Name: ____

Year 12 Differentiation, stationary points, tangents and normal, IAL C12 adapted ACH

Date:

Time:

Total marks available:

Total marks achieved: ______

ACH

Questions

Q1.

$$f(x) = \frac{(4 + 3\sqrt{x})^2}{x}, \qquad x > 0$$

(a) Show that $f(x) = Ax^{-1} + Bx^{k} + C$, where A, B, C and k are constants to be determined.

(4)

(b) Hence find $f'(x)$.

(2)

(c) Find an equation of the tangent to the curve $y = f(x)$ at the point where $x = 4$

(4)

(Total for question = 10 marks)

Q2.

The curve C has equation

$$y = 12x^{\frac{5}{4}} - \frac{5}{18}x^2 - 1000, \qquad x > 0$$

(a) Find $\frac{dy}{dx}$

(2)

(b) Hence find the coordinates of the stationary point on C.

(5)

(c) Use $\frac{d^2y}{dx^2}$ to determine the nature of this stationary point.

(3)

(Total for question = 10 marks)

Q3.

The curve C has equation

$$y = \frac{(x-3)(3x-25)}{x}, \quad x > 0$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in a fully simplified form.

(3)

(b) Hence find the coordinates of the turning point on the curve C.

(4)

(c) Determine whether this turning point is a minimum or maximum, justifying your answer.

(2)

The point P, with x coordinate $2\frac{1}{2}$, lies on the curve C.

(d) Find the equation of the normal at P, in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

(Total for question = 14 marks)

Q4.

The curve C has equation

$$y = 2x^2 - \frac{1}{4x} - 3 \qquad x > 0$$

(a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ giving the answer in its simplest form.

(3)

The point $P\left(\frac{1}{2}, -3\right)$ lies on C.

(b) Find the equation of the tangent to C at the point P. Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

(Total for question = 6 marks)

Q5.

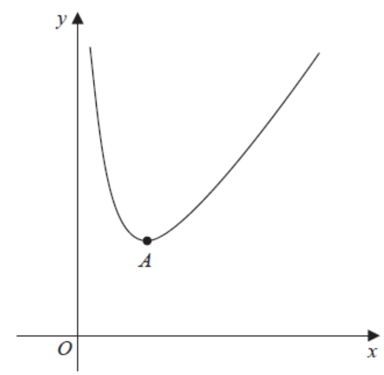


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = x^2 + \frac{16}{x}, \quad x > 0$$

The curve has a minimum turning point at A.

(a) Find $\mathrm{f}'(x)$.

(2)

(b) Hence find the coordinates of A.

(4)

(c) Use your answer to part (b) to write down the turning point of the curve with equation

(i) $y = \mathrm{f}(x + 1)$,

(ii) $y = \frac{1}{2}\mathrm{f}(x)$.

(2)

(Total for question = 8 marks)

Q6.

The curve C has equation $y = 4x\sqrt{x} + \frac{48}{\sqrt{x}} - \sqrt{8}, \quad x > 0$

(a) Find, simplifying each term,

(i) $\frac{dy}{dx}$

(ii) $\frac{d^2y}{dx^2}$

(5)

(b) Use part (a) to find the exact coordinates of the stationary point of C.

(5)

(c) Determine whether the stationary point of C is a maximum or minimum, giving a reason for your answer.

(2)

(Total for question = 12 marks)

Q7.

The curve C has equation

$$y = 3x^2 - 4x + 2$$

The line l_1 is the normal to the curve C at the point $P(1, 1)$

(a) Show that l_1 has equation

$$x + 2y - 3 = 0$$

(5)

The line l_1 meets curve C again at the point Q.

(b) By solving simultaneous equations, determine the coordinates of the point Q.

(4)

Another line l_2 has equation $kx + 2y - 3 = 0$, where k is a constant.

(c) Show that the line l_2 meets the curve C once only when

$$k^2 - 16k + 40 = 0$$

(4)

(d) Find the two exact values of k for which l_2 is a tangent to C.

(2)

(Total for question = 15 marks)

Q8.

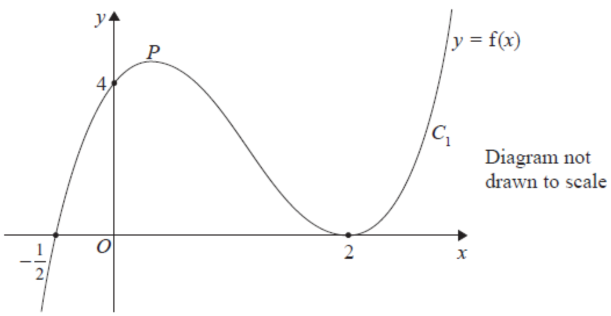


Figure 2

Figure 2 shows a sketch of the curve C_1 with equation $y = f(x)$ where

$$f(x) = (x - 2)^2(2x + 1), \quad x \in \mathbb{R}$$

The curve crosses the x-axis at $\left(-\frac{1}{2}, 0\right)$, touches it at (2, 0) and crosses the y-axis at (0, 4).
There is a maximum turning point at the point marked P.

