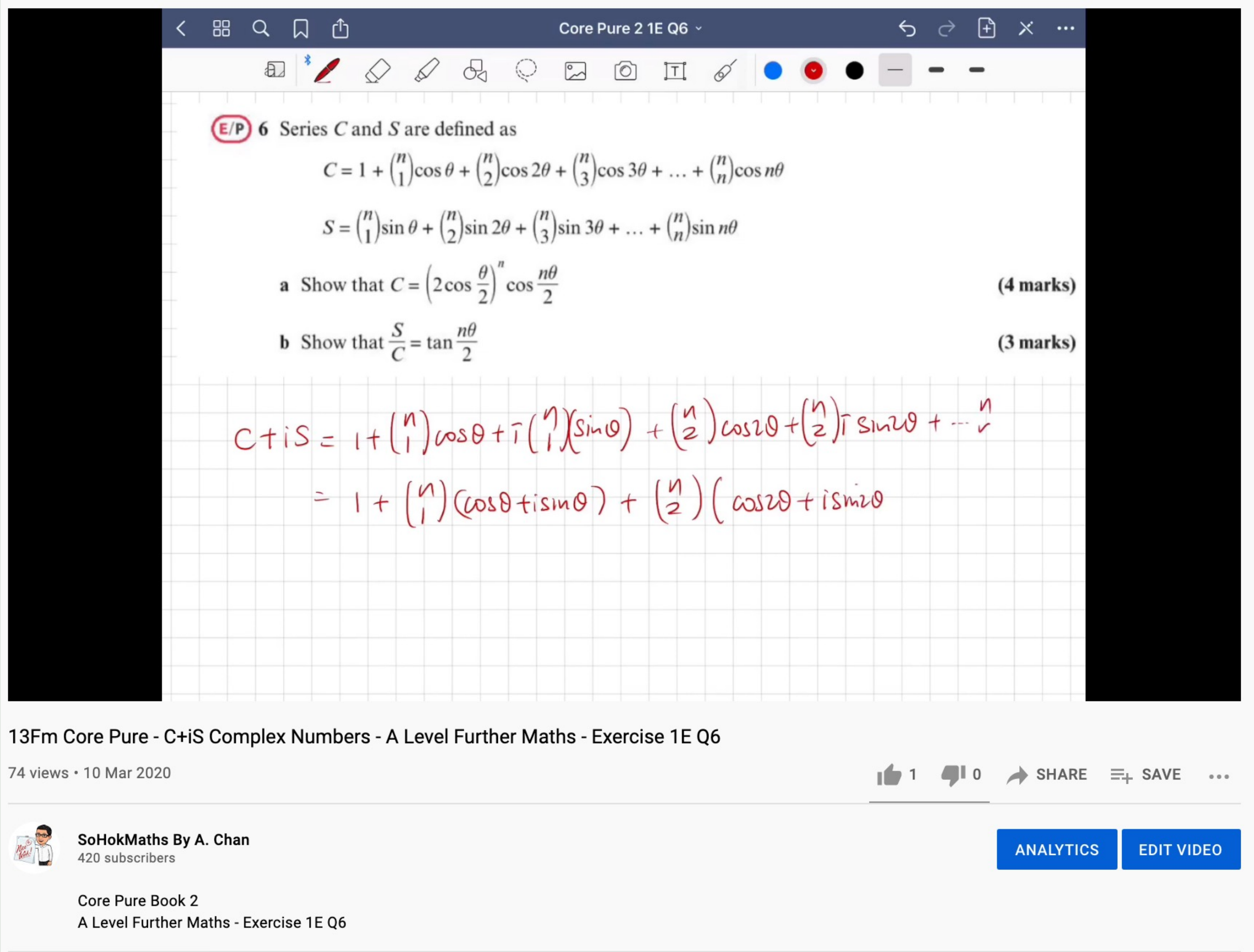
Mr. Chan's 13FM Complex Numbers Questions by Topic Pack

https://www.youtube.com/watch?v=MPIcHpTs_gI



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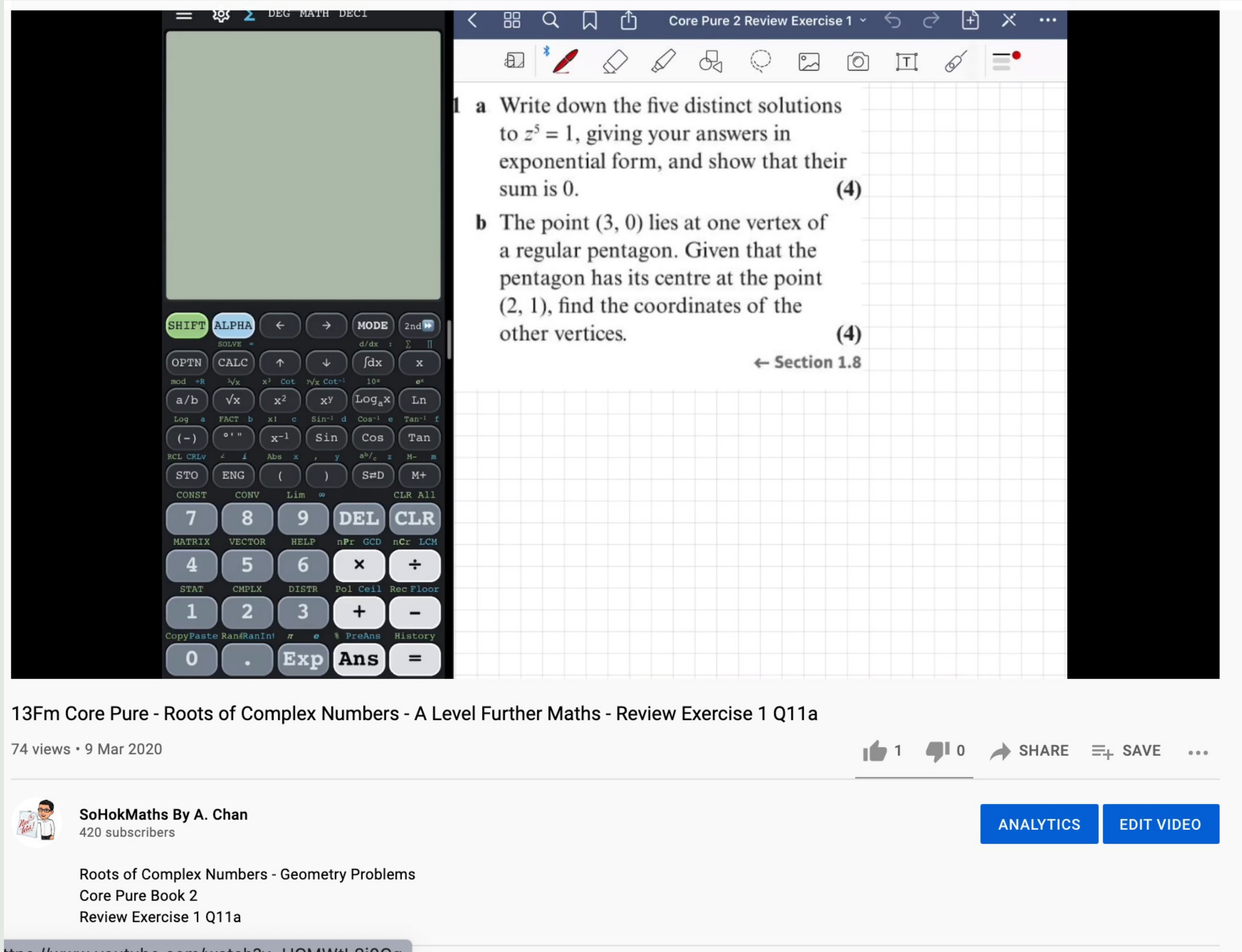

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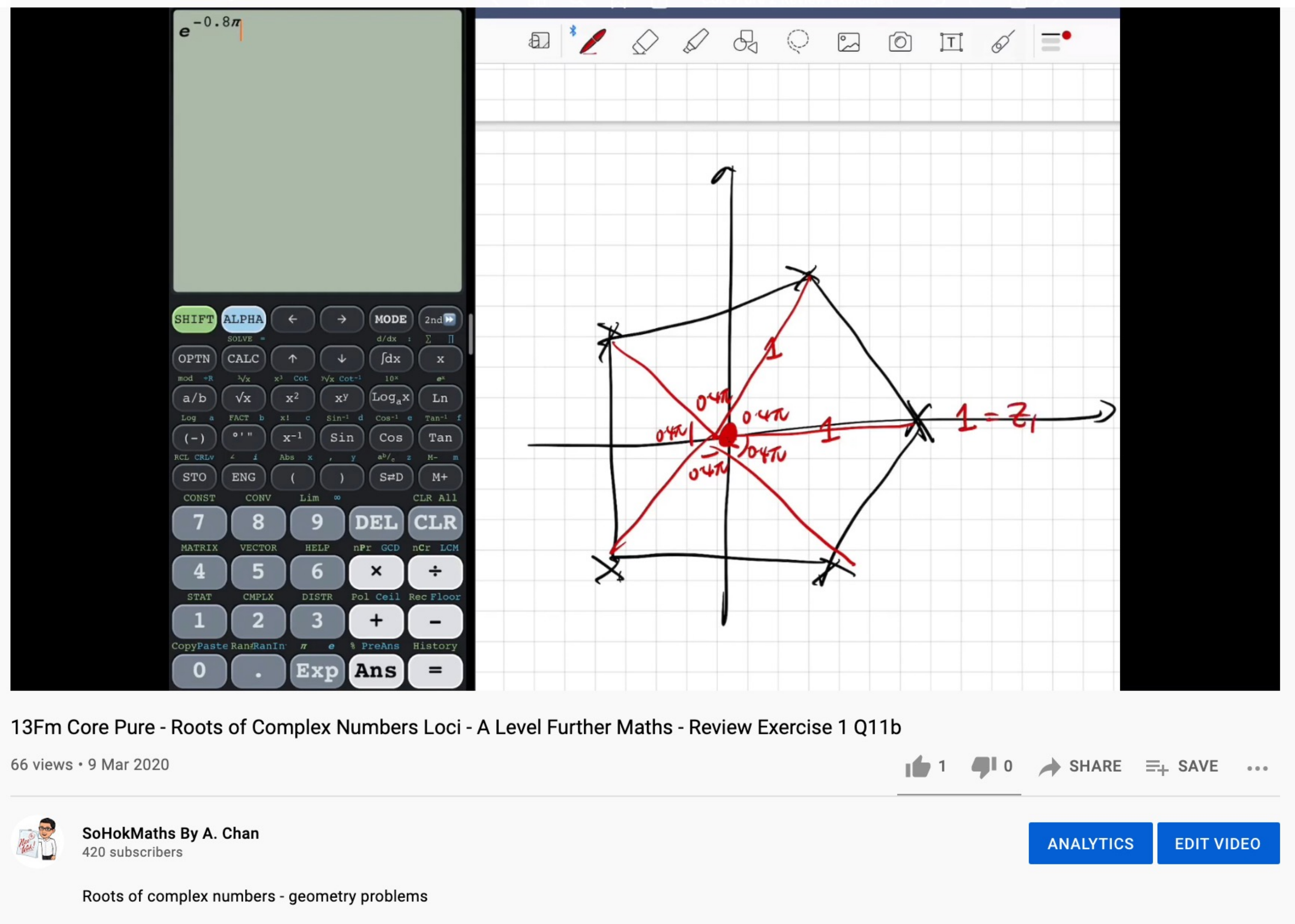

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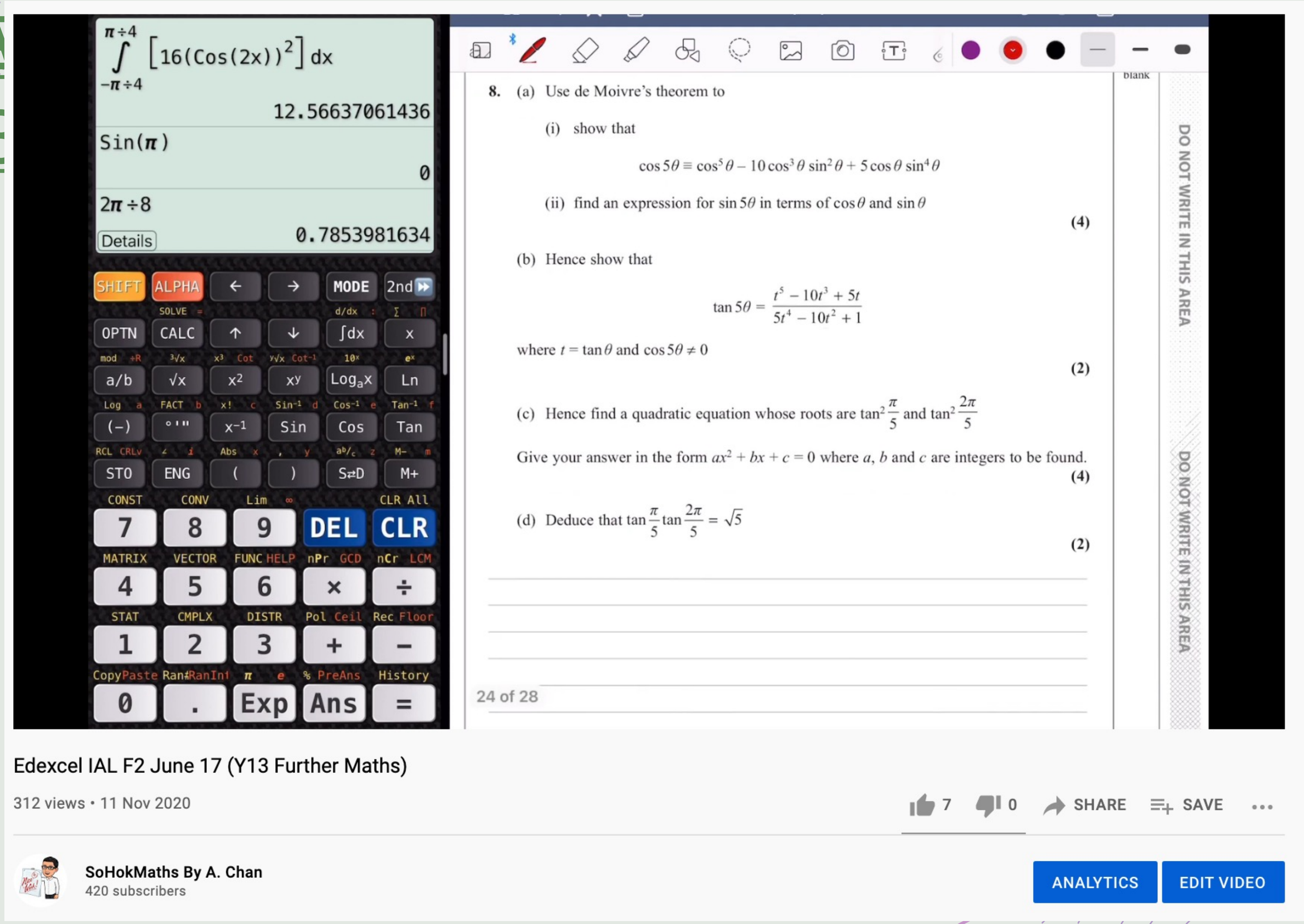
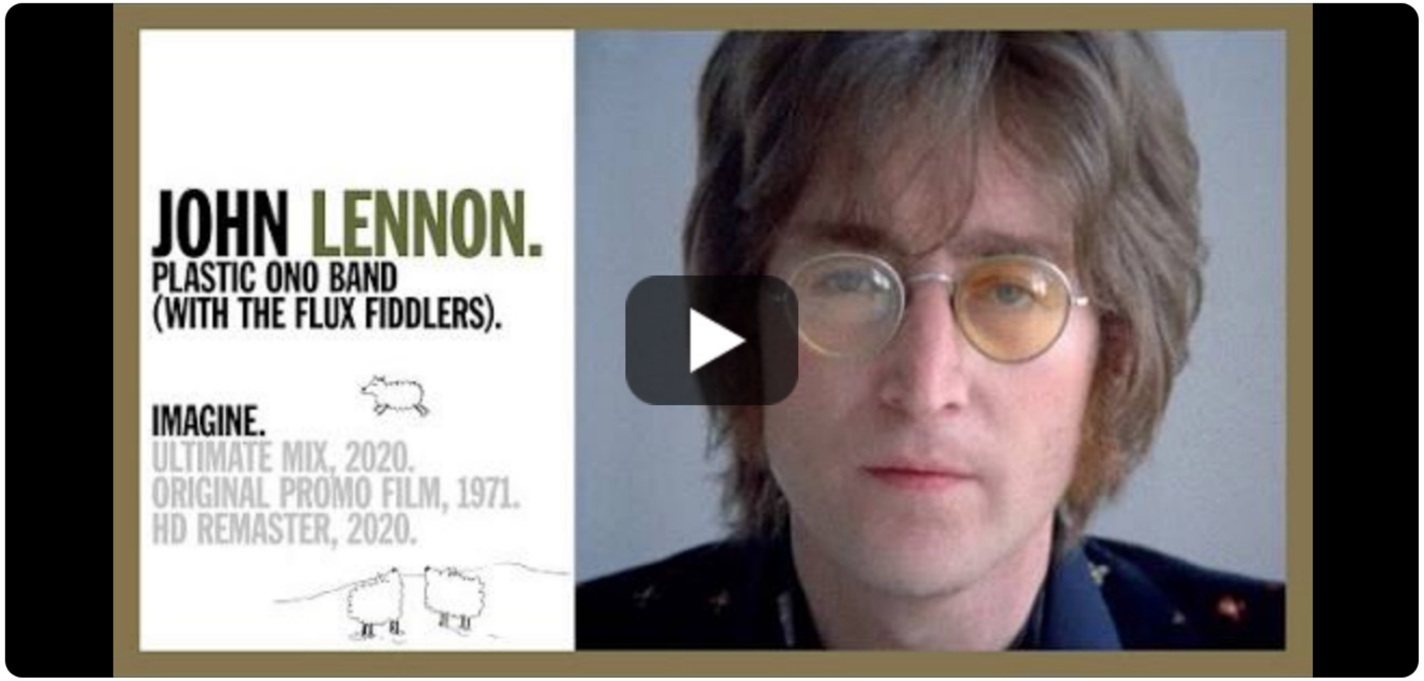


