation

$$(x-1)(3x+2)\frac{\mathrm{d}y}{\mathrm{d}x} = 5y \qquad x > 1$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = f(x)$

Question 2:

(a) **[4 marks]** Express $\dfrac{5-4x}{2x^2+x-1}$ in partial fractions.

(b) **[7 marks]**

(i) Find a general solution of the differential equation

$$(2x^2+x-1)\frac{\mathrm{d}y}{\mathrm{d}x} = 5y - 4xy \qquad x > \frac{1}{2}$$

(ii) Given that $y = 4$ when $x = 2$, find the particular solution of this differential equation. Give your answer in the form $y = f(x)$.

Question 3:

(a) **[3 marks]** Express $\dfrac{1}{P(5-P)}$ in partial fractions.

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{15}P(5-P), \qquad t \geq 0$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

(b) **[8 marks]** Given that $P = 1$ when $t = 0$, solve the differential equation, giving your answer in the form

$$P = \frac{a}{b + ce^{-t/3}}$$

where a, b and c are integers.

(c) **[1 mark]** Hence, show that the population cannot exceed 5000.

Question 4:

(a) [**6 marks**] Use the substitution $u = 4 - \sqrt{h}$ to show that

$$\int \frac{\mathrm{d}h}{4-\sqrt{h}} = -8\ln|4-\sqrt{h}| - 2\sqrt{h} + k$$

where k is a constant.

A team of scientists is studying a species of slow growing tree.

The rate of change in height of a tree in this species is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{t^{0.25}(4-\sqrt{h})}{20}$$

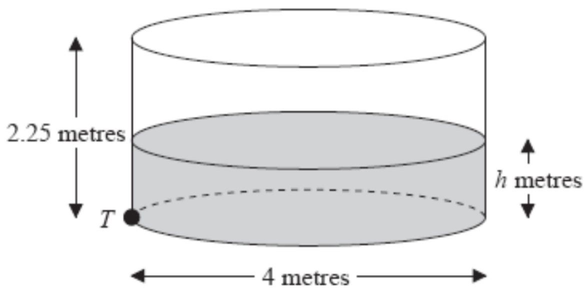
where h is the height in metres and t is the time, measured in years, after the tree is planted.

(b) [**2 marks**] Find, according to the model, the range in heights of the trees in this species.

One of these trees is one metre high when it is first planted.

(c) [**7 marks**] According to the model, calculate the time this tree would take to reach a height of 12 metres, giving your answer to 3sf.

Question 5:



Water is flowing into a cylindrical tank at a constant rate of 0.4π m^3 min^{-1}. The diameter of the circular cross section of the tank is 4 m and the height is 2.25 m. There is a tap at a point T at the base of the tank. When the tap is open, water leaves the tank at a rate of $0.2\pi\sqrt{h}$ m^3 min^{-1}, where h is the height of the water in metres.

(a) [**5 marks**] Show that at time t minutes after the tap has been opened, the height h m of the water in the tank satisfies the differential equation

$$20\frac{\mathrm{d}h}{\mathrm{d}t} = 2 - \sqrt{h}$$

At the instant when the tap is opened, $t = 0$ and $h = 0.16$.

(b) [**2 marks**] Use the differential equation to show that the time taken to fill the tank to a height of 2.25 m is given by

$$\int_{0.16}^{2.25} \frac{20}{2-\sqrt{h}}\,\mathrm{d}h$$

(c) [**7 marks**] Using the substitution $h = (2-x)^2$, or otherwise, find the time taken to fill the tank to a height of 2.25 m. Give your answer in minutes to the nearest minute.

Numerical Answers:

(1) (a) $\dfrac{1}{x-1}+\dfrac{-3}{3x+2}$

(b) $\ln(x-1)-\ln(3x+2)+C$

(c) $y=\dfrac{64(x-1)}{3x+2}$

(2) (a) $\dfrac{2}{2x-1}+\dfrac{-3}{x=1}$

(b) (i) $\ln(y)=\ln(2x-1)-3\ln(x+1)+C$

(ii) $y=\dfrac{36(2x-1)}{(x+1)^3}$

(3) (a) $\dfrac{1/5}{P}+\dfrac{1/5}{5-P}$

(b) $P=\dfrac{5}{1+4e^{-t/3}}$

(c) $P<5$

(4) (a) $-8\ln|4-\sqrt{h}|-2\sqrt{h}+k$

(b) $0\le h\le 16$

(c) $t=75.2$ years

(5) (a) $20\dfrac{\mathrm{d}h}{\mathrm{d}t}=2-\sqrt{h}$

(b) $\displaystyle\int_{0.16}^{2.25}\frac{20}{2-\sqrt{h}}\,\mathrm{d}h$

(c) 49 minutes