(a) Use $f'(x)$ to find the exact coordinates of the turning point P.

(7)

A second curve C_2 has equation $y = f(x + 1)$.

(b) Write down an equation of the curve C_2

You may leave your equation in a factorised form.

(1)

(c) Use your answer to part (b) to find the coordinates of the point where the curve C_2 meets the y-axis.

(2)

(d) Write down the coordinates of the two turning points on the curve C_2

(2)

(e) Sketch the curve C_2, with equation $y = f(x + 1)$, giving the coordinates of the points where the curve crosses or touches the x-axis.

(3)

(Total for question = 15 marks)

Q9.

A curve has equation

$$y = 16x\sqrt{x} - 3x^2 - 78 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the equation of the normal to the curve at the point where $x = 4$, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(5)

(Total for question = 8 marks)

Q10.

The curve C has equation $y = f(x)$, $x > 0$, where

$$f'(x) = \frac{5x^2 + 4}{2\sqrt{x}} - 5$$

It is given that the point $P(4, 14)$ lies on C.

(a) Find $f(x)$, writing each term in a simplified form.

(6)

(b) Find the equation of the tangent to C at the point P, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

(Total for question = 10 marks)

Q11.

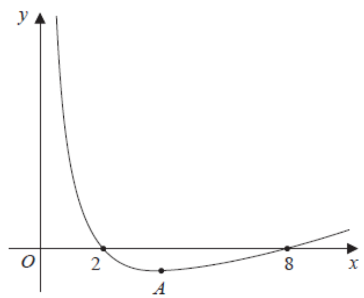


Figure 3

Figure 3 shows a sketch of the curve with equation $y = f(x)$ where

$$f(x) = \frac{8}{x} + \frac{1}{2}x - 5, \qquad 0 < x \leqslant 12$$

The curve crosses the x-axis at $(2, 0)$ and $(8, 0)$ and has a minimum point at A.

(a) Use calculus to find the coordinates of point A.

(5)

(b) State

(i) the roots of the equation $2\mathrm{f}(x) = 0$

(ii) the coordinates of the turning point on the curve $y = \mathrm{f}(x) + 2$

(iii) the roots of the equation $\mathrm{f}(4x) = 0$

(3)

(Total for question = 8 marks)

Mark Scheme

Q1.

Question Number	Scheme	Marks
	(a) and (b) can be marked together	
(a)	$\mathrm{f}(x) = \dfrac{16 + 24\sqrt{x} + 9x}{x}$	M1
	$\mathrm{f}(x) = 16x^{-1} + 24x^{-\frac{1}{2}} + 9$	M1A1A1
		[4]
(b)	$\mathrm{f}'(x) = -16x^{-2} - 12x^{-\frac{3}{2}}$	M1 A1
		[2]
(c)	When $x = 4$, $y = 25$	B1
	$\mathrm{f}'(4) = -1 - \dfrac{12}{8} = -2\frac{1}{2}$	M1
	Equation of tangent is $y - 25 = -\dfrac{5}{2}(x - 4)$	M1 A1
		[4]
		10 marks

	Notes	
(a)	**M1:** expands numerator into a three (or four) term quadratic in $\sqrt{x}$ (allow $\left(\sqrt{x}\right)^2$ for x) **M1:** Divides at least one term in numerator by x correctly following an attempt at expansion. May just be $\dfrac{16}{x}$. **A1:** Two correct terms **A1:** All terms correct	
(b)	**M1:** Evidence of differentiation $x^n \to x^{n-1}$ of an expression of the form Ax^{-1} or Bx^k so $x^{-1} \to x^{-2}$ or $x^k \to x^{k-1}$ $(k \neq 1)$ and not just $C \to 0$. Differentiating top and bottom separately is M0. Note this is a hence and so attempts at e.g. use of the quotient rule scores M0. **A1:** cao and cso (May be un-simplified) **Note: An incorrect constant in part (a) (e.g. 3 instead of 9) will fortuitously give the same derivative so scores M1A0 if otherwise correct.**	
(c)	**B1:** 25 only **M1:** Substitute $x = 4$ into their derived function **M1:** Uses their "25" and their "gradient" which has come from calculus (**not the normal gradient**) and $x = 4$ to give correct ft equation of line. If using $y = mx + c$ must at least obtain a value for c **A1:** any correct form e.g. $y = -\dfrac{5}{2}x + 35$, $5x + 2y - 70 = 0$ **BUT NOT JUST** $\dfrac{y - 25}{x - 4} = -\dfrac{5}{2}$, **this scores M1A0** **Note: An incorrect constant in part (a) (e.g. 3 instead of 9) will fortuitously give the correct answer in (c) and will lose the final A mark if otherwise correct.**	

Q2.