https://www.youtube.com/watch?v=YkgkThdzX-8



IMAGINE. (Ultimate Mix, 2020) - John Lennon & The Plastic ...

https://www.youtube.com › watch

Lyrics

Imagine there's no heaven
It's easy if you try
No hell below us
Above us only sky… More

There are three big style of A2 complex numbers questions:
1) General Trigonometry/Binomial
2) C+iS
3) Geometric Problems (nth Roots)

+ MEI uses **j** instead **i**
+ I have included the full question so other "parts" of the question may be "cross topic" links
+ Questions from Old Spec MEI FP2, OCR FP3, AQA FP2, IAL Edexcel F2, Edexcel FP2
+ Not all questions are included

General Trigonometry/Binomial

- 4. Edexcel IAL June 2015 FP2
- 8. Edexcel IAL June 2016 F2
- 12. Edexcel June 2011 FP2
- 15. Edexcel June 2013 FP2
- 19. MEI JUNE 2006 FP2
- 22. AQA JAN 2006 FP2
- 25. AQA JUNE 2006 FP2
- 28. AQA JAN 2007 FP2
- 31. AQA JAN 2008 FP2
- 34. AQA JUNE 2008 FP2
- 37. AQA JUNE 2009 FP2
- 39. AQA JUNE 2011 FP2
- 41. Aqa June 2012 FP2
- 44. AQA JUNE 2013 FP2

C+IS

- 48. MEI Jan 2006 FP2
- 51. MEI Jan 2007 FP2
- 54. MEI Jan 2008 FP2
- 57. MEI JUNE 2009 FP2
- 60. MEI JAN 2010 FP2
- 63. MEI JAN 2012 FP2
- 66. MEI JUNE 2012 FP2
- 69. OCR JAN 2008 FP3
- 71. Ocr June 2010 FP3
- 73. OCR JAN 2013 FP3

Geometric Problems (nth Roots)

- 77. OCR JUNE 2007 FP3
- 79. OCR JUNE 2010 FP3
- 81. OCR JAN 2011 FP3
- 83. AQA JAN 2010 FP2
- 85. AQA JAN 2011 FP2
- 88. AQA JAN 2013 FP2
- 91. AQA JAN 2007 FP2
- 94. AQA JAN 2010 Q8 FP2
- 96. AQA JAN 2013 FP2
- 99. MEI JUNE 2007 FP2
- 102. MEI JAN 2009 FP2
- 105. MEI JUNE 2010 FP2
- 108. MEI JUNE 2011 FP2
- 111. MEI JUNE 2012 FP2

(a) Show that

$$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3 = z^6 - \frac{1}{z^6} - k\left(z^2 - \frac{1}{z^2}\right)$$

where k is a constant to be found.

(3)

Given that $z = \cos\theta + i\sin\theta$, where θ is real,

(b) show that

(i) $z^n + \frac{1}{z^n} = 2\cos n\theta$

(ii) $z^n - \frac{1}{z^n} = 2i\sin n\theta$

(3)

(c) Hence show that

$$\cos^3\theta\sin^3\theta = \frac{1}{32}(3\sin 2\theta - \sin 6\theta)$$

(4)

(d) Find the exact value of

$$\int_0^{\frac{\pi}{8}} \cos^3\theta\sin^3\theta\, d\theta$$

(4)

Edexcel IAL June 2015 FP2

Imagine having general Trigonometry/ Binomial in your complex HOME...

Question Number	Scheme	Notes	Marks
(a)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=\left(z^2-\frac{1}{z^2}\right)^3$		
	$=z^6-3z^2+\frac{3}{z^2}-z^{-6}$	M1: Attempt to expand A1: Correct expansion	M1A1
	$=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	Correct answer with no errors seen	A1
			(3)
(a) ALT	$\left(z+\frac{1}{z}\right)^3=z^3+3z+\frac{3}{z}+\frac{1}{z^3},\ \left(z-\frac{1}{z}\right)^3=z^3-3z+\frac{3}{z}-\frac{1}{z^3}$		M1A1
	M1: Attempt to expand both cubic brackets A1: Correct expansions		
	$=z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)$	Correct answer with no errors	A1
			(3)
			(3)

			(3)
(b)(i)(ii)	$z^n = \cos n\theta + i\sin n\theta$	Correct application of de Moivre	B1
	$z^{-n} = \cos(-n\theta) + i\sin(-n\theta) = \pm\cos n\theta \pm \sin n\theta$ but must be different from their z^n	Attempt z^{-n}	M1
	$z^n + \frac{1}{z^n} = 2\cos n\theta$*, $z^n - \frac{1}{z^n} = 2i\sin n\theta$*	$z^{-n} = \cos n\theta - i\sin n\theta$ must be seen	A1*
			(3)
(c)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3 = (2\cos\theta)^3(2i\sin\theta)^3$		B1
	$z^6 - \frac{1}{z^6} - 3\left(z^2 - \frac{1}{z^2}\right) = 2i\sin 6\theta - 6i\sin 2\theta$	Follow through their k in place of 3	B1ft
	$-64i\sin^3\theta\cos^3\theta = 2i\sin 6\theta - 6i\sin 2\theta$	Equating right hand sides and simplifying $2^3 \times (2i)^3$ (B mark needed for each side to gain M mark)	M1
	$\cos^3\theta\sin^3\theta = \frac{1}{32}(3\sin 2\theta - \sin 6\theta)$ *		A1cso
			(4)

(c)	$\left(z+\frac{1}{z}\right)^3\left(z-\frac{1}{z}\right)^3=(2\cos\theta)^3(2i\sin\theta)^3$		B1
	$z^6-\frac{1}{z^6}-3\left(z^2-\frac{1}{z^2}\right)=2i\sin 6\theta-6i\sin 2\theta$	Follow through their k in place of 3	B1ft
	$-64i\sin^3\theta\cos^3\theta=2i\sin 6\theta-6i\sin 2\theta$	Equating right hand sides and simplifying $2^3\times(2i)^3$ (B mark needed for each side to gain M mark)	M1
	$\cos^3\theta\sin^3\theta=\frac{1}{32}(3\sin 2\theta-\sin 6\theta)$ *		A1cso
			(4)

(d)	$\int_0^{\frac{\pi}{8}}\cos^3\theta\sin^3\theta\,d\theta=\int_0^{\frac{\pi}{8}}\frac{1}{32}(3\sin 2\theta-\sin 6\theta)\,d\theta$		
	$=\frac{1}{32}\left[-\frac{3}{2}\cos 2\theta+\frac{1}{6}\cos 6\theta\right]_0^{\frac{\pi}{8}}$	M1: $p\cos 2\theta+q\cos 6\theta$ A1: Correct integration Differentiation scores M0A0	M1A1
	$=\frac{1}{32}\left[\left(-\frac{3}{2\sqrt{2}}-\frac{1}{6\sqrt{2}}\right)-\left(-\frac{3}{2}+\frac{1}{6}\right)\right]=\frac{1}{32}\left(\frac{4}{3}-\frac{5\sqrt{2}}{6}\right)$	dM1: Correct use of limits – lower limit to have non-zero result. Dep on previous M mark A1: Cao (oe) but must be exact	dM1A1
			(4)
			Total 14

Edexcel IAL June 2016 F2

Imagine having general Trigonometry/ Binomial in your complex HOME…

(a) Use de Moivre's theorem to show that

$$\cos^5\theta \equiv p\cos 5\theta + q\cos 3\theta + r\cos\theta$$

where p, q and r are rational numbers to be found.

(6)

(b) Hence, showing all your working, find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos^5\theta \, d\theta$$

(4)

(Total for question = 10 marks)

(a) WAY 1	$\left(z+\frac{1}{z}\right)^5 = z^5+5z^3+10z+\frac{10}{z}+\frac{5}{z^3}+\frac{1}{z^5}$	M1: Attempt to expand $\left(z\pm\frac{1}{z}\right)^5$ A1: Correct expansion with correct powers of z.	M1A1
	$z=\cos\theta+\mathrm{i}\sin\theta \Rightarrow z+\frac{1}{z}=2\cos\theta$	May be implied	B1
	$=z^5+\frac{1}{z^5}+5\left(z^3+\frac{1}{z^3}\right)+10\left(z+\frac{1}{z}\right)=2\cos 5\theta+10\cos 3\theta+20\cos\theta$ Uses at least one of $z^5+\frac{1}{z^5}=2\cos 5\theta$ or $z^3+\frac{1}{z^3}=2\cos 3\theta$		M1
	$\left(z+\frac{1}{z}\right)^5=32\cos^5\theta$		B1
	$\cos^5\theta=\frac{1}{16}\cos 5\theta+\frac{5}{16}\cos 3\theta+\frac{5}{8}\cos\theta$	Correct expression	A1
			(6)

	WAY 2 (Using $e^{i\theta}$)		
	$\left(e^{i\theta}+e^{-i\theta}\right)^5=e^{5i\theta}+5e^{3i\theta}+10e^{i\theta}+10e^{-i\theta}+5e^{-3i\theta}+e^{-5i\theta}$	M1: Attempt to expand $\left(e^{i\theta}\pm e^{-i\theta}\right)^5$ A1: Correct expansion	M1A1
	$2\cos\theta=e^{i\theta}+e^{-i\theta}$	May be implied	B1
	$=e^{5i\theta}+e^{-5i\theta}+5\left(e^{3i\theta}+e^{-3i\theta}\right)+10\left(e^{i\theta}+e^{-i\theta}\right)=2\cos 5\theta+10\cos 3\theta+20\cos\theta$ Uses one of $e^{5i\theta}+e^{-5i\theta}=2\cos 5\theta$ or $e^{3i\theta}+e^{-3i\theta}=2\cos 3\theta$		M1
	$\left(e^{i\theta}+e^{-i\theta}\right)^5=32\cos^5\theta$		B1
	$\cos^5\theta=\frac{1}{16}\cos 5\theta+\frac{5}{16}\cos 3\theta+\frac{5}{8}\cos\theta$	Correct expression	A1

	WAY 3 (Using De Moivre on cos 5θ and identity for cos 3θ)		
	$(\cos\theta + i\sin\theta)^5 = c^5 + 5ic^4s + 10c^3i^2s^2 + 10c^2i^3s^3 + 5ci^4s^4 + i^5s^5$ M1: Attempts to expand. NB may only consider real parts here. A1: Correct real terms (may include i's) (Ignore imaginary parts for this mark)		M1A1
	$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$	Correct real terms with no i's	B1
	$= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$	Uses $\sin^2\theta = 1-\cos^2\theta$ to eliminate $\sin\theta$	M1
	$16\cos^5\theta = \cos 5\theta + 20\cos^3\theta - 5\cos\theta$		
	$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$	Correct identity for cos 3θ	B1
	$16\cos^5\theta = \cos 5\theta + 5\cos 3\theta + 10\cos\theta$		
	$\cos^5\theta = \frac{1}{16}\cos 5\theta + \frac{5}{16}\cos 3\theta + \frac{5}{8}\cos\theta$	Correct expression	A1
			(6)

(b)	$\int\left(\frac{1}{16}\cos 5\theta+\frac{5}{16}\cos 3\theta+\frac{5}{8}\cos\theta\right)\mathrm{d}\theta=\frac{1}{80}\sin 5\theta+\frac{5}{48}\sin 3\theta+\frac{5}{8}\sin\theta$ M1: Attempt to integrate – Evidence of $\cos n\theta\rightarrow\pm\frac{1}{n}\sin n\theta$ where $n=5$ or 3 A1ft: Correct integration (ft their p, q, r)		M1A1ft
	$\left[\frac{1}{80}\sin 5\theta+\frac{5}{48}\sin 3\theta+\frac{5}{8}\sin\theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}}=\left(\frac{1}{80}\sin\frac{5\pi}{3}+\frac{5}{48}\sin\pi+\frac{5}{8}\sin\frac{\pi}{3}\right)-\left(\frac{1}{80}\sin\frac{5\pi}{6}+\frac{5}{48}\sin\frac{\pi}{2}+\frac{5}{8}\sin\frac{\pi}{6}\right)$ Substitutes the given limits into a changed function and subtracts the right way round. There should be evidence of the substitution of $\frac{\pi}{3}$ and $\frac{\pi}{6}$ into their changed function for at least 2 of their terms and subtraction. If there is no evidence of substitution and the answer is incorrect, score M0 here.		M1
	$=\frac{49\sqrt{3}}{160}-\frac{203}{480}$	Allow exact equivalents e.g. $=\frac{1}{16}\left(4.9\sqrt{3}-\frac{203}{30}\right)$	A1
	If they use the letters *p*, *q* and *r* or their values of *p*, *q* and *r*, even from no working, the M marks are available in (b) but <u>not</u> the A marks.		
			(4)
			Total 10

Edexcel June 2011 FP2

Imagine having general Trigonometry/ Binomial in your complex HOME…

(a) Use de Moivre's theorem to show that

$$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$$

(5)

Hence, given also that $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$,

(b) find all the solutions of

$$\sin 5\theta = 5\sin 3\theta,$$

in the interval 0 ≤ θ < 2π. Give your answers to 3 decimal places.

