Question 1:

(a) Show that the equation $\log_2(3) = \frac{a}{b}$ can be written as $2^a = 3^b$.

[2]

(b) Prove by contradiction that $\log_2(3)$ is an irrational number.

[4]

Question 2:

Express the following in partial fractions:

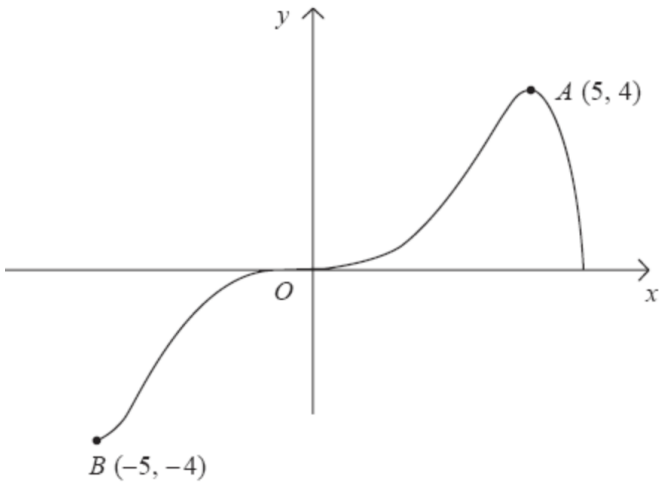
(a) $\dfrac{x^2 + 4x + 5}{x^2 + 4x + 3}$

[4]

(b) $\dfrac{x^2 + 5x + 5}{x^2 + 4x + 4}$

[5]

Question 3:



A sketch of the curve with equation $y = f(x)$ is shown above. The curve passes through the origin O and the points $A(5, 4)$ and $B(-5, -4)$.

(a) Sketch $y = |f(x)|$.

[3]

(b) Sketch $y = f(|x|)$.

[3]

(c) Sketch $y = f(2x + 1)$.

[4]

On each sketch, show the coordinates of the points corresponding to A and B.

Question 4:

It is given that

$$f(x) = e^{-3x} - 4 \qquad x \geq 0$$
$$g(x) = \ln\left(\frac{1}{x+2}\right) \qquad x \geq -1$$

(a) Sketch $y = f(x), x \geq 0$. On your sketch, state the equation of any asymptotes, and the coordinates of any points where the curve crosses the coordinate axes.

[3]

(b) Find $f^{-1}(x)$.

[3]

(c) State the domain of f^{-1}.

[1]

(d) Show that $fg(x)$ can be written in the form $x^3 + ax^2 + bx + c$ where a, b, c are constants to be determined.

[3]

Question 5:

The second term of a geometric sequences is 18, and the fifth term is 13.122.

(a) Find the sum of the first 8 terms, giving your answer to one decimal place.

[4]

(b) Given that $S_\infty - S_N < 0.04$ where S_N is the sum of the first N terms of the sequence, show that $0.9^N < 0.0002$

[4]

(c) Hence find the smallest possible value of N.

[2]

Question 6:

A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_1 = 5$$
$$u_{n+1} = ku_n + 2 \qquad \text{for } n \geq 1$$

where k is a **non-zero** constant.

(a) Find u_2 and u_3 in terms of k, simplifying your answers where appropriate.

[2]

(b) Given that $u_3 = 2$, find the value of $\sum_{n=1}^{3} u_n$

[3]

Numerical Answers:

(1) (a) Show

(b) Prove (use part (a))

(2) (a) $1 + \frac{1}{x+1} - \frac{1}{x+3}$

(b) $1 + \frac{1}{x+2} - \frac{1}{(x+2)^2}$

(3) (a) $A \to (5, 4)$, $B \to (-5, 4)$

(b) $A \to (5, 4)$ and $(-5, 4)$, B disappears

(c) $A \to (2, 4)$, $B \to (-3, -4)$

(4) (a) Asymptote: $y = -4$; Intersection: $(0, -3)$

(b) $f^{-1}(x) = -\frac{1}{3}\ln(x + 4)$

(c) Domain $= \{x : -4 < x \leq -3\}$

(d) $fg(x) = x^3 + 6x^2 + 12x + 4$

(5) (a) $S_8 = 113.9$ ($a = 20$ and $r = 0.9$)

(b) Show

(c) $N = 81$

(6) (a) $u_2 = 5k + 2$, $u_3 = 5k^2 + 2k + 2$

(b) $\sum_{n=1}^{3} = 7$