Question Number	Scheme	Marks
	$y = 12x^{\frac{5}{4}} - \dfrac{5}{18}x^2 - 1000$	
(a)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} = 12 \times \dfrac{5}{4}x^{\frac{1}{4}} - \dfrac{10}{18}x$	M1 A1
		[2]
(b)	Put $12 \times \frac{5}{4}x^{\frac{1}{4}} - \frac{10}{18}x = 0$ so $x^n = k$ $(n \in \mathbb{R},\ k \neq 0)$	M1
	$\therefore\ x = (\ )^{\frac{4}{3}}$	dM1
	$\therefore\ x = 81$ **(Ignore $x = 0$ if given as a second solution)**	A1
	So $y = 12(81)^{\frac{5}{4}} - \frac{5}{18}(81)^2 - 1000$ i.e. $y = 93.5$	dM1A1
		[5]
(c)	$\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{15}{4}x^{-\frac{3}{4}} - \dfrac{5}{9}$	B1ft
	Substitutes their non-zero x (positive or negative) into their second derivative.	M1
	Obtains maximum after correctly substituting 81 into correct second derivative to give correct negative quantity $-\dfrac{15}{36}$ o.e. or decimal e.g. -0.4…. (see note below) and considers negative sign deducing maximum. Note that a correct second derivative followed by $x = 81 \Rightarrow \dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} = \dfrac{15}{4}81^{-\frac{1}{4}} - \dfrac{5}{9} = -\dfrac{5}{12}$ therefore maximum scores B1M1A0 here.	A1
		[3]
		10 marks

	Notes
(a)	**M1:** Attempt to differentiate – power reduced by one $x^n \to x^{n-1}$ (but not just $1000 \to 0$) **A1:** Two correct terms and no extra terms. Terms may be un-simplified.
(b)	**M1:** Puts derivative = 0 and attempts to solve to obtain an equation of the form $x^n = k$ where n is real and k is non-zero **dM1:** Correct processing to obtain a value for x. (Dependent on the first method mark). This mark can only be awarded for processing an equation of the form $ax^{\frac{1}{4}} - bx = 0$ i.e. their derivative must have the **correct powers of** x. E.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow x^{\frac{1}{4}}\left(a - bx^{\frac{3}{4}}\right) \Rightarrow x = k^{\frac{4}{3}}$ or $ax^{\frac{1}{4}} - bx = 0 \Rightarrow ax^{\frac{1}{4}} = bx \Rightarrow px = qx^4 \Rightarrow x = \sqrt[3]{k}$ Do not allow incorrect squaring e.g. $ax^{\frac{1}{4}} - bx = 0 \Rightarrow px - qx^4 = 0$ etc. **A1:** cao **dM1:** Substitutes their **positive** value for x into $y = \ldots$ **and not into** $\frac{dy}{dx} = \ldots$ (Dependent on the first method mark) **A1:** cao If $x = 81$ appears from **no working** following a correct derivative score M1M0A0 then allow full recovery.
(c)	**B1ft:** Correct **follow through** second derivative **M1:** Substitutes their non-zero x (**positive or negative**) into their second derivative. **Note**: Solving $\frac{d^2y}{dx^2} = 0$ is M0 **A1cso:** Completely correct work ($-\frac{5}{12}$ o.e.). Note that o.e. could be $= \frac{15}{4} \times \frac{1}{27} - \frac{5}{9}$ or $\frac{15}{108} - \frac{5}{9}$ or $\frac{5}{36} - \frac{5}{9}$ or -0.4….**but it has to be correct for the final mark.**

Q3.

Question Number	Scheme	Marks
(a)	So $y = 3x - 34 + \frac{75}{x}$	B1
	$\frac{dy}{dx} = 3 - 75x^{-2} + \{0\}$ $(x > 0)$ Accept $\frac{dy}{dx} = \frac{3x^2 - 75}{x^2}$ or equivalent	M1 A1
		[3]
(b)	Put $\frac{dy}{dx} = 3 - 75x^{-2} = 0$	M1
	$x = 5$	A1
	Substitute to give $y = -4$	M1 A1
		[4]
(c)	Consider $\frac{d^2y}{dx^2} = 150x^{-3} > 0$	M1
	So minimum	A1
		[2]
(d)	When $x = 2.5,\ y = 3.5$	B1
	Also gradient of curve found by substituting 2.5 into their $\frac{dy}{dx}$ (= -9)	M1
	So gradient of normal is $-\frac{1}{m}\ (= \frac{1}{9})$	dM1
	Either: $y - "3.5" = "\tfrac{1}{9}"(x - 2.5)$ **or**: $y = "\tfrac{1}{9}"x + c$ and $"3.5" = "\tfrac{1}{9}"(2.5) + c \Rightarrow c = "3\tfrac{2}{9}"$	dM1
	So $\underline{x - 9y + 29 = 0}$ or $\underline{9y - x - 29 = 0}$ or any multiple of these answers	A1
		[5]
		14 marks

Q4.