(6)

(Total 11 marks)

Question Number	Scheme	Marks
(a)	$\sin 5\theta = \text{Im}(\cos\theta + \text{i}\sin\theta)^5$	B1
	$5\cos^4\theta(\text{i}\sin\theta) + 10\cos^2\theta(\text{i}^3\sin^3\theta) + \text{i}^5\sin^5\theta$	M1
	$= \text{i}(5\cos^4\theta\sin\theta - 10\cos^2\theta\sin^3\theta + \sin^5\theta)$	A1
	$\left(\text{Im}(\cos\theta + \text{i}\sin\theta)^5\right) = 5\sin\theta(1-\sin^2\theta)^2 - 10\sin^3\theta(1-\sin^2\theta) + \sin^5\theta$	M1
	$\sin 5\theta = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta \quad (*)$	A1cso
		(5)

(b)	$16\sin^5\theta - 20\sin^3\theta + 5\sin\theta = 5\left(3\sin\theta - 4\sin^3\theta\right)$	M1	
	$16\sin^5\theta - 10\sin\theta = 0$	M1	
	$\sin^4\theta = \frac{5}{8} \qquad \theta = 1.095$	A1	
	Inclusion of solutions from $\sin\theta = -\sqrt[4]{\frac{5}{8}}$	M1	
	Other solutions: $\theta = 2.046,\ 4.237,\ 5.188$	A1	
	$\sin\theta = 0 \Rightarrow \theta = 0,\ \theta = \pi\ (3.142)$	B1	
			(6) **11**
(a) (b)	Award B if solution considers Imaginary parts and equates to $\sin 5\theta$ 1st M1 for correct attempt at expansion and collection of imaginary parts 2nd M1 for substitution powers of $\cos\theta$ 1st M for substituting correct expressions 2nd M for attempting to form equation Imply 3rd M if 4.237 or 5.188 seen. Award for their negative root. Ignore 2π but 2nd A0 if other extra solutions given.		

Edexcel June 2013 FP2

The complex number $z = e^{i\theta}$, where θ is real.

(a) Use de Moivre's theorem to show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

where n is a positive integer.

(2)

(b) Show that

$$\cos^5\theta = \frac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$$

(5)

(c) Hence find all the solutions of

$$\cos 5\theta + 5\cos 3\theta + 12\cos\theta = 0$$

in the interval $0 \leq \theta < 2\pi$

(4)

(Total 11 marks)

Imagine having general Trigonometry/ Binomial in your complex HOME...

(a)	$z^n + z^{-n} = e^{in\theta} + e^{-in\theta}$ $= \cos n\theta + i\sin n\theta + \cos n\theta - i\sin n\theta$ $= 2\cos n\theta$ *	M1A1 (2)

Notes for Question
Question a M1 for using de Moivre's theorem to show that either $z^n = \cos n\theta + i\sin n\theta$ or $z^{-n} = \cos n\theta - i\sin n\theta$ A1 for completing to the given result $z^n + z^n = 2\cos n\theta$ *

(b)	$\left(z+z^{-1}\right)^5 = 32\cos^5\theta$	B1
	$\left(z+z^{-1}\right)^5 = z^5+5z^3+10z+10z^{-1}+5z^{-3}+z^{-5}$	M1A1
	$32\cos^5\theta = \left(z^5+z^{-5}\right)+5\left(z^3+z^{-3}\right)+10\left(z+z^{-1}\right)$	
	$= 2\cos 5\theta + 10\cos 3\theta + 20\cos\theta$	M1
	$\cos^5\theta = \frac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$ *	A1
		(5)

Question b

B1 for using the result in (a) to obtain $\left(z+z^{-1}\right)^5 = 32\cos^5\theta$ Need not be shown explicitly.

M1 for attempting to expand $\left(z+z^{-1}\right)^5$ by binomial, Pascal's triangle or multiplying out the brackets. If ${}^nC_r$ is used do not award marks until changed to numbers

A1 for a correct expansion $\left(z+z^{-1}\right)^5 = z^5+5z^3+10z+10z^{-1}+5z^{-3}+z^{-5}$

M1 for replacing $\left(z^5+z^{-5}\right), \left(z^3+z^{-3}\right), \left(z+z^{-1}\right)$ with $2\cos 5\theta, 2\cos 3\theta, 2\cos\theta$ and equating their revised expression to their result for $\left(z+z^{-1}\right)^5 = 32\cos^5\theta$

A1cso for $\cos^5\theta = \frac{1}{16}\left(\cos 5\theta + 5\cos 3\theta + 10\cos\theta\right)$ *

(c)	$\cos 5\theta + 5\cos 3\theta + 10\cos\theta = -2\cos\theta$	M1
	$16\cos^5\theta = -2\cos\theta$	A1
	$2\cos\theta\left(8\cos^4\theta + 1\right) = 0$	
	$8\cos^4\theta + 1 = 0$ no solution	B1
	$\cos\theta = 0$	
	$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$	A1
		(4)
		11 Marks

Question c

M1 for attempting re-arrange the equation with one side matching the bracket in the result in (b) Question states "hence", so no other method is allowed.

A1 for using the result in (b) to obtain $16\cos^5\theta = -2\cos\theta$ oe

B1 for stating that there is no solution for $8\cos^4\theta + 1 = 0$ oe eg $8\cos^4\theta + 1 \neq 0$ $8\cos^4\theta + 1 > 0$ or "ignore" but $\cos\theta = \sqrt[4]{-\frac{1}{8}}$ without comment gets B0

A1 for $\theta = \frac{\pi}{2}$ **and** $\frac{3\pi}{2}$ and no more in the range. **Must** be in radians, can be in decimals (1.57..., 4.71... 3 sf or better)

Imagine having general Trigonometry/Binomial in your complex HOME…

MEI JUNE 2006 FP2

2 **(a)** **(i)** Given that $z = \cos\theta + \mathrm{j}\sin\theta$, express $z^n + \frac{1}{z^n}$ and $z^n - \frac{1}{z^n}$ in simplified trigonometric form. [2]

(ii) By considering $\left(z - \frac{1}{z}\right)^4 \left(z + \frac{1}{z}\right)^2$, find A, B, C and D such that

$$\sin^4\theta\cos^2\theta = A\cos 6\theta + B\cos 4\theta + C\cos 2\theta + D.$$ [6]

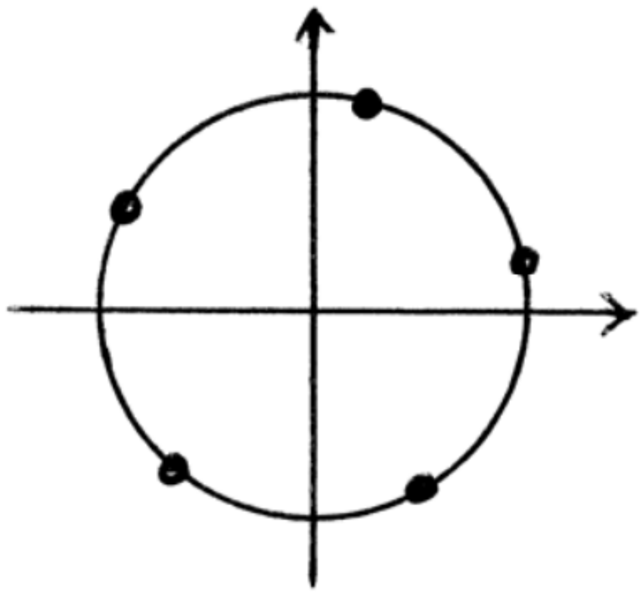
(b) **(i)** Find the modulus and argument of $4 + 4\mathrm{j}$. [2]

(ii) Find the fifth roots of $4 + 4\mathrm{j}$ in the form $r\mathrm{e}^{\mathrm{j}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$.

Illustrate these fifth roots on an Argand diagram. [6]

(iii) Find integers p and q such that $(p + q\mathrm{j})^5 = 4 + 4\mathrm{j}$. [2]

2 (a)(i)	$z^n + \frac{1}{z^n} = 2\cos n\theta$, $z^n - \frac{1}{z^n} = 2\mathrm{j}\sin n\theta$	B1B1 **2**	
(ii)	$\left(z - \frac{1}{z}\right)^4 \left(z + \frac{1}{z}\right)^2 = 64\sin^4\theta\cos^2\theta$ $= z^6 - 2z^4 - z^2 + 4 - \frac{1}{z^2} - \frac{2}{z^4} + \frac{1}{z^6}$ $= 2\cos 6\theta - 4\cos 4\theta - 2\cos 2\theta + 4$ $\sin^4\theta\cos^2\theta = \frac{1}{32}\cos 6\theta - \frac{1}{16}\cos 4\theta - \frac{1}{32}\cos 2\theta + \frac{1}{16}$ $(A = \frac{1}{32},\ B = -\frac{1}{16},\ C = -\frac{1}{32},\ D = \frac{1}{16})$	B1 M1 A1 M1 A1 ft A1 **6**	Expansion $z^6 + \ldots + z^{-6}$ Using $z^n + \frac{1}{z^n} = 2\cos n\theta$ with $n = 2,\ 4$ or 6. Allow M1 if used in partial expansion, or if 2 omitted, etc
(b)(i)	$\lvert 4 + 4\mathrm{j} \rvert = \sqrt{32}$, $\arg(4 + 4\mathrm{j}) = \frac{1}{4}\pi$	B1B1 **2**	Accept 5.7; 0.79, $45°$

(ii)	$r=\sqrt{2}$ $\theta=-\frac{3}{4}\pi,\quad -\frac{7}{20}\pi,\quad \frac{1}{20}\pi,\quad \frac{9}{20}\pi,\quad \frac{17}{20}\pi$ 	B1 B3 B2	 **6**	Accept $32^{\frac{1}{10}}$, 1.4, $\sqrt[5]{4\sqrt{2}}$ etc Accept $-2.4, -1.1, 0.16, 1.4, 2.7$ Give B2 for three correct Give B1 for one correct Deduct 1 mark (maximum) if degrees used $(-135°,\ -63°,\ 9°,\ 81°,\ 153°)$ $\frac{1}{20}\pi+\frac{2}{5}k\pi$ earns B2; with $k=-2,\ -1,\ 0,\ 1,\ 2$ earns B3 Give B1 for four points correct, or B1 ft for five points
(iii)	$\sqrt{2}e^{-\frac{3}{4}\pi\,j}=\sqrt{2}\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}j\right)$ $=-1-j$ $p=-1,\ q=-1$	M1 A1	 **2**	Exact evaluation of a fifth root Give B2 for correct answer stated or obtained by any other method

AQA JAN 2006 FP2

Imagine having general Trigonometry/Binomial in your complex HOME...

6 It is given that $z = e^{i\theta}$.

(a) (i) Show that

$$z + \frac{1}{z} = 2\cos\theta$$

(2 marks)

(ii) Find a similar expression for

$$z^2 + \frac{1}{z^2}$$

(2 marks)

(iii) Hence show that

$$z^2 - z + 2 - \frac{1}{z} + \frac{1}{z^2} = 4\cos^2\theta - 2\cos\theta$$

(3 marks)

(b) Hence solve the quartic equation

$$z^4 - z^3 + 2z^2 - z + 1 = 0$$

giving the roots in the form $a + ib$.

(5 marks)

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$z+\frac{1}{z}=\cos\theta+\mathrm{i}\sin\theta+$			Or $z+\frac{1}{z}=\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}$
	$\cos(-\theta)+\mathrm{i}\sin(-\theta)$	M1		
	$=2\cos\theta$	A1	2	AG
(ii)	$z^2+\frac{1}{z^2}=\cos 2\theta+i\sin 2\theta$			
	$+\cos(-2\theta)+\mathrm{i}\sin(-2\theta)$	M1		
	$=2\cos 2\theta$	A1	2	OE
(iii)	$z^2-z+2-\frac{1}{z}+\frac{1}{z^2}$			
	$=2\cos 2\theta-2\cos\theta+2$	M1		
	Use of $\cos 2\theta=2\cos^2\theta-1$	m1		
	$=4\cos^2\theta-2\cos\theta$	A1	3	AG

(b)	$z+\frac{1}{z}=0 \qquad z=\pm\mathrm{i}$	M1A1		**Alternative:**
	$z+\frac{1}{z}=1 \qquad z^2-z+1=0$	M1A1		$\cos\theta=0 \qquad \theta=\pm\frac{1}{2}\pi$ M1 $z=\pm\mathrm{i}$ A1 $\cos\theta=\frac{1}{2} \qquad \theta=\pm\frac{1}{3}\pi$ M1 $z=\mathrm{e}^{\pm\frac{1}{3}\pi\mathrm{i}}=\frac{1}{2}\left(1\pm\mathrm{i}\sqrt{3}\right)$ A1 A1
	$z=\frac{1\pm\mathrm{i}\sqrt{3}}{2}$ Accept solution to (b) if done otherwise	A1F	5	
	Alternative If $\theta=+\frac{1}{2}\pi \quad \theta=\frac{1}{3}\pi$	M1		
	$z=\mathrm{i} \quad z=\frac{1+\sqrt{3}\,\mathrm{i}}{2}$	A1		
	Or any correct z values of θ Any 2 correct answers One correct answer only	M1 A1 B1		
	Total		**12**	

7 (a) six roots of the equation $z^6 = 1$, giving your answers in the form $e^{i\phi}$, where $-\pi < \phi \leqslant \pi$. *(3 marks)*

(b) It is given that $w = e^{i\theta}$, where $\theta \neq n\pi$.

(i) Show that $\dfrac{w^2 - 1}{w} = 2i\sin\theta$. *(2 marks)*

(ii) Show that $\dfrac{w}{w^2 - 1} = -\dfrac{i}{2\sin\theta}$. *(2 marks)*

(iii) Show that $\dfrac{2i}{w^2 - 1} = \cot\theta - i$. *(3 marks)*

(iv) Given that $z = \cot\theta - i$, show that $z + 2i = zw^2$. *(2 marks)*

(c) (i) Explain why the equation

$$(z + 2i)^6 = z^6$$

has five roots. *(1 mark)*

(ii) Find the five roots of the equation

$$(z + 2i)^6 = z^6$$

giving your answers in the form $a + ib$. *(4 marks)*

Imagine having general Trigonometry/Binomial in your complex HOME...

AQA JUNE 2006 FP2

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$z = e^{\frac{2k\pi i}{6}}, \quad k = 0, \pm 1, \pm 2, 3$	M1 A2,1,0	3	OE M1A1 only if: (1) range for k is incorrect eg 0,1,2,3,4,5 (2) i is missing
(b)(i)	$\frac{w^2-1}{w} = w - \frac{1}{w} = 2i\sin\theta$	M1A1	2	AG
(ii)	$\frac{w}{w^2-1} = \frac{1}{2i\sin\theta}$	M1		
	$= -\frac{i}{2\sin\theta}$	A1	2	AG
(iii)	$\frac{2i}{w^2-1} = \frac{-2iw^{-1}i}{2\sin\theta}$	M1		Or for $\frac{1}{\sin\theta\, e^{i\theta}}$
	$= \frac{1}{\sin\theta}(\cos\theta - i\sin\theta)$	A1		
	$= \cot\theta - i$	A1	3	AG
(iv)	$z = \frac{2i}{w^2-1}$ Or $z + 2i = \frac{2i}{w^2-1} + 2i$	M1		ie any correct method
	$z + 2i = zw^2$	A1	2	AG

(c)(i)	No coefficient of z^6	E1	1	
(ii)	$\left(w^2\right)^6 = 1 \quad w^2 = e^{\frac{k\pi i}{3}}$	B1		**Alternatively:** $z + 2i = e^{\frac{k\pi i}{3}} z$ B1
	$z = \cot\frac{k\pi}{6} - i\ , \quad k = \pm 1, \pm 2, 3$	M1 A2,1,0	4	$z = \dfrac{2i}{e^{\frac{k\pi i}{3}} - 1}$ M1 roots A2,1,0 (NB roots are $\pm\sqrt{3} - i; \pm\dfrac{1}{\sqrt{3}} - i; -i$)
	Total		**17**	

Imagine having general Trigonometry/Binomial in your complex HOME...