Question Number	Scheme	Marks
(a)	$\frac{dy}{dx} = 2 \times 2x - -\frac{1}{4}x^{-2}$	M1A1
	$\frac{dy}{dx} = 4x + \frac{1}{4}x^{-2}$ oe	A1
		(3)
(b)	$\left.\frac{dy}{dx}\right\rvert_{x=\frac{1}{2}} = 4 \times \frac{1}{2} + \frac{1}{4 \times \left(\frac{1}{2}\right)^2} = (3)$	M1
	$y + 3 = 3\left(x - \frac{1}{2}\right) \Rightarrow y = 3x - \frac{9}{2}$	dM1 A1
		(3)
		(6 marks)

(a)

M1 For reducing a correct power by one on either x term.

The indices must be processed and not left as, for example, x^{2-1}

Look for either x or x^1 or x^{-2} or $\frac{1}{x^2}$

A1 Correct (but may be un simplified) See line 1 scheme for possible expression

Allow here a correct simplified / unsimplified expression with an additional '+ c'.

A1 $\frac{dy}{dx}=4x+\frac{1}{4}x^{-2}$ or exact simplified equivalent. Allow $4x \leftrightarrow 4x^1$

ISW after a correct answer. They may attempt to write as a single fraction or write e.g. $\frac{dy}{dx}=4x+\frac{1}{4\sqrt{x}}$

(b)

M1 For substituting $x=\frac{1}{2}$ into their $\frac{dy}{dx}$ and finding a numerical answer. Unlikely to be scored if there is a '+ c'

dM1 For correct method of finding the equation of the tangent. Eg $y+3=\left.\frac{dy}{dx}\right|_{x=\frac{1}{2}}\left(x-\frac{1}{2}\right)$

Condone one error on the sign of the $\frac{1}{2}$ or the -3.

If the form $y=mx+c$ is used they must proceed to $c=...$

A1 $y=3x-\frac{9}{2}$ or $y=3x-4.5$. ISW after the correct answer.

It must be written in this form and not left $m=3,\ c=-\frac{9}{2}$

SC. If a calculator is used to find $\left.\frac{dy}{dx}\right|_{x=\frac{1}{2}}=3$ **without sight of** $\frac{dy}{dx}=4x+\frac{1}{4}x^{-2}$ then you may allow the final two marks in (b) for correct method to find a correct tangent.

Q5.

Question Number	Scheme	Marks
(a)	$f(x)=x^2+\frac{16}{x} \Rightarrow f'(x)=2x-\frac{16}{x^2}$	M1A1
		(2)
(b)	Setting $2x-\frac{16}{x^2}=0 \Rightarrow x=..$	M1
	$x^3=8 \Rightarrow x=2$	dM1A1
	$A=(2,12)$	A1
		(4)
(c)(i)	$A'=(1,12)$	B1ft
(ii)	$A'=(2,6)$	B1ft
		(2)
		(8 marks)

(a)

M1 $x^n \to x^{n-1}$ for either term. Accept $x^2 \to x$ or $\frac{1}{x} \to \frac{1}{x^2}(x^{-1} \to x^{-2})$

A1 A correct unsimplified $f'(x)=2x-\frac{16}{x^2}$. Accept versions such as $f'(x)=2\times x+16\times -1x^{-2}$

(b)

M1 Sets their $f'(x)=0$ and proceeds to $x=$.... Don't be overly concerned with how they get to x=..

dM1 Dependent upon the previous M mark. It is scored for $\times x^2$ to reach $x^3=k \Rightarrow x=\sqrt[3]{k}$

This may be implied by the correct answer to their equation.

A1 Correctly achieving $x=2$. Ignore any additional solutions.

A1 Correctly achieving $A=(2,12)$. Accept $x=2, y=12$.

If any additional solutions are given $(x>0)$ this mark will be withheld.

Accept $y=12$ appearing in part (c) as long as you are convinced that it is for $y=f(x)$

(c)(i)

B1 ft $A'=(1,12)$. Accept this on a sketch graph or as $x=1, y=12$

If $A=(p,q)$ was incorrect follow through on their value from part (b), $A'=(p-1,q)$

If part (b) was not attempted this can be scored from an algebraic or 'made up' answer

(c)(ii)

B1 ft $A'=(2,6)$ Accept this on a sketch graph or as $x=2, y=6$

If $A=(p,q)$ was incorrect follow through on their value from part (b), $A'=(p,\frac{1}{2}q)$

If part (b) was not attempted this can be scored from an algebraic or 'made up' answer

Do Not allow multiple attempts, mark in the order given if not clearly labelled.
The isw rule is suspended for this part of the question.

Q6.

Question Number	Scheme	Marks
(a)(i)	$\frac{dy}{dx}=6x^{0.5}-24x^{-1.5}$	M1A1A1
(ii)	$\frac{d^2y}{dx^2}=3x^{-0.5}+36x^{-2.5}$	M1A1
		(5)
(b)	$\frac{dy}{dx}=0 \Rightarrow 6x^{0.5}-24x^{-1.5}=0$	M1
	$x^2=4 \Rightarrow x=2$	dM1, A1
	Substitutes their $x=2$ into $y=4x\sqrt{x}+\frac{48}{\sqrt{x}}-\sqrt{8} \Rightarrow y=30\sqrt{2}$	M1,A1
		(5)
(c)	Substitutes their $x=2$ into their $\frac{d^2y}{dx^2}=3x^{-0.5}+36x^{-2.5}$	M1
	Statement +reason. ie $\frac{d^2y}{dx^2}>0 \Rightarrow$ minimum	A1 cso
		(2)
		(12 marks)

(a)(i)

M1 For a correct power on any of the 'three' terms including the $\sqrt{8} \to 0$.

A1 Two of the three terms correctly differentiated (can be unsimplified)

You may accept $6x^{0.5}$ as $4\times1.5x^{1.5-1}$ and $-24x^{-1.5}$ as $+48\times-\frac{1}{2}x^{-0.5-1}$

A1 Cao but remember to isw. Accept alternatives for the terms in x such as $x^{0.5} = \sqrt{x} = x^{\frac{1}{2}}$

Allow expressions given in the form of the question $\left(\frac{dy}{dx} = \right) 6\sqrt{x} - \frac{24}{x\sqrt{x}}$

(a)(ii)

M1 Differentiating again. Scored for reducing any **fractional** power by one (seen once allowing follow through)

A1 Cao. See part (i) notes for acceptable alternatives. Eg accept $\left(\frac{d^2y}{dx^2} = \right) \frac{3}{\sqrt{x}} - \frac{36}{x^2\sqrt{x}}$

(b)

M1 Sets (or implies that) their $\frac{dy}{dx} = 0$

dM1 Dependent upon the previous M. For forming an equation of the type $x^n = A$, **following correct index work.**

A1 $x = 2$ (Ignore any reference to $x = -2$). Part (a) must be correct and both M's must have been scored.

M1 For substituting their solution (of $\frac{dy}{dx} = 0$) into $y = 4x\sqrt{x} + \frac{48}{\sqrt{x}} - \sqrt{8} \Rightarrow y = ...$

A1 $(y) = 30\sqrt{2}$ Part (a) must be correct and all three M's must have been scored.

(c)

M1 For substituting their $x = 2$ into their $\left(\frac{d^2y}{dx^2} = \right) 3x^{-0.5} + 36x^{-2.5}$ and finding (or implying to find) a numerical result. Alternatively, for substituting their $x = 2$ into their $\left(\frac{d^2y}{dx^2} = \right) 3x^{-0.5} + 36x^{-2.5}$ and considering the sign. Eg When $x = 2 \Rightarrow 3\times2^{-0.5} + 36\times2^{-2.5} > 0$

A1 CSO Requires a correct $x = 2$ and a correct $\left(\frac{d^2y}{dx^2}\right) = 3x^{-0.5} + 36x^{-2.5}$

A statement and a conclusion is required to score this mark.

Allow the candidate to state that when $x = 2$ $\frac{d^2y}{dx^2} = 3\times2^{-0.5} + 36\times2^{-2.5} > 0 \Rightarrow$ minimum

If the candidate gives the numerical value to $\frac{d^2y}{dx^2}$, it must be correct. Accept $6\sqrt{2}$ oe or awrt 8.5

Alternatives in part (c)

M1 Finding the value of 'y' at $x = 2$, left of 2 and right of 2.

Alternatively finding the $\frac{dy}{dx}$ at $x = 2$, left of 2 and right of 2

A1 A statement and a conclusion is required to score this mark. A sketch graph can be used instead of a statement. Numerical values must be correct.

Q7.