AQA JAN 2007 FP2

5 (a) Prove by induction that, if n is a positive integer,

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$$ *(5 marks)*

(b) Find the value of $\left(\cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right)^6$. *(2 marks)*

(c) Show that

$$(\cos\theta + \mathrm{i}\sin\theta)(1 + \cos\theta - \mathrm{i}\sin\theta) = 1 + \cos\theta + \mathrm{i}\sin\theta$$ *(3 marks)*

(d) Hence show that

$$\left(1 + \cos\frac{\pi}{6} + \mathrm{i}\sin\frac{\pi}{6}\right)^6 + \left(1 + \cos\frac{\pi}{6} - \mathrm{i}\sin\frac{\pi}{6}\right)^6 = 0$$ *(4 marks)*

MFP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)	Assume true for $n=k$			
	$(\cos\theta+\mathrm{i}\sin\theta)^{k+1}$			
	$=(\cos k\theta+\mathrm{i}\sin k\theta)(\cos\theta+\mathrm{i}\sin\theta)$	M1		
	Multiply out	A1		Any form
	$=\cos(k+1)\theta+\mathrm{i}\sin(k+1)\theta$	A1		
	True for $n=1$ shown	B1		
	$P(k)\Rightarrow P(k+1)$ and $P(1)$ true	E1	5	Allow E1 only if previous 4 marks earned
(b)	$\left(\cos\frac{\pi}{6}+\mathrm{i}\sin\frac{\pi}{6}\right)^6=\cos\frac{6\pi}{6}+\mathrm{i}\sin\frac{6\pi}{6}$	M1		
	$=-1$	A1	2	
(c)	$(\cos\theta+\mathrm{i}\sin\theta)(1+\cos\theta-\mathrm{i}\sin\theta)$	M1		
	$=\cos\theta+\cos^2\theta-\mathrm{i}\sin\theta\cos\theta$			
	$+\mathrm{i}\sin\theta+\mathrm{i}\sin\theta\cos\theta+\sin^2\theta$	A1		(Accept $-\mathrm{i}^2\sin^2\theta$)
				Or $\mathrm{e}^{\mathrm{i}\theta}\left(1+\mathrm{e}^{-\mathrm{i}\theta}\right)$
	$=1+\cos\theta+\mathrm{i}\sin\theta$	A1	3	AG

(c)	$(\cos\theta + i\sin\theta)(1+\cos\theta - i\sin\theta)$	M1		
	$= \cos\theta + \cos^2\theta - i\sin\theta\cos\theta$ $+ i\sin\theta + i\sin\theta\cos\theta + \sin^2\theta$	A1		(Accept $-i^2\sin^2\theta$) Or $e^{i\theta}\left(1+e^{-i\theta}\right)$
	$= 1 + \cos\theta + i\sin\theta$	A1	3	AG
(d)	$\theta = \frac{\pi}{6}$ used	M1		In the context of part (c)
	Part (c) raised to power 6	M1		
	Use of result in part (b)	A1		
	$\left(1+\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^6 +$ $\left(1+\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)^6 = 0$	A1	4	AG
	Total		**14**	

6 (a) (i) By applying De Moivre's theorem to $(\cos\theta + i\sin\theta)^3$, show that

$$\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$$ *(3 marks)*

(ii) Find a similar expression for $\sin 3\theta$. *(1 mark)*

(iii) Deduce that

$$\tan 3\theta = \frac{\tan^3\theta - 3\tan\theta}{3\tan^2\theta - 1}$$ *(3 marks)*

(b) (i) Hence show that $\tan\frac{\pi}{12}$ is a root of the cubic equation

$$x^3 - 3x^2 - 3x + 1 = 0$$ *(3 marks)*

(ii) Find two other values of θ, where $0 < \theta < \pi$, for which $\tan\theta$ is a root of this cubic equation. *(2 marks)*

(c) Hence show that

$$\tan\frac{\pi}{12} + \tan\frac{5\pi}{12} = 4$$ *(2 marks)*

AQA JAN 2008 FP2

Imagine having general Trigonometry/ Binomial in your complex HOME...

6(a)(i)	$\cos 3\theta + i\sin 3\theta = (\cos\theta + i\sin\theta)^3$	M1		
	$= \cos^3\theta + 3i\cos^2\theta\sin\theta + 3i^2\cos\theta\sin^2\theta$ $+ i^3\sin^3\theta$	A1		
	Real parts: $\cos 3\theta = \cos^3\theta - 3\cos\theta\sin^2\theta$	A1	3	AG
(ii)	Imaginary parts: $\sin 3\theta = 3\cos^2\theta\sin\theta - \sin^3\theta$	A1F	1	
(iii)	$\tan 3\theta = \dfrac{\sin 3\theta}{\cos 3\theta}$	M1		Used
	$= \dfrac{3\cos^2\theta\sin\theta - \sin^3\theta}{\cos^3\theta - 3\sin^2\theta\cos\theta}$	A1F		Error in $\sin 3\theta$
	$= \dfrac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$			
	$= \dfrac{\tan^3\theta - 3\tan\theta}{3\tan^2\theta - 1}$	A1	3	AG

(b)(i)	$\tan\frac{3\pi}{12}=1$	B1		Used (possibly implied)
	$\tan\frac{\pi}{12}$ is a root of $1=\frac{x^3-3x}{3x^2-1}$	M1		Must be hence
	$x^3-3x^2-3x+1=0$	A1	3	
(ii)	Other roots are $\tan\frac{5\pi}{12}$, $\tan\frac{9\pi}{12}$	B1B1	2	
(c)	$\tan\frac{\pi}{12}+\tan\frac{5\pi}{12}+\tan\frac{9\pi}{12}=3$	M1		Must be hence
	$\tan\frac{\pi}{12}+\tan\frac{5\pi}{12}=4$	A1	2	
	Total		**14**	

8 (a) (i) Expand

$$\left(z+\frac{1}{z}\right)\left(z-\frac{1}{z}\right)$$ *(1 mark)*

(ii) Hence, or otherwise, expand

$$\left(z+\frac{1}{z}\right)^4\left(z-\frac{1}{z}\right)^2$$ *(3 marks)*

(b) (i) Use De Moivre's theorem to show that if $z = \cos\theta + \mathrm{i}\sin\theta$ then

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$ *(3 marks)*

(ii) Write down a corresponding result for $z^n - \dfrac{1}{z^n}$. *(1 mark)*

(c) Hence express $\cos^4\theta\sin^2\theta$ in the form

$$A\cos 6\theta + B\cos 4\theta + C\cos 2\theta + D$$

where A, B, C and D are rational numbers. *(4 marks)*

(d) Find $\displaystyle\int \cos^4\theta\sin^2\theta\,\mathrm{d}\theta$. *(2 marks)*

AQA JUNE 2008 FP2

Imagine having general Trigonometry/ Binomial in your complex HOME...

MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\left(z+\frac{1}{z}\right)\left(z-\frac{1}{z}\right)=z^2-\frac{1}{z^2}$	B1	1	
(ii)	$\left(z^2-\frac{1}{z^2}\right)^2\left(z+\frac{1}{z}\right)^2$			
	$=\left(z^4-2+\frac{1}{z^4}\right)\left(z^2+2+\frac{1}{z^2}\right)$	M1A1		Alternatives for M1A1: $\left(z^4+4z^2+6+\frac{4}{z^2}+\frac{1}{z^4}\right)\left(z^2-2+\frac{1}{z^2}\right)$ or $\left(z^3-\frac{1}{z^3}\right)^2-2\left(z^3-\frac{1}{z^3}\right)\left(z-\frac{1}{z}\right)+\left(z-\frac{1}{z}\right)^2$
	$=z^6+\frac{1}{z^6}+2\left(z^4+\frac{1}{z^4}\right)-\left(z^2+\frac{1}{z^2}\right)-4$	A1	3	CAO (not necessarily in this form)

(b)(i)	$z^n + \frac{1}{z^n} = \cos n\theta + \mathrm{i}\sin n\theta$ $+ \cos(-n\theta) + \mathrm{i}\sin(-n\theta)$	M1A1		
	$= 2\cos n\theta$	A1	3	AG SC: if solution is incomplete and $(\cos\theta + \mathrm{i}\sin\theta)^{-n}$ is written as $\cos n\theta - \mathrm{i}\sin n\theta$, award M1A0A1
(ii)	$z^n - z^{-n} = 2\mathrm{i}\sin n\theta$	B1	1	
(c)	RHS = $2\cos 6\theta + 4\cos 4\theta - 2\cos 2\theta - 4$	M1 A1F		ft incorrect values in (a)(ii) provided they are cosines
	LHS = $-64\cos^4\theta\,\sin^2\theta$	M1		
	$\cos^4\theta\sin^2\theta$ $= -\frac{1}{32}\cos 6\theta - \frac{1}{16}\cos 4\theta + \frac{1}{32}\cos 2\theta + \frac{1}{16}$	A1	4	
(d)	$-\frac{\sin 6\theta}{192} - \frac{\sin 4\theta}{64} + \frac{\sin 2\theta}{64} + \frac{\theta}{16}$ $(+k)$	M1 A1F	2	ft incorrect coefficients but not letters *A*, *B*, *C*, *D*
	Total		**14**	

AQA JUNE 2009 FP2

Imagine having general Trigonometry/Binomial in your complex HOME...

5 (a) Prove by induction that, if n is a positive integer,

$$(\cos\theta + \mathrm{i}\sin\theta)^n = \cos n\theta + \mathrm{i}\sin n\theta$$

(5 marks)

(b) Hence, given that

$$z = \cos\theta + \mathrm{i}\sin\theta$$

show that

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

(3 marks)

(c) Given further that $z + \frac{1}{z} = \sqrt{2}$, find the value of

$$z^{10} + \frac{1}{z^{10}}$$

(4 marks)

5(a)	$(\cos\theta + \mathrm{i}\sin\theta)^{k+1} =$ $(\cos k\theta + \mathrm{i}\sin k\theta)(\cos\theta + \mathrm{i}\sin\theta)$	M1		
	Multiply out	A1		Any form
	$= \cos(k+1)\theta + \mathrm{i}\sin(k+1)\theta$	A1		Clearly shown
	True for $n = 1$ shown	B1		
	$P(k) \Rightarrow P(k+1)$ and $P(1)$ true	E1	5	provided previous 4 marks earned
(b)	$\frac{1}{z^n} = \frac{1}{\cos n\theta + \mathrm{i}\sin n\theta} = \cos n\theta - \mathrm{i}\sin n\theta$	M1A1		or $z^{-n} = \cos(-n\theta) + \mathrm{i}\sin(-n\theta)$ SC $(\cos\theta + \mathrm{i}\sin\theta)^{-n}$ quoted as $\cos n\theta - \mathrm{i}\sin n\theta$ earns M1A1 only
	$z^n + \frac{1}{z^n} = 2\cos n\theta$	A1	3	AG
(c)	$z + \frac{1}{z} = \sqrt{2}$			
	$2\cos\theta = \sqrt{2}$	M1		
	$\theta = \frac{\pi}{4}$	A1		
	$z^{10} + \frac{1}{z^{10}} = 2\cos\left(\frac{10\pi}{4}\right)$	M1		M0 for merely writing $z^{10} + \frac{1}{z^{10}} = 2\cos 10\theta$
	$= 0$	A1F	4	
	Total		**12**	

7 (a) (i) Use de Moivre's Theorem to show that

$$\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$$

and find a similar expression for $\sin 5\theta$. *(5 marks)*

(ii) Deduce that

$$\tan 5\theta = \frac{\tan\theta(5 - 10\tan^2\theta + \tan^4\theta)}{1 - 10\tan^2\theta + 5\tan^4\theta}$$ *(3 marks)*

(b) Explain why $t = \tan\frac{\pi}{5}$ is a root of the equation

$$t^4 - 10t^2 + 5 = 0$$

and write down the three other roots of this equation in trigonometrical form.
(3 marks)

(c) Deduce that

$$\tan\frac{\pi}{5}\tan\frac{2\pi}{5} = \sqrt{5}$$ *(5 marks)*

AQA JUNE 2011 FP2

Imagine having general Trigonometry/ Binomial in your complex HOME...

MFP2 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)	$1+\sqrt{3}\,\mathrm{i} = 2\mathrm{e}^{\frac{\pi \mathrm{i}}{3}}$	B1		B1 both correct
	$1-\mathrm{i} = \sqrt{2}\,\mathrm{e}^{-\frac{\pi \mathrm{i}}{4}}$	B1B1	3	OE
(ii)	$2^{\frac{21}{2}}$ or equivalent single expression	B1F		No decimals; must include fractional powers
	Raising and adding powers of e	M1		
	$\frac{17\pi}{12}$ or equivalent angle	AIF	3	Denominators of angles must be different
(b)	$z = \sqrt[3]{2^{10}\sqrt{2}}\,\mathrm{e}^{\frac{17\pi \mathrm{i}}{36}+\frac{2k\pi \mathrm{i}}{3}}$	M1		
	$\sqrt[3]{2^{10}\sqrt{2}} = 8\sqrt{2}$	B1		CAO
	$\theta = \frac{17\pi}{36}, -\frac{7\pi}{36}, -\frac{31\pi}{36}$	A2,1F	4	Correct answers outside range: deduct 1 mark only
	Total		**10**	

8 (a) Use De Moivre's Theorem to show that, if $z = \cos\theta + \mathrm{i}\sin\theta$, then

$$z^n + \frac{1}{z^n} = 2\cos n\theta$$

(3 marks)

(b) (i) Expand $\left(z^2 + \frac{1}{z^2}\right)^4$. *(1 mark)*

(ii) Show that

$$\cos^4 2\theta = A\cos 8\theta + B\cos 4\theta + C$$

where A, B and C are rational numbers. *(4 marks)*

(c) Hence solve the equation

$$8\cos^4 2\theta = \cos 8\theta + 5$$

for $0 \leqslant \theta \leqslant \pi$, giving each solution in the form $k\pi$. *(3 marks)*

(d) Show that

$$\int_0^{\frac{\pi}{2}} \cos^4 2\theta \, \mathrm{d}\theta = \frac{3\pi}{16}$$

(3 marks)

Aqa June 2012 FP2

Imagine having general Trigonometry/ Binomial in your complex HOME…

MFP2

Q	Solution	Marks	Total	Comments
8(a)	Use of $(\cos\theta+\mathrm{i}\sin\theta)^n=\cos n\theta+\mathrm{i}\sin n\theta$	M1		Stated or used
	$\cos(-n\theta)+\mathrm{i}\sin(-n\theta)=\cos n\theta-\mathrm{i}\sin n\theta$	A1		allow $\frac{2}{3}$ if this line is assumed allow if complex conjugate used
	$z^n+\frac{1}{z^n}=2\cos n\theta$	A1	3	AG
(b)(i)	$z^8+4z^4+6+4z^{-4}+z^{-8}$	B1	1	allow in retrospect
(ii)	$z^2+\frac{1}{z^2}=2\cos 2\theta$ used	B1		Can be implied from (b)(i)
	$(2\cos 2\theta)^4=2\cos 8\theta+8\cos 4\theta+6$	M1A1		M1 for RHS A1 for whole line
	$\cos^4 2\theta=\frac{1}{8}\cos 8\theta+\frac{1}{2}\cos 4\theta+\frac{3}{8}$	A1F	4	ft coefficients on previous line
	Alternative to (b)(ii)			
	$\cos^4 2\theta=\left(\frac{1+\cos 4\theta}{2}\right)^2$	(M1) (A1)		
	$\cos^2 4\theta=\frac{1}{2}(1+\cos 8\theta)$	(B1)		
	Final result	(A1)		

(c)	$8\cos^4 2\theta = \cos 8\theta + 5 \to \cos 4\theta = \frac{1}{2}$	M1 A1F		ft provided simplifies to cos 4 $\theta = p$
	$k = \frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{11}{12}$	A1	3	CAO
(d)	$\int_0^{\frac{\pi}{2}} \cos^4 2\theta \, \mathrm{d}\theta =$			
	$\left[\frac{\sin 8\theta}{64} + \frac{\sin 4\theta}{8} + \frac{3}{8}\theta\right]_0^{\frac{\pi}{2}}$	M1 A1F		ie their $\cos^4 2\theta$
	$= \frac{3\pi}{16}$	A1	3	AG
	Total		**14**	

8 (a) (i) Use de Moivre's theorem to show that

$$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$$

and find a similar expression for $\sin 4\theta$. *(5 marks)*

(ii) Deduce that

$$\tan 4\theta = \frac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$$ *(3 marks)*

(b) Explain why $t = \tan\frac{\pi}{16}$ is a root of the equation

$$t^4 + 4t^3 - 6t^2 - 4t + 1 = 0$$

and write down the three other roots in trigonometric form. *(4 marks)*

(c) Hence show that

$$\tan^2\frac{\pi}{16} + \tan^2\frac{3\pi}{16} + \tan^2\frac{5\pi}{16} + \tan^2\frac{7\pi}{16} = 28$$ *(5 marks)*

AQA JUNE 2013 FP2

Imagine having general Trigonometry/ Binomial in your complex HOME...