Question Number	Scheme	Marks
(a)	$y = 3x^2 - 4x + 2$ $\frac{dy}{dx} = 6x - 4 + \{0\}$	M1A1
	At (1, 1) gradient of curve is 2 and so gradient of normal is $-\frac{1}{2}$	M1
	$\therefore (y-1) = -\frac{1}{2}(x-1)$ and so $x + 2y - 3 = 0$*	M1 A1* **[5]**
(b)	Eliminate x or y to give $2(3x^2 - 4x + 2) + x - 3 = 0$ or $y = 3(3-2y)^2 - 4(3-2y) + 2$	M1
	Solve three term quadratic e.g $6x^2 - 7x + 1 = 0$ or $12y^2 - 29y + 17 = 0$ to give $x =$ or y =	M1
	$x = \frac{1}{6}$ **or** $y = 1\frac{5}{12}$	A1
	Both $x = \frac{1}{6}$ **and** $y = 1\frac{5}{12}$ i.e. $(\frac{1}{6}, 1\frac{5}{12})$ *or* (0.17,1.42) { Ignore (1 , 1) listed as well }	A1 **[4]**
(c)	When this line meets the curve $2(3x^2 - 4x + 2) + kx - 3 = 0$	M1
	So $6x^2 + (k-8)x + 1 = 0$	dM1
	Uses condition for equal roots "$b^2 = 4ac$" on their three term quadratic to get expression in k	ddM1
	So obtain $(k-8)^2 = 24$ i.e. $k^2 - 16k + 40 = 0$ *	A1 *
	If they use **gradient of tangent** to do part (c) see the end of the notes below*.	**[4]**
(d)	Solve the given quadratic or their quadratic by formula or completion of the square to give	M1A1
	$k = 8 \pm \sqrt{24}$ or $8 \pm 2\sqrt{6}$ or $\frac{16 \pm \sqrt{96}}{2}$	**[2]**
		15 marks
	Notes	

(a) **M1:** Evidence of differentiation, so $x^n \to x^{n-1}$ at least once

A1: Both terms correct

M1: Substitutes $x = 1$ into their derivative and uses perpendicular property

M1: Correct method for Linear equation, using (1,1) and their changed gradient

A1: Should conclude with printed answer (this answer is given in the question)

(b) M1: May make sign slips in their algebra; {e.g. substitute 3 + 2y }– does not need to be simplified so isw.

But putting $3(3-2y)^2 - 4(3-2y) + 2 = 0$ instead of $= y$ is M0

M1: Solve three term quadratic to give one of the two variables

A1: One Correct coordinate – accept any equivalent

A1: Both correct – any equivalent form. Allow decimals if correct awrt (0.17, 1.42) (ignore (1,1) given as well)

(c) M1: Eliminate y (condone small copying errors)

dM1: Collect into 3 term quadratic in x or identifies "a", "b" and "c" clearly (may be implied by later work).

ddM1: Uses condition "$b^2 = 4ac$" on quadratic in x (dependent on both previous M marks)

NB **M0** for $b^2 > 4ac$ or $b^2 \geq 4ac$ or $b^2 < 4ac$ or $b^2 \leq 4ac$

A1: Need $(k-8)^2 = 24$ or equivalent before stating printed answer

*Alternative method for part (c)

M1: Use gradient of line = gradient of curve so "$6x - 4$" = "$-\frac{k}{2}$"

M1: Find $x = \frac{2}{3} - \frac{k}{12}$ and use line equation to get $y = \frac{3}{2} - \frac{1}{3}k + \frac{k^2}{24}$ (these equations do not need to be simplified)

M1: Find $x = \frac{2}{3} - \frac{k}{12}$ and use curve equation to get $y = \frac{2}{3} + \frac{k^2}{48}$ (these equations do not need to be simplified)

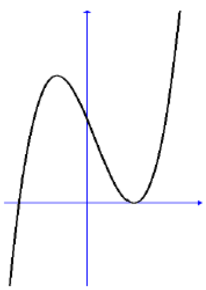
A1: Puts two correct expressions for y equal and obtains printed answer without error.

(d) M1: Solve by formula or completion of the square to give $k =$ (Attempt at factorization is M0)

A1: Correct answer – should be one of the forms given in the main scheme or equivalent exact form

Answers only with no working 2 marks (exact and correct) or 0 marks (approximate or wrong)

Q8.

Question Number	Scheme	Marks
(a)	$f(x)=(x-2)^2(2x+1)=2x^3-7x^2+4x+4$	M1
	So $f'(x)=6x^2-14x\ +4$	M1 A1
	Puts $f'(x)=0$ and solves three term quadratic to obtain for example $2(3x-1)(x-2)=0$ so $x=$	M1
	$x=\frac{1}{3}$ (with $x=2$)	A1
	Calculates f(their x) and find y $\Rightarrow\left(\frac{1}{3},\frac{125}{27}\right)$ Allow $x=\frac{1}{3}, y=4\frac{17}{27}$	dM1 A1 [7]
(b)	$y=(x-1)^2(2x+3)$	B1 [1]
(c)	When $x=0, y=3$	M1 A1 [2]
(d)	$(1, 0)$ and $\left(-\frac{2}{3},\text{"}\frac{125}{27}\text{"}\right)$	M1 A1ft [2]
(e)	 M1: Shape same as before, +ve cubic, but moved. Don't be overly concerned about the position of the maximum point.	M1
	A1: Shape same as before but moved to the **left** (maximum must be in second quadrant and minimum on +ve x - axis) and graph lies in three quadrants	A1
	A1: (1,0) and (-1.5,0) or marked on the x axis as 1 and -1.5	A1 [3]
		15 marks
	Notes	