Q	Solution	Marks	Total	Comments
8(a)(i)	$\cos 4\theta + i\sin 4\theta = (\cos\theta + i\sin\theta)^4$	M1		De Moivre & attempt to expand RHS
	$\cos^4\theta + 4i\cos^3\theta\sin\theta + 6i^2\cos^2\theta\sin^2\theta$ $+ 4i^3\cos\theta\sin^3\theta + i^4\sin^4\theta$	A1		any correct expansion
	Equating "their" real parts	m1		or imaginary parts
	$\cos 4\theta = \cos^4\theta - 6\cos^2\theta\sin^2\theta + \sin^4\theta$	A1		**AG** be convinced
	$\sin 4\theta = 4\cos^3\theta\sin\theta - 4\cos\theta\sin^3\theta$	B1	5	correct
(ii)	$\tan 4\theta = \dfrac{\text{“their expression for ” } \sin 4\theta}{\text{“their expression for ” } \cos 4\theta}$	M1		
	Division by $\cos^4\theta$	m1		
	$\tan 4\theta = \dfrac{4\tan\theta - 4\tan^3\theta}{1 - 6\tan^2\theta + \tan^4\theta}$	A1	3	**AG** be convinced

(b)	$(\tan 4\theta = 1 \Rightarrow) \quad 1 = \dfrac{4t - 4t^3}{1 - 6t^2 + t^4}$	M1		when $\theta = \dfrac{\pi}{16}$
	$1 - 6t^2 + t^4 = 4t - 4t^3$ $\Rightarrow t^4 + 4t^3 - 6t^2 - 4t + 1 = 0$	A1		**AG** be convinced
	$\theta = \dfrac{\pi}{16}$ *satisfies* $\tan 4\theta = 1$ $\Rightarrow \tan\dfrac{\pi}{16}$ is root of quartic equation	E1		both statements required
	(other roots are) $\tan\dfrac{5\pi}{16}, \tan\dfrac{9\pi}{16}, \tan\dfrac{13\pi}{16}$	B1	4	or equivalent tan expressions
(c)	$\sum\alpha = -4$ **and** $\sum\alpha\beta = -6$	B1		watch for minus signs
	$\sum\alpha^2 = (\sum\alpha)^2 - 2\sum\alpha\beta$	M1		correct formula
	$(= 16 + 12) = 28$	A1cso		
	$\tan\dfrac{9\pi}{16} = -\tan\dfrac{7\pi}{16}$, $\tan\dfrac{13\pi}{16} = -\tan\dfrac{3\pi}{16}$	B1		explicitly seen
	$\tan^2\dfrac{\pi}{16} + \tan^2\dfrac{3\pi}{16} + \tan^2\dfrac{5\pi}{16} + \tan^2\dfrac{7\pi}{16} = 28$	A1cso	5	**AG** must earn previous 4 marks
	Total		**17**	

Imagine having general C+iS in your complex HOME...

MEI Jan 2006 FP2

2 In this question, θ is a real number with $0 < \theta < \frac{1}{6}\pi$, and $w = \frac{1}{2}e^{3j\theta}$.

(i) State the modulus and argument of each of the complex numbers

$$w, \quad w^* \quad \text{and} \quad jw.$$

Illustrate these three complex numbers on an Argand diagram. [6]

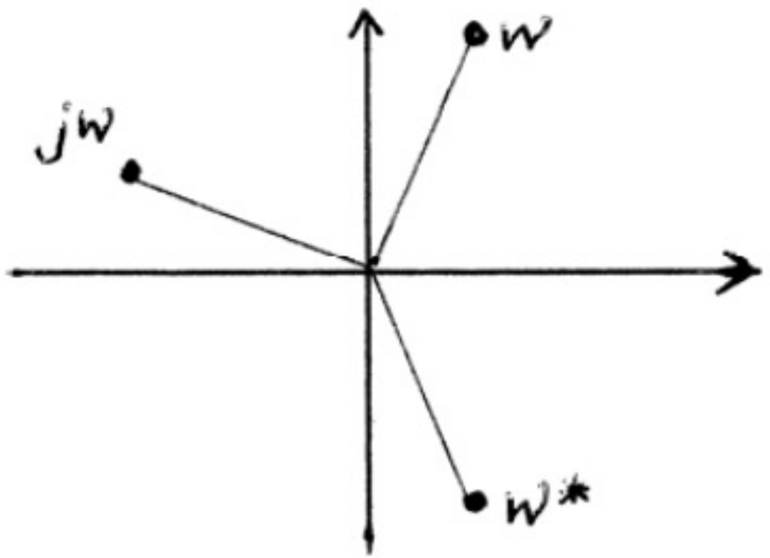
(ii) Show that $(1 + w)(1 + w^*) = \frac{5}{4} + \cos 3\theta$. [4]

Infinite series C and S are defined by

$$C = \cos 2\theta - \frac{1}{2}\cos 5\theta + \frac{1}{4}\cos 8\theta - \frac{1}{8}\cos 11\theta + \dots ,$$

$$S = \sin 2\theta - \frac{1}{2}\sin 5\theta + \frac{1}{4}\sin 8\theta - \frac{1}{8}\sin 11\theta + \dots .$$

(iii) Show that $C = \dfrac{4\cos 2\theta + 2\cos\theta}{5 + 4\cos 3\theta}$, and find a similar expression for S. [8]

2 (i)	$\lvert w\rvert = \frac{1}{2}$, $\arg w = 3\theta$ $\lvert w^*\rvert = \frac{1}{2}$, $\arg w^* = -3\theta$ $\lvert jw\rvert = \frac{1}{2}$, $\arg jw = 3\theta + \frac{1}{2}\pi$ 	B1 B1 ft B1B1 ft B2 **6**	w^* and jw in correct positions relative to their w in first quadrant Give B1 for at least two points in correct quadrants
(ii)	$(1+w)(1+w^*) = 1 + \frac{1}{2}e^{3j\theta} + \frac{1}{2}e^{-3j\theta} + (\frac{1}{2}e^{3j\theta})(\frac{1}{2}e^{-3j\theta})$ $= 1 + \frac{1}{2}(\cos 3\theta + j\sin 3\theta) + \frac{1}{2}(\cos 3\theta - j\sin 3\theta) + \frac{1}{4}$ $= \frac{5}{4} + \cos 3\theta$	M1 A1 M1 A1 (ag) **4**	for $w^* = \frac{1}{2}e^{-3j\theta}$ for $1 + \frac{1}{4}$ correctly obtained for $w = \frac{1}{2}(\cos 3\theta + j\sin 3\theta)$ for $\cos 3\theta$ correctly obtained

(iii)	$C + \mathrm{j}S = \mathrm{e}^{2\mathrm{j}\theta} - \frac{1}{2}\mathrm{e}^{5\mathrm{j}\theta} + \frac{1}{4}\mathrm{e}^{8\mathrm{j}\theta} - \ldots$	M1	Obtaining a geometric series
	$= \dfrac{\mathrm{e}^{2\mathrm{j}\theta}}{1 + \frac{1}{2}\mathrm{e}^{3\mathrm{j}\theta}}$	M1 A1	Summing an infinite geometric series
	$= \dfrac{\mathrm{e}^{2\mathrm{j}\theta}(1 + \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta})}{(1 + \frac{1}{2}\mathrm{e}^{3\mathrm{j}\theta})(1 + \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta})}$	M1	Using complex conjugate of denom
	$= \dfrac{\mathrm{e}^{2\mathrm{j}\theta}(1 + \frac{1}{2}\mathrm{e}^{-3\mathrm{j}\theta})}{\frac{5}{4} + \cos 3\theta}$	A1	
	$= \dfrac{\mathrm{e}^{2\mathrm{j}\theta} + \frac{1}{2}\mathrm{e}^{-\mathrm{j}\theta}}{\frac{5}{4} + \cos 3\theta} \quad \left(= \dfrac{4\mathrm{e}^{2\mathrm{j}\theta} + 2\mathrm{e}^{-\mathrm{j}\theta}}{5 + 4\cos 3\theta} \right)$		
	$C = \dfrac{4\cos 2\theta + 2\cos\theta}{5 + 4\cos 3\theta}$	M1 A1 (ag)	Equating real or imaginary parts
	$S = \dfrac{4\sin 2\theta - 2\sin\theta}{5 + 4\cos 3\theta}$	A1	Correctly obtained
		8	

2 **(a)** You are given the complex numbers $w = 3e^{-\frac{1}{12}\pi j}$ and $z = 1 - \sqrt{3}j$.

(i) Find the modulus and argument of each of the complex numbers w, z and $\frac{w}{z}$. [5]

(ii) Hence write $\frac{w}{z}$ in the form $a + bj$, giving the exact values of a and b. [2]

(b) In this part of the question, n is a positive integer and θ is a real number with $0 < \theta < \frac{\pi}{n}$.

(i) Express $e^{-\frac{1}{2}j\theta} + e^{\frac{1}{2}j\theta}$ in simplified trigonometric form, and hence, or otherwise, show that

$$1 + e^{j\theta} = 2e^{\frac{1}{2}j\theta}\cos\tfrac{1}{2}\theta.$$

[4]

Series C and S are defined by

$$C = 1 + \binom{n}{1}\cos\theta + \binom{n}{2}\cos 2\theta + \binom{n}{3}\cos 3\theta + \ldots + \binom{n}{n}\cos n\theta,$$

$$S = \binom{n}{1}\sin\theta + \binom{n}{2}\sin 2\theta + \binom{n}{3}\sin 3\theta + \ldots + \binom{n}{n}\sin n\theta.$$

(ii) Find C and S, and show that $\frac{S}{C} = \tan\frac{1}{2}n\theta$. [7]

MEI Jan 2007 FP2

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2(a)(i)	$\lvert w \rvert = 3$, $\arg w = -\frac{1}{12}\pi$ $\lvert z \rvert = 2$, $\arg z = -\frac{1}{3}\pi$ $\left\lvert \frac{w}{z} \right\rvert = \frac{3}{2}$, $\arg \frac{w}{z} = (-\frac{1}{12}\pi) - (-\frac{1}{3}\pi) = \frac{1}{4}\pi$	B1 B1B1 B1B1 ft **5**	*Deduct 1 mark if answers given in form $r(\cos\theta + \mathrm{j}\sin\theta)$ but modulus and argument not stated.* Accept degrees and decimal approxs
(ii)	$\frac{w}{z} = \frac{3}{2}(\cos\frac{1}{4}\pi + \mathrm{j}\sin\frac{1}{4}\pi)$ $= \frac{3}{2\sqrt{2}} + \frac{3}{2\sqrt{2}}\mathrm{j}$	M1 A1 **2**	Accept $\sqrt{1.125} + \sqrt{1.125}\,\mathrm{j}$
(b)(i)	$\mathrm{e}^{-\frac{1}{2}\mathrm{j}\theta} + \mathrm{e}^{\frac{1}{2}\mathrm{j}\theta}$ $= (\cos\frac{1}{2}\theta - \mathrm{j}\sin\frac{1}{2}\theta) + (\cos\frac{1}{2}\theta + \mathrm{j}\sin\frac{1}{2}\theta)$ $= 2\cos\frac{1}{2}\theta$	M1 A1	For either bracketed expression
	$1 + \mathrm{e}^{\mathrm{j}\theta} = \mathrm{e}^{\frac{1}{2}\mathrm{j}\theta}(\mathrm{e}^{-\frac{1}{2}\mathrm{j}\theta} + \mathrm{e}^{\frac{1}{2}\mathrm{j}\theta})$ $= \mathrm{e}^{\frac{1}{2}\mathrm{j}\theta}(2\cos\frac{1}{2}\theta)$	M1 A1 ag **4**	
	OR $1 + \mathrm{e}^{\mathrm{j}\theta} = 1 + \cos\theta + \mathrm{j}\sin\theta$ $= 2\cos^2\frac{1}{2}\theta + 2\mathrm{j}\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta$ M1 $= 2\cos\frac{1}{2}\theta(\cos\frac{1}{2}\theta + \mathrm{j}\sin\frac{1}{2}\theta)$ $= 2\mathrm{e}^{\frac{1}{2}\mathrm{j}\theta}\cos\frac{1}{2}\theta$ A1		

(ii)	$C + jS = 1 + \binom{n}{1}e^{j\theta} + \binom{n}{2}e^{2j\theta} + ... + \binom{n}{n}e^{nj\theta}$	M1	
	$= (1 + e^{j\theta})^n$	M1A1	
	$= 2^n e^{\frac{1}{2}n\theta j} \cos^n \frac{1}{2}\theta$	M1	Using (i) to obtain a form from which the real and imaginary parts can be written down
	$C = 2^n \cos(\frac{1}{2}n\theta)\cos^n \frac{1}{2}\theta$	A1	
	$S = 2^n \sin(\frac{1}{2}n\theta)\cos^n \frac{1}{2}\theta$	A1	
	$\frac{S}{C} = \frac{2^n \sin(\frac{1}{2}n\theta)\cos^n \frac{1}{2}\theta}{2^n \cos(\frac{1}{2}n\theta)\cos^n \frac{1}{2}\theta} = \frac{\sin(\frac{1}{2}n\theta)}{\cos(\frac{1}{2}n\theta)} = \tan(\frac{1}{2}n\theta)$	B1 ag **7**	Allow ft from $C + jS = e^{\frac{1}{2}n\theta j} \times$ any real function of *n* and θ

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MEI Jan 2008 FP2

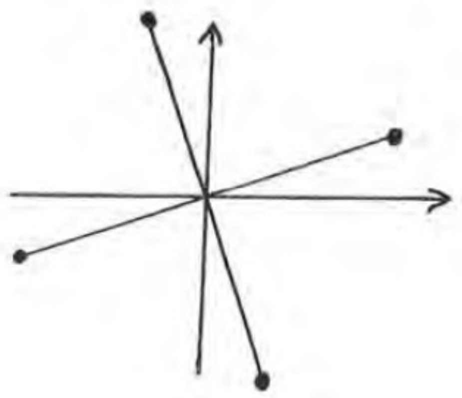
2 **(a)** Find the 4th roots of 16j, in the form $re^{j\theta}$ where $r > 0$ and $-\pi < \theta \leqslant \pi$. Illustrate the 4th roots on an Argand diagram. [6]

(b) **(i)** Show that $(1 - 2e^{j\theta})(1 - 2e^{-j\theta}) = 5 - 4\cos\theta$. [3]

Series C and S are defined by

$$C = 2\cos\theta + 4\cos 2\theta + 8\cos 3\theta + \ldots + 2^n \cos n\theta,$$
$$S = 2\sin\theta + 4\sin 2\theta + 8\sin 3\theta + \ldots + 2^n \sin n\theta.$$