(a)

M1: Expand brackets, must have a four term cubic with or without collected terms.
M1: Differentiates to a quadratic– reduction of a power by one seen at least once
A1: Completely correct $f'(x)=6x^2-14x\ +4$
M1: Puts their derivative = 0 and solves to find the other root to '2'. The derivative must be a 3TQ expression.
A1: Allow exact equivalences including recurring decimals. May include $x = 2$
dM1: Substitutes their 1/3 into f(x) to find the y coordinates. Implied by y = awrt 4.63 Dependent upon previous M
A1: $x=\frac{1}{3}, y=\frac{125}{27}$ must be exact. Allow mixed numbers, allow recurring decimals

The first 3 marks could be done by the product rule
M1: For $f'(x)=A(x-2)^2+B(2x+1)(x-2)$
M1 A1: For $f'(x)=2(x-2)^2+2(2x+1)(x-2)$

(b)
B1: cao. Must be in the form y =....or f (x) = or f (x+1) =
Allow $y=2(x+1)^3-7(x+1)^2+4(x+1)+4$ You may isw after seeing this
Do not allow the mark if the function is left in the form $y=(x+1-2)^2(2(x+1)+1)$

(c)
M1: Puts $x = 0$ into their new function. Allow embedded values or correct ft.
A1: $y = 3$ The function must have been correct, but not necessarily simplified, to score this mark,
Condone lack of $y =$ if the candidates work implies that y is being found at $x = 0$

(d)
M1: Either coordinate pair correct. Follow through their point P.
So (1,0) or (a – 1, b) where P had coordinates (a, b)
A1ft: Both pairs correct, follow through **only** on the y **coordinate** of P
You may condone a decimal approximation such as 0.33
So if $P=\left(\frac{1}{3},2\right)$ the answer of $(1,0)$ *and* $\left(-\frac{2}{3},2\right)$ would score M1 A1ft

Note: If they do differentiate again they only score the marks as above. They cannot be awarded from the sketch in (e)

(e)
M1: Curve moved in any way. Evidence could be, for example, the maximum to the left of the y axis or the minimum not on the x axis or a point adapted. Be tolerant on slips in shape.
A1: Shape same as before but translated to the **left** (maximum must be in second quadrant and minimum on +ve x - axis) and graph lies in three quadrants. If the maximum looks on the y - axis, do not allow.
A1: For the new curve having a minimum point on the x axis at (1,0) and passing through the x axis at -1.5. Allow this mark if it just stops at the x axis at -1.5. (It would lose the earlier A1 for not appearing in quadrant 3)

Watch for the curve been superimposed on Figure 2. If it appears twice, on blank page and on Figure 2, the blank page takes precedence. Be tolerant of slips on shape especially for the M1. Also do not penalise changes in height as we need to mark this attempt in exactly the same way as an attempt on its own.

Q9.

Question Number	Scheme		Marks
(a)	$y = 16x\sqrt{x} - 3x^2 - 78 = 16x^{\frac{3}{2}} - 3x^2 - 78$		
	$\frac{dy}{dx} = 24x^{\frac{1}{2}} - 6x$		
	Correct index for either term in x so $16x\sqrt{x} \to \alpha x^{\frac{1}{2}}$ or $-3x^2 \to \beta x$		M1
	Any one term correct and simplified e.g. $24x^{\frac{1}{2}}$ (or $24\sqrt{x}$) or $-6x$		A1
	$\left(\frac{dy}{dx} =\right) 24x^{\frac{1}{2}} - 6x$ Correct expression with no 'extra' terms e.g. '+ c' Allow $24\sqrt{x}$ for $24x^{\frac{1}{2}}$ and allow $-6x^1$ Apply isw once a correct answer is seen		A1
			[3]
(b)	$x = 4 \Rightarrow y = 2$	States or uses $y = 2$	B1
	$x = 4 \Rightarrow \frac{dy}{dx} = 24 \times 4^{\frac{1}{2}} - 6 \times 4 (= 24)$	Substitutes $x = 4$ into their $\frac{dy}{dx}$	M1
	$m_N = -\frac{1}{\frac{dy}{dx}} = \left(-\frac{1}{24}\right)$	Correct method for finding gradient of normal. **Dependent on the previous method mark.**	dM1
	E.g. $y - "2" = "-\frac{1}{24}"(x-4)$ or $\frac{y - "2"}{x-4} = "-\frac{1}{24}"$ or $y = mx + c \Rightarrow "2" = "-\frac{1}{24}" \times 4 + c \Rightarrow c = ...$ Correct method for finding the equation of the normal **<u>with $x = 4$ and their $y = 2$, which has come from an attempt at y when $x = 4$, correctly placed.</u>** **Dependent on both previous method marks.**		ddM1
	$x + 24y - 52 = 0$	$x + 24y - 52 = 0$ or $\pm k(x + 24y - 52) = 0, \quad k \in \mathbb{N}$ <u>Must see the equation not just values of a, b, c stated.</u>	A1
			[5]
			(8 marks)

Q10.

Question Number	Scheme	Marks
(a)	$\frac{5x^2+4}{2\sqrt{x}} = \frac{5}{2}x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$	B1
	$f(x) = \frac{\frac{5}{2}x^{\frac{5}{2}}}{\frac{5}{2}} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} - 5x \quad (+c)$	M1 A1A1
	Uses f(4) = 14 to find $c =$	
	$c = -6$ and so $f(x) = x^{\frac{5}{2}} + 4x^{\frac{1}{2}} - 5x - 6$ o.e. e.g $x^{\frac{1}{2}}(x^2+4) - 5x - 6$	dM1A1
		[6]
(b)	Gradient of curve at (4, 14) is $f'(4) = \frac{84}{4} - 5 = 16$	M1 A1
	So $(y - 14) =$ '16' $(x - 4)$ and $y = 16x - 50$	dM1 A1
		[4]
		10 marks

(a)

B1: $\frac{5x^2+4}{2\sqrt{x}} = \frac{5}{2}x^{\frac{3}{2}} + 2x^{-\frac{1}{2}}$ which may have un simplified coefficients. Allow decimal indices. This B mark may be implied by later work.

M1: Attempt to integrate - one power, even if incorrect, increased by one. Usually scored for $-5 \to -5x$

Allow for $x^{\frac{3}{2}} \to x^{\frac{3}{2}+1}$

Do not award if the candidate integrates the numerator and denominator without first attempting division.

A1: **Two** of the three terms in x correct un-simplified or simplified– (ignore no constant here).

The indices must now be simplified / calculated.

A1: **All three terms** correct **un-simplified. There is no need to have +** c

dM1: Uses $x = 4$ when f(x) = 14 to find numerical value for c (may make slips). They must have attempted to integrate.

A1cao: All four terms **correct simplified** with **-6 included.** You may condone the omission of f (x) =

(b)

M1: For an attempt to substitute $x = 4$ into $f'(x) = \frac{5x^2+4}{2\sqrt{x}} - 5$ or their 'simplified' function from (a). Also allow a candidate to differentiate their answer to part (a) and substitute $x = 4$ in the result.

Look for evidence but allow f ' (4) =... Condone slips (eg. forgetting to subtract 5) BUT do not allow this if an incorrect value just appears from nowhere.

A1: Get f '(4) = 16

dM1: Linear equation with their gradient through (4,14). It must be their $f'(4)$ and not a "normal"

If they use the form $y = mx + c$ they must proceed as far as $c = \ldots$

A1: cao: $y = 16x - 50$

Q11.

Question Number	Scheme		Marks
(a)	$f(x)=8x^{-1}+\frac{1}{2}x-5$ $\Rightarrow f'(x)=-8x^{-2}+\frac{1}{2}$	M1: $-8x^{-2}$ or $\frac{1}{2}$ A1: Fully correct $f'(x)=-8x^{-2}+\frac{1}{2}$ **(may be un-simplified)**	M1A1
	Sets $-8x^{-2}+\frac{1}{2}=0 \Rightarrow x=4$	M1: Sets their $f'(x)=0$ i.e. a "changed" function (may be implied by their work) and proceeds to find x. A1: $x=4$ (Allow $x=\pm 4$)	M1A1
	$(4,-1)$	Correct coordinates (allow $x=4$, $y=-1$). Ignore their (-4, …)	A1
			(5)
(b)(i)	$(x=)2, 8$	$x=2$ and $x=8$ **only**. Do **not** accept as coordinates here.	B1
(b)(ii)	$(4,\ 1)$	$(4,\ 1)$ or follow through on their solution in (a). Accept $(x, y+2)$ from their (x, y). **With no other points.**	B1ft
(b)(iii)	$(x=)2, \frac{1}{2}$	Both answers are needed and **accept** $(2,\ 0), \left(\frac{1}{2}, 0\right)$ here. Ignore any reference to the image of the turning point.	B1
			(3)
			(8 marks)