(ii) Show that $C = \dfrac{2\cos\theta - 4 - 2^{n+1}\cos(n+1)\theta + 2^{n+2}\cos n\theta}{5 - 4\cos\theta}$, and find a similar expression for S. [9]

2(a)	4th roots of $16\mathrm{j} = 16\,\mathrm{e}^{\frac{1}{2}\pi\mathrm{j}}$ are $r\,\mathrm{e}^{\mathrm{j}\theta}$ where $r = 2$ $\theta = \frac{1}{8}\pi$ $\theta = \dfrac{\pi}{8} + \dfrac{2k\pi}{4}$ $\theta = -\frac{7}{8}\pi,\ -\frac{3}{8}\pi,\ \frac{5}{8}\pi$ 	B1 B1 M1 A1 M1 A1 **6**	*Accept* $16^{\frac{1}{4}}$ Implied by at least two correct (ft) further values or stating $k = -2,\ -1,\ (0),\ 1$ Points at vertices of a square centre O *or* 3 correct points (ft) *or* 1 point in each quadrant
(b)(i)	$(1-2\,\mathrm{e}^{\mathrm{j}\theta})(1-2\,\mathrm{e}^{-\mathrm{j}\theta}) = 1-2\,\mathrm{e}^{\mathrm{j}\theta} - 2\,\mathrm{e}^{-\mathrm{j}\theta} + 4$ $= 5 - 2(\mathrm{e}^{\mathrm{j}\theta} + \mathrm{e}^{-\mathrm{j}\theta})$ $= 5 - 4\cos\theta$	M1 A1 A1 ag **3**	For $\mathrm{e}^{\mathrm{j}\theta}\mathrm{e}^{-\mathrm{j}\theta} = 1$
	OR $(1-2\cos\theta - 2\mathrm{j}\sin\theta)(1-2\cos\theta + 2\mathrm{j}\sin\theta)$ M1 $= (1-2\cos\theta)^2 + 4\sin^2\theta$ A1 $= 1 - 4\cos\theta + 4(\cos^2\theta + \sin^2\theta)$ $= 5 - 4\cos\theta$ A1		

(ii)	$C + \mathrm{j}S = 2\mathrm{e}^{\mathrm{j}\theta} + 4\mathrm{e}^{2\mathrm{j}\theta} + 8\mathrm{e}^{3\mathrm{j}\theta} + \ldots + 2^n \mathrm{e}^{n\mathrm{j}\theta}$ $= \dfrac{2\mathrm{e}^{\mathrm{j}\theta}\left(1-(2\mathrm{e}^{\mathrm{j}\theta})^n\right)}{1-2\mathrm{e}^{\mathrm{j}\theta}}$ $= \dfrac{2\mathrm{e}^{\mathrm{j}\theta}(1-2^n\mathrm{e}^{n\mathrm{j}\theta})(1-2\mathrm{e}^{-\mathrm{j}\theta})}{(1-2\mathrm{e}^{\mathrm{j}\theta})(1-2\mathrm{e}^{-\mathrm{j}\theta})}$ $= \dfrac{2\mathrm{e}^{\mathrm{j}\theta} - 4 - 2^{n+1}\mathrm{e}^{(n+1)\mathrm{j}\theta} + 2^{n+2}\mathrm{e}^{n\mathrm{j}\theta}}{5-4\cos\theta}$ $C = \dfrac{2\cos\theta - 4 - 2^{n+1}\cos(n+1)\theta + 2^{n+2}\cos n\theta}{5-4\cos\theta}$ $S = \dfrac{2\sin\theta - 2^{n+1}\sin(n+1)\theta + 2^{n+2}\sin n\theta}{5-4\cos\theta}$	M1 M1 A1 M1 A2 M1 A1 ag A1 **9**	Obtaining a geometric series Summing (M0 for sum to infinity) Give A1 for two correct terms in numerator Equating real (or imaginary) parts

MEI JUNE 2009 FP2

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3 **(a)** **(i)** Sketch the graph of $y = \arcsin x$ for $-1 \leqslant x \leqslant 1$. **[1]**

Find $\dfrac{dy}{dx}$, justifying the sign of your answer by reference to your sketch. **[4]**

(ii) Find the exact value of the integral $\displaystyle\int_0^1 \frac{1}{\sqrt{2-x^2}}\,dx$. **[3]**

(b) The infinite series C and S are defined as follows.

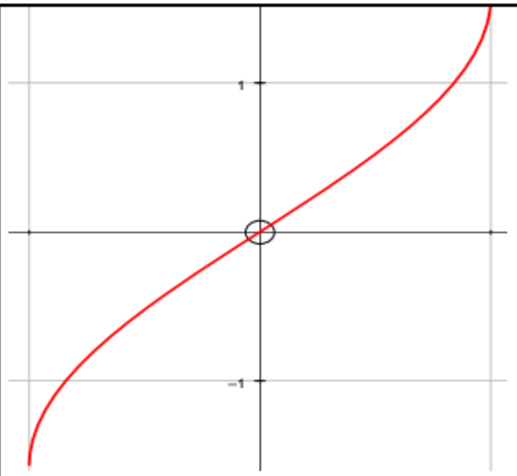
$$C = \cos\theta + \tfrac{1}{3}\cos 3\theta + \tfrac{1}{9}\cos 5\theta + \ \ldots$$

$$S = \sin\theta + \tfrac{1}{3}\sin 3\theta + \tfrac{1}{9}\sin 5\theta + \ \ldots$$

By considering $C + jS$, show that

$$C = \frac{3\cos\theta}{5 - 3\cos 2\theta},$$

and find a similar expression for S. **[11]**

3(a)(i)		G1	Correct basic shape (positive gradient, through (0, 0))
		1	
	$y = \arcsin x \Rightarrow \sin y = x$	M1	sin y = and attempt to diff. both sides
	$\Rightarrow \dfrac{dx}{dy} = \cos y$	A1	Or $\cos y \dfrac{dy}{dx} = 1$
	$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{\cos y} = \dfrac{1}{\sqrt{1-x^2}}$	A1	www. SC1 if quoted without working
	Positive square root because gradient positive	B1	Dep. on graph of an increasing function
		4	
(ii)	$\int_0^1 \dfrac{1}{\sqrt{2-x^2}}dx = \left[\arcsin\dfrac{x}{\sqrt{2}}\right]_0^1$	M1	arcsin function alone, or any sine substitution
		A1	$\dfrac{x}{\sqrt{2}}$, or $\int 1\, d\theta$ www without limits
	$= \dfrac{\pi}{4}$	A1	Evaluated in terms of π
		3	

(b)	$C + jS = e^{j\theta} + \frac{1}{3}e^{3j\theta} + \frac{1}{9}e^{5j\theta} + \ldots$	M1	Forming $C + jS$ as a series of powers
	This is a geometric series	M1	Identifying geometric series and attempting sum to infinity or to n terms
	with first term $a = e^{j\theta}$, common ratio $r = \frac{1}{3}e^{2j\theta}$	A1	Correct a and r
	Sum to infinity = $\frac{a}{1-r} = \frac{e^{j\theta}}{1-\frac{1}{3}e^{2j\theta}}$ $\left(= \frac{3e^{j\theta}}{3-e^{2j\theta}}\right)$	A1	Sum to infinity
	$= \frac{3e^{j\theta}}{3-e^{2j\theta}} \times \frac{3-e^{-2j\theta}}{3-e^{-2j\theta}}$	M1*	Multiplying numerator and denominator by $1-\frac{1}{3}e^{-2j\theta}$ o.e. Or writing in terms of trig functions and realising the denominator
	$= \frac{9e^{j\theta} - 3e^{-j\theta}}{9 - 3e^{-2j\theta} - 3e^{2j\theta} + 1}$	M1	Multiplying out numerator and denominator. Dep. on M1*
	$= \frac{9(\cos\theta + j\sin\theta) - 3(\cos\theta - j\sin\theta)}{10 - 3(\cos 2\theta - j\sin 2\theta) - 3(\cos 2\theta + j\sin 2\theta)}$	M1	Valid attempt to express in terms of trig functions. If trig functions used from start, M1 for using the compound angle formulae and Pythagoras Dep. on M1*
	$= \frac{6\cos\theta + 12j\sin\theta}{10 - 6\cos 2\theta}$	A1	
	$\Rightarrow \quad C = \frac{6\cos\theta}{10 - 6\cos 2\theta}$	M1	Equating real and imaginary parts. Dep. on M1*

MEI JAN 2010 FP2

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2 **(a)** Use de Moivre's theorem to find the constants a, b, c in the identity

$$\cos 5\theta \equiv a\cos^5\theta + b\cos^3\theta + c\cos\theta.$$ **[6]**

(b) Let

$$C = \cos\theta + \cos\left(\theta + \frac{2\pi}{n}\right) + \cos\left(\theta + \frac{4\pi}{n}\right) + \ldots + \cos\left(\theta + \frac{(2n-2)\pi}{n}\right),$$

and $$S = \sin\theta + \sin\left(\theta + \frac{2\pi}{n}\right) + \sin\left(\theta + \frac{4\pi}{n}\right) + \ldots + \sin\left(\theta + \frac{(2n-2)\pi}{n}\right),$$

where n is an integer greater than 1.

By considering $C + \mathrm{j}S$, show that $C = 0$ and $S = 0$. **[7]**

(c) Write down the Maclaurin series for e^t as far as the term in t^2.

Hence show that, for t close to zero,

$$\frac{t}{e^t - 1} \approx 1 - \tfrac{1}{2}t.$$ **[5]**

2 (a)	$\cos 5\theta + j\sin 5\theta = (\cos\theta + j\sin\theta)^5$ $= \cos^5\theta + 5\cos^4\theta\, j\sin\theta + 10\cos^3\theta\, j^2\sin^2\theta$ $+ 10\cos^2\theta\, j^3\sin^3\theta + 5\cos\theta\, j^4\sin^4\theta + j^5\sin^5\theta$ $= \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta + j(\ldots)$ $\cos 5\theta = \cos^5\theta - 10\cos^3\theta\sin^2\theta + 5\cos\theta\sin^4\theta$ $= \cos^5\theta - 10\cos^3\theta(1-\cos^2\theta) + 5\cos\theta(1-\cos^2\theta)^2$ $= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$	M1 M1 A1 M1 M1 A1 **6**	Using de Moivre Using binomial theorem appropriately Correct real part. Must evaluate powers of j Equating real parts Replacing $\sin^2\theta$ by $1-\cos^2\theta$ $a = 16$, $b = -20$, $c = 5$
(b)	$C + jS$ $= e^{j\theta} + e^{j\left(\theta+\frac{2\pi}{n}\right)} + \ldots + e^{j\left(\theta+\frac{(2n-2)\pi}{n}\right)}$ This is a G.P. $a = e^{j\theta}$, $r = e^{j\frac{2\pi}{n}}$ Sum $= \dfrac{e^{j\theta}\left(1-\left(e^{j\frac{2\pi}{n}}\right)^n\right)}{1-e^{j\frac{2\pi}{n}}}$ Numerator $= e^{j\theta}\left(1-e^{2\pi j}\right)$ and $e^{2\pi j} = 1$ so sum = 0 $\Rightarrow \quad C = 0$ and $S = 0$	M1 A1 M1 A1 A1 E1 E1 **7**	Forming series $C + jS$ as exponentials Need not see whole series Attempting to sum finite or infinite G.P. Correct a, r used or stated, and n terms Must see j Convincing explanation that sum = 0 $C = S = 0$. Dep. on previous E1 Both E marks dep. on 5 marks above

(c)	$e^t \approx 1+t+\frac{1}{2}t^2$	B1	Ignore terms in higher powers
	$\frac{t}{e^t-1} \approx \frac{t}{t+\frac{1}{2}t^2}$	M1 A1	Substituting Maclaurin series
	$\frac{t}{t+\frac{1}{2}t^2} = \frac{1}{1+\frac{1}{2}t} = \left(1+\frac{1}{2}t\right)^{-1} = 1-\frac{1}{2}t+\ldots$	M1	Suitable manipulation and use of binomial theorem
	OR $\frac{1}{1+\frac{1}{2}t} = \frac{1}{1+\frac{1}{2}t} \times \frac{1-\frac{1}{2}t}{1-\frac{1}{2}t} = \frac{1-\frac{1}{2}t}{1-\frac{1}{4}t^2}$ M1		
	Hence $\frac{t}{e^t-1} \approx 1-\frac{1}{2}t$	A1 (ag)	
	OR $\left(e^t-1\right)\left(1-\frac{1}{2}t\right) = \left(t+\frac{1}{2}t^2+\ldots\right)\left(1-\frac{1}{2}t\right)$ M1 A1 $\approx t +$ terms in t^3 M1 $\Rightarrow \frac{t}{e^t-1} \approx 1-\frac{1}{2}t$ A1		Substituting Maclaurin series Correct expression Multiplying out Convincing explanation
		5	**18**

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MEI JAN 2012 FP2

2 **(a)** The infinite series C and S are defined as follows.

$$C = 1 + a\cos\theta + a^2\cos 2\theta + \ldots,$$

$$S = a\sin\theta + a^2\sin 2\theta + a^3\sin 3\theta + \ldots,$$

where a is a real number and $|a| < 1$.

By considering $C + \mathrm{j}S$, show that $C = \dfrac{1 - a\cos\theta}{1 + a^2 - 2a\cos\theta}$ and find a corresponding expression for S. **[8]**

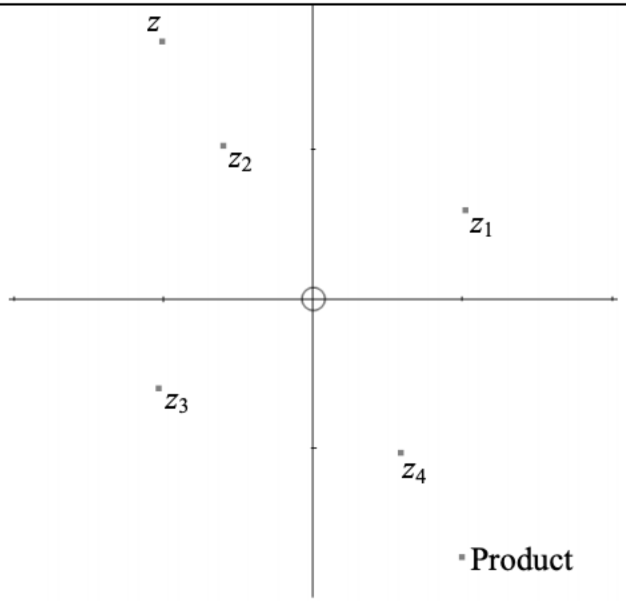
(b) Express the complex number $z = -1 + \mathrm{j}\sqrt{3}$ in the form $r\,\mathrm{e}^{\mathrm{j}\theta}$.

Find the 4th roots of z in the form $r\,\mathrm{e}^{\mathrm{j}\theta}$.

Show z and its 4th roots in an Argand diagram.

Find the product of the 4th roots and mark this as a point on your Argand diagram. **[10]**

Question			Answer	Marks	Guidance	
2	(a)		$C+jS=1+ae^{j\theta}+a^2e^{2j\theta}+\ldots$	M1	Forming $C+jS$ as a series of powers	$\ldots a^2(\cos 2\theta+j\sin 2\theta)$ insufficient. Powers must be correct
			This is a geometric series with $r=ae^{j\theta}$	M1	Identifying G.P. and attempting sum. Dependent on first M1	
			Sum to infinity $=\dfrac{1}{1-ae^{j\theta}}$	A1		
			$=\dfrac{1}{1-ae^{j\theta}}\times\dfrac{1-ae^{-j\theta}}{1-ae^{-j\theta}}$	M1*	Multiplying numerator and denominator by $1-ae^{-j\theta}$ o.e.	
			$=\dfrac{1-ae^{-j\theta}}{1-ae^{j\theta}-ae^{-j\theta}+a^2}$	M1	Multiplying out denominator. Dependent on M1*	Use of FOIL with powers combined correctly (allow one slip)
			$=\dfrac{1-a(\cos\theta-j\sin\theta)}{1-2a\cos\theta+a^2}$	M1	Introducing trig functions. Dependent on M1*	Condone e.g. $e^{-j\theta}=\cos\theta+j\sin\theta$
			$=\dfrac{1-a\cos\theta}{1-2a\cos\theta+a^2}+\dfrac{aj\sin\theta}{1-2a\cos\theta+a^2}$			
			$\Rightarrow C=\dfrac{1-a\cos\theta}{1-2a\cos\theta+a^2}$	E1		Answer given. www which leads to C
			and $S=\dfrac{a\sin\theta}{1-2a\cos\theta+a^2}$	A1		
				[8]		

2	(b)		Modulus = 2	B1		
			Argument $= \frac{2\pi}{3}$	B1		
			$\Rightarrow -1 + j\sqrt{3} = 2e^{j\frac{2\pi}{3}}$			
			$\Rightarrow$ fourth roots have $r = \sqrt[4]{2}$	B1		Allow 1.19 or better
			and $\theta = \frac{\pi}{6}$			
				M1	$\div$ arg z by 4 and adding $\frac{\pi}{2}$	$\theta = \frac{\pi}{6} + \frac{2k\pi}{4}$ scores M1;
			$\Rightarrow$ roots are $\sqrt[4]{2}e^{j\frac{\pi}{6}}, \sqrt[4]{2}e^{j\frac{2\pi}{3}}, \sqrt[4]{2}e^{j\frac{7\pi}{6}}, \sqrt[4]{2}e^{j\frac{5\pi}{3}}$	A1	All arguments correct	$k = 0, 1, 2, 3$ (or $-2, -1, 0, 1$) A1
				G1 G1ft G1ft	Position of z Roots forming square Position of product	In 2nd quadrant Ignore marked angles Correct or their $-z$
			Product of 4th roots $= 2e^{j(1+4+7+10)\frac{\pi}{6}}$	M1	Attempting to find product	Must consider both modulus and argument
			$= 2e^{j\frac{5\pi}{3}}$	A1	Or $-\frac{\pi}{3}$ o.e.	
				[10]		

MEI
JAN
2013
FP2

Imagine having general C+iS in your complex HOME...

2 **(a)** **(i)** Show that

$$1 + e^{j2\theta} = 2\cos\theta(\cos\theta + j\sin\theta).$$ **[2]**

(ii) The series C and S are defined as follows.

$$C = 1 + \binom{n}{1}\cos 2\theta + \binom{n}{2}\cos 4\theta + \ldots + \cos 2n\theta$$

$$S = \binom{n}{1}\sin 2\theta + \binom{n}{2}\sin 4\theta + \ldots + \sin 2n\theta$$

By considering $C + jS$, show that

$$C = 2^n\cos^n\theta\cos n\theta,$$

and find a corresponding expression for S. **[7]**

(b) **(i)** Express $e^{j2\pi/3}$ in the form $x + jy$, where the real numbers x and y should be given exactly. **[1]**

(ii) An equilateral triangle in the Argand diagram has its centre at the origin. One vertex of the triangle is at the point representing $2 + 4j$. Obtain the complex numbers representing the other two vertices, giving your answers in the form $x + jy$, where the real numbers x and y should be given exactly. **[6]**

(iii) Show that the length of a side of the triangle is $2\sqrt{15}$. **[2]**

Question			Answer	Marks	Guidance	
2	(a)	(ii)	$C+\mathrm{j}S = 1+\binom{n}{1}e^{\mathrm{j}2\theta}+\binom{n}{2}e^{\mathrm{j}4\theta}+\ldots+e^{\mathrm{j}2n\theta}$	M1	Forming $C+\mathrm{j}S$	
			$=\left(1+e^{\mathrm{j}2\theta}\right)^n$	M1 A1	Recognising as binomial expansion	
			$=2^n\cos^n\theta(\cos\theta+\mathrm{j}\sin\theta)^n$			
			$=2^n\cos^n\theta(\cos n\theta+\mathrm{j}\sin n\theta)$	M1 A1	Applying (i) and De Moivre o.e.	Dependent on M1M1 above
			$\Rightarrow C=2^n\cos^n\theta\cos n\theta$	A1(ag)	Completion www	Need to see $e^{\mathrm{j}n\theta}=\cos n\theta+\mathrm{j}\sin n\theta$ o.e.
			and $S=2^n\cos^n\theta\sin n\theta$	A1 [7]		
2	(b)	(i)	$e^{\mathrm{j}\frac{2\pi}{3}}=\cos\frac{2\pi}{3}+\mathrm{j}\sin\frac{2\pi}{3}=-\frac{1}{2}+\mathrm{j}\frac{\sqrt{3}}{2}$	B1 [1]	Must evaluate trigonometric functions	
2	(b)	(ii)	Other two vertices are $(2+4\mathrm{j})e^{\mathrm{j}\frac{2\pi}{3}}$	M1	Award for idea of rotation by $\frac{2\pi}{3}$	e.g. use of $\arctan 2+\frac{2\pi}{3}$ (3.202 rad) (must be 2)
			$=(2+4\mathrm{j})\left(-\frac{1}{2}+\mathrm{j}\frac{\sqrt{3}}{2}\right)$			
			$=\left(-1-2\sqrt{3}\right)+\mathrm{j}\left(-2+\sqrt{3}\right)$	A1A1	May be given as co-ordinates	
			and $(2+4\mathrm{j})e^{\mathrm{j}\frac{4\pi}{3}}=(2+4\mathrm{j})e^{-\mathrm{j}\frac{2\pi}{3}}$	M1	Award for idea of rotation by $-\frac{2\pi}{3}$	e.g. use of $\arctan 2+\frac{4\pi}{3}$ (5.296 rad) (must be 2)
			$=(2+4\mathrm{j})\left(-\frac{1}{2}-\mathrm{j}\frac{\sqrt{3}}{2}\right)$			
			$=\left(-1+2\sqrt{3}\right)+\mathrm{j}\left(-2-\sqrt{3}\right)$	A1A1	May be given as co-ordinates	If A0A0A0A0 award SC1 for awrt $-4.46-0.27\mathrm{j}$ and $2.46-3.73\mathrm{j}$

Question			Answer	Marks	Guidance	
2	(b)	(iii)	Length of (2 + 4j) = $\sqrt{20}$ So length of side = $2\sqrt{20}\cos\frac{\pi}{6} = 2\sqrt{20}\times\frac{\sqrt{3}}{2}$ $= 2\sqrt{15}$	 M1 A1(ag) **[2]**	 Complete method Completion www	Alternative: finding distance between (2, 4) and $\left(-1-2\sqrt{3}, -2+\sqrt{3}\right)$ o.e.

Imagine having general C+iS in your complex HOME…

OCR JAN 2008 FP3

Jan 2008

4 The integrals C and S are defined by

$$C = \int_0^{\frac{1}{2}\pi} e^{2x} \cos 3x \, dx \qquad \text{and} \qquad S = \int_0^{\frac{1}{2}\pi} e^{2x} \sin 3x \, dx.$$

By considering $C + iS$ as a single integral, show that

$$C = -\tfrac{1}{13}(2 + 3e^{\pi}),$$

and obtain a similar expression for S. [8]

(You may assume that the standard result for $\int e^{kx} \, dx$ remains true when k is a complex constant, so that $\int e^{(a+ib)x} \, dx = \frac{1}{a + ib} e^{(a+ib)x}$.)

4 $(C+\mathrm{i}S=)\ \int_0^{\frac{1}{2}\pi} e^{2x}(\cos 3x+\mathrm{i}\sin 3x)(\mathrm{d}x)$		
$\cos 3x+\mathrm{i}\sin 3x=e^{3\mathrm{i}x}$	B1	For using de Moivre, seen or implied
$\int_0^{\frac{1}{2}\pi} e^{(2+3\mathrm{i})x}(\mathrm{d}x)=\frac{1}{2+3\mathrm{i}}\left[e^{(2+3\mathrm{i})x}\right]_0^{\frac{1}{2}\pi}$	M1* A1	For writing as a single integral in exp form For correct integration (ignore limits)
$=\frac{2-3\mathrm{i}}{4+9}\left(e^{(2+3\mathrm{i})\frac{1}{2}\pi}-e^0\right)=\frac{2-3\mathrm{i}}{13}\left(-\mathrm{i}e^{\pi}-1\right)$	A1 M1 (dep*)	For substituting limits correctly (unsimplified) (may be earned at any stage) For multiplying by complex conjugate of 2+3i
$=\left\{\frac{1}{13}\left(-2-3e^{\pi}+\mathrm{i}(3-2e^{\pi})\right)\right\}$	M1 (dep*)	For equating real and/or imaginary parts
$C=-\frac{1}{13}\left(2+3e^{\pi}\right)$	A1	For correct expression **AG**
$S=\frac{1}{13}\left(3-2e^{\pi}\right)$	A1	For correct expression
	8	

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Ocr June 2010 FP3

5 Convergent infinite series C and S are defined by

$$C = 1 + \tfrac{1}{2}\cos\theta + \tfrac{1}{4}\cos 2\theta + \tfrac{1}{8}\cos 3\theta + \ldots ,$$
$$S = \tfrac{1}{2}\sin\theta + \tfrac{1}{4}\sin 2\theta + \tfrac{1}{8}\sin 3\theta + \ldots .$$

(i) Show that $C + \mathrm{i}S = \dfrac{2}{2 - \mathrm{e}^{\mathrm{i}\theta}}$. **[4]**

(ii) Hence show that $C = \dfrac{4 - 2\cos\theta}{5 - 4\cos\theta}$, and find a similar expression for S. **[4]**

5	(i)	$C+\mathrm{i}S=1+\frac{1}{2}\mathrm{e}^{\mathrm{i}\theta}+\frac{1}{4}\mathrm{e}^{2\mathrm{i}\theta}+\frac{1}{8}\mathrm{e}^{3\mathrm{i}\theta}+\ldots$	M1 A1		For using $\cos n\theta+\mathrm{i}\sin n\theta=\mathrm{e}^{\mathrm{i}n\theta}$ at least once for $n \geqslant 2$ For correct series
		$=\dfrac{1}{1-\frac{1}{2}\mathrm{e}^{\mathrm{i}\theta}}=\dfrac{2}{2-\mathrm{e}^{\mathrm{i}\theta}}$	M1 A1	4	For using sum of infinite GP For correct expression **AG** **SR** For omission of 1st stage award up to M0 A0 M1 A1 **OEW**
	(ii)	$C+\mathrm{i}S=\dfrac{2\left(2-\mathrm{e}^{-\mathrm{i}\theta}\right)}{\left(2-\mathrm{e}^{\mathrm{i}\theta}\right)\left(2-\mathrm{e}^{-\mathrm{i}\theta}\right)}$	M1		For multiplying top and bottom by complex conjugate
		$=\dfrac{4-2\mathrm{e}^{-\mathrm{i}\theta}}{4-2\left(\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{-\mathrm{i}\theta}\right)+1}=\dfrac{4-2\cos\theta+2\mathrm{i}\sin\theta}{4-4\cos\theta+1}$	M1		For reverting to cosθ and sinθ and equating Re *OR* Im parts
		$\Rightarrow C=\dfrac{4-2\cos\theta}{5-4\cos\theta},\quad S=\dfrac{2\sin\theta}{5-4\cos\theta}$	A1 A1	4	For correct expression for C **AG** For correct expression for S
				8	

OCR JAN 2013 FP3

Imagine having general C+iS in your complex HOME...

Jan 2013

7 Let $S = e^{i\theta} + e^{2i\theta} + e^{3i\theta} + \ldots + e^{10i\theta}$.

(i) (a) Show that, for $\theta \neq 2n\pi$, where n is an integer,

$$S = \frac{e^{\frac{1}{2}i\theta}\left(e^{10i\theta} - 1\right)}{2i\sin\left(\frac{1}{2}\theta\right)}.$$ **[4]**

(b) State the value of S for $\theta = 2n\pi$, where n is an integer. **[1]**

(ii) Hence show that, for $\theta \neq 2n\pi$, where n is an integer,

$$\cos\theta + \cos 2\theta + \cos 3\theta + \ldots + \cos 10\theta = \frac{\sin\left(\frac{21}{2}\theta\right)}{2\sin\left(\frac{1}{2}\theta\right)} - \frac{1}{2}.$$ **[3]**

(iii) Hence show that $\theta = \frac{1}{11}\pi$ is a root of $\cos\theta + \cos 2\theta + \cos 3\theta + \ldots + \cos 10\theta = 0$ and find another root in the interval $0 < \theta < \frac{1}{4}\pi$. **[4]**

7	**(i)**	**(a)**	$\mathrm{e}^{\mathrm{i}\theta}+\mathrm{e}^{2\mathrm{i}\theta}+\ldots+\mathrm{e}^{10\mathrm{i}\theta}=\dfrac{\mathrm{e}^{\mathrm{i}\theta}\left(\left(\mathrm{e}^{\mathrm{i}\theta}\right)^{10}-1\right)}{\mathrm{e}^{\mathrm{i}\theta}-1}$ $=\dfrac{\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta}\left(\mathrm{e}^{10\mathrm{i}\theta}-1\right)}{\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta}-\mathrm{e}^{-\frac{1}{2}\mathrm{i}\theta}}$ $=\dfrac{\mathrm{e}^{\frac{1}{2}\mathrm{i}\theta}\left(\mathrm{e}^{10\mathrm{i}\theta}-1\right)}{2\mathrm{i}\sin\left(\frac{1}{2}\theta\right)}$	M1 A1 M1 A1 **[4]**	Sum of a GP AG	
7	**(i)**	**(b)**	$\theta=2n\pi\Rightarrow \text{sum}=10$	B1 **[1]**		

Question			Answer	Marks	Guidance	
7	(ii)		$\cos\theta + \cos 2\theta + \ldots + \cos 10\theta = \text{Re}\left(\dfrac{e^{\frac{1}{2}i\theta}\left(e^{10i\theta}-1\right)}{2i\sin\left(\frac{1}{2}\theta\right)}\right)$	M1	Take real parts	
			$= \dfrac{\text{Re}\left(-ie^{\frac{1}{2}i\theta}\left(e^{10i\theta}-1\right)\right)}{2\sin\left(\frac{1}{2}\theta\right)} = \dfrac{\text{Re}\left(-ie^{\frac{21}{2}i\theta}+ie^{\frac{1}{2}i\theta}\right)}{2\sin\left(\frac{1}{2}\theta\right)}$	M1	Manipulate expression	Must at least make genuine progress in sorting real part of numerator, or in converting numerator to trig terms.
			$= \dfrac{\sin\left(\frac{21}{2}\theta\right)-\sin\left(\frac{1}{2}\theta\right)}{2\sin\left(\frac{1}{2}\theta\right)}$ $= \dfrac{\sin\left(\frac{21}{2}\theta\right)}{2\sin\left(\frac{1}{2}\theta\right)} - \dfrac{1}{2}$	A1 **[3]**	AG	
7	(iii)		$\cos\frac{1}{11}\pi + \cos\frac{2}{11}\pi + \ldots + \cos\frac{10}{11}\pi = \dfrac{\sin\left(\frac{21}{22}\pi\right)}{2\sin\left(\frac{1}{22}\pi\right)} - \dfrac{1}{2}$	M1		For second M1, must convince that solution is exact and not simply from calculator.
			But $\sin\frac{21}{22}\pi = \sin\left(\pi - \frac{21}{22}\pi\right) = \sin\frac{1}{22}\pi$	M1		
			So RHS $= \frac{1}{2} - \frac{1}{2} = 0$, so $\frac{1}{11}\pi$ is a root	A1	AG	
			Using $\sin(2\pi + x) = \sin x$ gives $2\pi + \frac{1}{2}\theta = \frac{21}{2}\theta \Rightarrow \theta = \frac{1}{5}\pi$	A1 **[4]**		

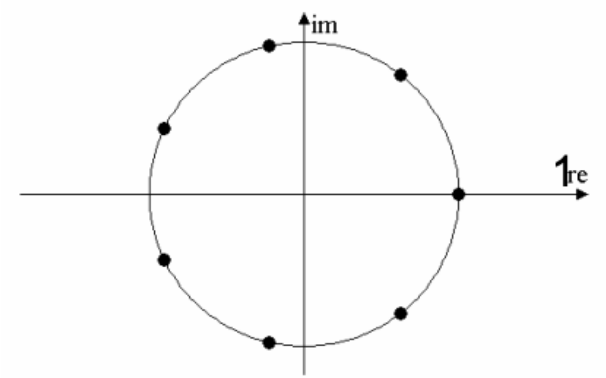
Imagine having a regular n-gon in your complex HOME...

OCR JUNE 2007 FP3

7 **(i)** Show that $(z - e^{i\phi})(z - e^{-i\phi}) \equiv z^2 - (2\cos\phi)z + 1$. [1]

(ii) Write down the seven roots of the equation $z^7 = 1$ in the form $e^{i\theta}$ and show their positions in an Argand diagram. [4]

(iii) Hence express $z^7 - 1$ as the product of one real linear factor and three real quadratic factors. [5]

7 **(i)** $(z-e^{i\phi})(z-e^{-i\phi}) \equiv z^2-(2)z\dfrac{(e^{i\phi}+e^{-i\phi})}{(2)}+1$ $\equiv z^2-(2\cos\phi)z+1$	B1	**1**	For correct justification **AG**
(ii) $z=e^{\frac{2}{7}k\pi i}$	B1		For general form *OR* any one non-real root
for $k=0, 1, 2, 3, 4, 5, 6$ *OR* $0, \pm1, \pm2, \pm3$	B1		For other roots specified (k=0 may be seen in any form, eg 1, e^0, $e^{2\pi i}$) For answers in form $\cos\theta+i\sin\theta$ allow maximum B1 B0
	B1		For any 7 points equally spaced round unit circle (circumference need not be shown)
	B1	**4**	For 1 point on $+^{ve}$ real axis, and other points in correct quadrants
(iii) $\left(z^7-1=\right)(z-1)(z-e^{\frac{2}{7}\pi i})(z-e^{\frac{4}{7}\pi i})$ $(z-e^{\frac{6}{7}\pi i})(z-e^{\frac{-2}{7}\pi i})(z-e^{\frac{-4}{7}\pi i})(z-e^{\frac{-6}{7}\pi i})$	M1		For using linear factors from **(ii),** seen or implied
$=(z-e^{\frac{2}{7}\pi i})(z-e^{\frac{-2}{7}\pi i})\times(z-e^{\frac{4}{7}\pi i})(z-e^{\frac{-4}{7}\pi i})$ $(z-e^{\frac{6}{7}\pi i})(z-e^{\frac{-6}{7}\pi i})\times$	M1		For identifying at least one pair of complex conjugate factors
$\times(z-1)$	B1		For linear factor seen
$=(z^2-(2\cos\frac{2}{7}\pi)z+1)\times$	A1		For any one quadratic factor seen
$(z^2-(2\cos\frac{4}{7}\pi)z+1)\times(z^2-(2\cos\frac{6}{7}\pi)z+1)\times$ $\times(z-1)$	A1	**5**	For the other 2 quadratic factors and expression written as product of 4 factors
		10	

Imagine having a regular n-gon in your complex HOME...

OCR JUNE 2010 FP3

3 In this question, w denotes the complex number $\cos\frac{2}{5}\pi + \mathrm{i}\sin\frac{2}{5}\pi$.

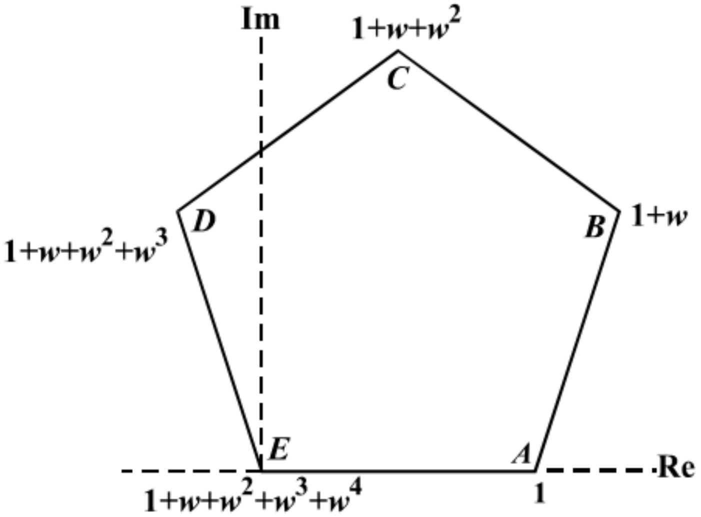
(i) Express w^2, w^3 and w^* in polar form, with arguments in the interval $0 \leqslant \theta < 2\pi$. **[4]**

(ii) The points in an Argand diagram which represent the numbers

$$1, \quad 1+w, \quad 1+w+w^2, \quad 1+w+w^2+w^3, \quad 1+w+w^2+w^3+w^4$$

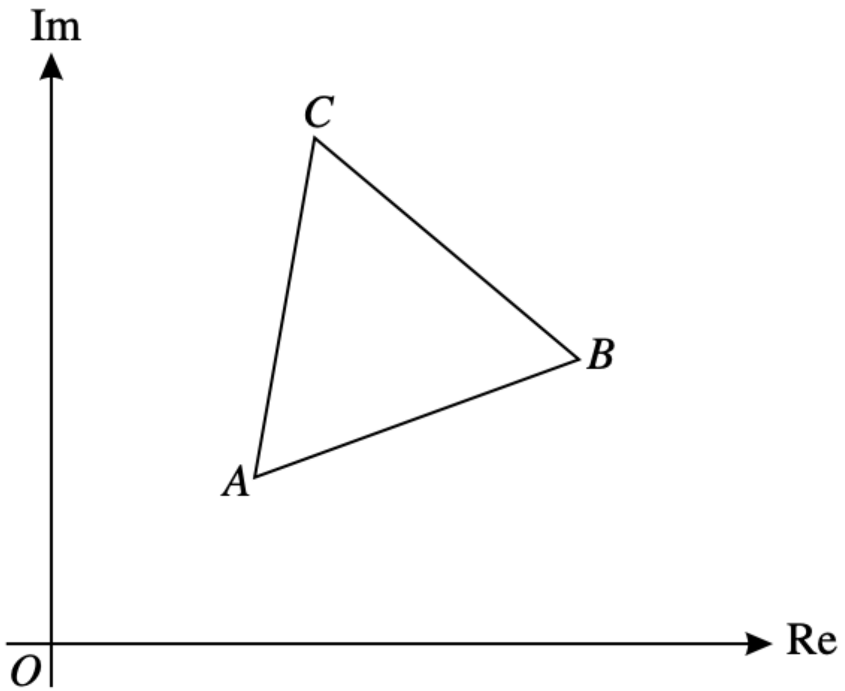
are denoted by A, B, C, D, E respectively. Sketch the Argand diagram to show these points and join them in the order stated. (Your diagram need not be exactly to scale, but it should show the important features.) **[4]**

(iii) Write down a polynomial equation of degree 5 which is satisfied by w. **[1]**

3				Allow $\operatorname{cis}\frac{k}{5}\pi$ and $e^{\frac{k}{5}\pi i}$ throughout
(i)	$w^2 = \cos\frac{4}{5}\pi + i\sin\frac{4}{5}\pi$	B1		For correct value
	$w^3 = \cos\frac{6}{5}\pi + i\sin\frac{6}{5}\pi$	B1		For correct value
	$w^* = \cos\frac{2}{5}\pi - i\sin\frac{2}{5}\pi$	B1		For w^* seen or implied
	$= \cos\frac{8}{5}\pi + i\sin\frac{8}{5}\pi$	B1	**4**	For correct value **SR** For exponential form with i missing, award B0 first time, allow others
(ii)		B1*		For $1+w$ in approximately correct position
		B1 (*dep)		For $AB \approx BC \approx CD$
		B1 (*dep)		For BC, CD equally inclined to Im axis
		B1	**4**	For E at the origin
				Allow points joined by arcs, or not joined Labels not essential
(iii)	$z^5 - 1 = 0$ *OR* $z^5 + z^4 + z^3 + z^2 + z = 0$	B1	**1**	For correct equation **AEF** (in any variable) Allow factorised forms using w, exp or trig
			9	

4 The cube roots of 1 are denoted by 1, ω and ω^2, where the imaginary part of ω is positive.

(i) Show that $1 + \omega + \omega^2 = 0$. **[2]**



In the diagram, ABC is an equilateral triangle, labelled anticlockwise. The points A, B and C represent the complex numbers z_1, z_2 and z_3 respectively.

(ii) State the geometrical effect of multiplication by ω and hence explain why $z_1 - z_3 = \omega(z_3 - z_2)$. **[4]**

(iii) Hence show that $z_1 + \omega z_2 + \omega^2 z_3 = 0$. **[2]**

OCR JAN 2011 FP3

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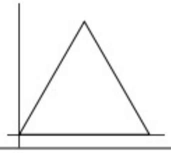
4	(i)	*EITHER* $1+\omega+\omega^2$ = sum of roots of ($z^3-1=0$) = 0	M1 A1	2	For result shown by any correct method **AG**
		OR $\omega^3=1 \Rightarrow (\omega-1)(\omega^2+\omega+1)=0$ $\Rightarrow 1+\omega+\omega^2=0$ (for $\omega \neq 1$)			
		OR sum of G.P. $1+\omega+\omega^2=\dfrac{1-\omega^3}{1-\omega}\left(=\dfrac{0}{1-\omega}\right)=0$			
		OR  shown on Argand diagram or explained in terms of vectors			
		OR $1+\text{cis}\,\frac{2}{3}\pi+\text{cis}\,\frac{4}{3}\pi=1+\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}\text{i}\right)+\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}\text{i}\right)=0$			
	(ii)	Multiplication by $\omega \Rightarrow$ rotation through $\frac{2}{3}\pi$ ↻	B1		For correct interpretation of × by ω (allow 120° and omission of, or error in, ↻)
		$z_1-z_3=\overrightarrow{CA}$, $z_3-z_2=\overrightarrow{BC}$	B1		For identification of vectors soi (ignore direction errors)
		$\overrightarrow{BC}$ rotates through $\frac{2}{3}\pi$ to direction of $\overrightarrow{CA}$	M1		For linking BC and CA by rotation of $\frac{2}{3}\pi$ *OR* ω
		$\Delta\, ABC$ has $BC = CA$, hence result	A1	4	For stating equal magnitudes $\Rightarrow$ **AG**
	(iii)	**(ii)** $\Rightarrow z_1+\omega z_2-(1+\omega)z_3=0$	M1		For using $1+\omega+\omega^2=0$ in **(ii)**
		$1+\omega+\omega^2=0 \Rightarrow z_1+\omega z_2+\omega^2 z_3=0$	A1	2	For obtaining **AG**
				8	

AQA JAN 2010 FP2

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8 (a) (i) Show that $\omega = e^{\frac{2\pi i}{7}}$ is a root of the equation $z^7 = 1$. *(1 mark)*

(ii) Write down the five other non-real roots in terms of ω. *(2 marks)*

(b) Show that

$$1 + \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 = 0$$ *(2 marks)*

(c) Show that:

(i) $\omega^2 + \omega^5 = 2\cos\frac{4\pi}{7}$; *(3 marks)*

(ii) $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}$. *(4 marks)*

MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\left(e^{\frac{2\pi i}{7}}\right)^7 = e^{2\pi i} = 1$	B1	1	Or $z^7 = e^{2k\pi i}$ $z = e^{\frac{2k\pi i}{7}}$ $k = 1$
(ii)	Roots are $\omega^2, \omega^3, \omega^4, \omega^5, \omega^6$	M1A1	2	OE; M1A0 for incomplete set **SC** B1 for a set of correct roots in terms of $e^{i\theta}$
(b)	Sum of roots considered $= 0$	M1 A1	 2	$\left\{ \text{or } \sum_{r=0}^{6} \omega^6 = \frac{\omega^7 - 1}{\omega - 1} = 0 \right.$
(c)(i)	$\omega^2 + \omega^5 = e^{\frac{4\pi i}{7}} + e^{\frac{10\pi i}{7}}$ $= e^{\frac{4\pi i}{7}} + e^{\frac{-4\pi i}{7}}$ $= 2\cos\frac{4\pi}{7}$	M1 A1 A1	 3	 Or $\cos\frac{4\pi}{7} + i\sin\frac{4\pi}{7} + \cos\frac{4\pi}{7} - i\sin\frac{4\pi}{7}$ AG
(ii)	$\omega + \omega^6 = 2\cos\frac{2\pi}{7}$; $\omega^3 + \omega^4 = 2\cos\frac{6\pi}{7}$ Using part (b) Result	B1,B1 M1 A1	 4	Allow these marks if seen earlier in the solution AG
	Total		**12**	

8 (a) Express in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$:

(i) $4(1 + \mathrm{i}\sqrt{3})$;

(ii) $4(1 - \mathrm{i}\sqrt{3})$. *(3 marks)*

(b) The complex number z satisfies the equation

$$(z^3 - 4)^2 = -48$$

Show that $z^3 = 4 \pm 4\sqrt{3}\,\mathrm{i}$. *(2 marks)*

(c) (i) Solve the equation

$$(z^3 - 4)^2 = -48$$

giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. *(5 marks)*

(ii) Illustrate the roots on an Argand diagram. *(3 marks)*

(d) (i) Explain why the sum of the roots of the equation

$$(z^3 - 4)^2 = -48$$

is zero. *(1 mark)*

(ii) Deduce that $\cos\frac{\pi}{9} + \cos\frac{3\pi}{9} + \cos\frac{5\pi}{9} + \cos\frac{7\pi}{9} = \frac{1}{2}$. *(3 marks)*

AQA JAN 2011 FP2

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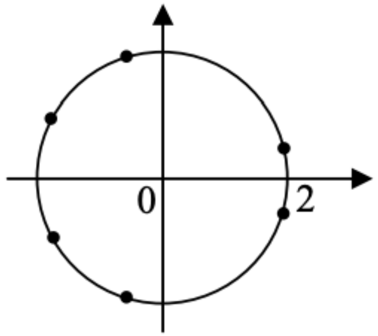
MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$4\left(1+\mathrm{i}\sqrt{3}\right)=8\left(\frac{1}{2}+\mathrm{i}\frac{\sqrt{3}}{2}\right)$	M1		for either $4\left(1+\mathrm{i}\sqrt{3}\right)$ or $4\left(1-\mathrm{i}\sqrt{3}\right)$ used
	$=8\mathrm{e}^{\frac{\pi\mathrm{i}}{3}}$	A1		If either r or θ is incorrect but the same value in both (i) and (ii) allow A1 but for θ only if it is given as $\frac{\pi}{6}$
(ii)	$4\left(1-\mathrm{i}\sqrt{3}\right)=8\mathrm{e}^{\frac{-\pi\mathrm{i}}{3}}$	A1	3	
(b)	$z^3-4=\pm\sqrt{-48}$	M1		taking square root
	$z^3=4\pm4\sqrt{3}\,\mathrm{i}$	A1	2	AG

(c)(i)	$z = 2e^{\frac{\frac{\pi i}{3}+2k\pi i}{3}}$ or $z = 2e^{\frac{\frac{-\pi i}{3}+2k\pi i}{3}}$	B1F M1		for the 2; ft incorrect 8, but no decimals for either, PI
	$z = 2e^{\frac{\pi i}{9}}, 2e^{\frac{7\pi i}{9}}, 2e^{\frac{5\pi i}{9}}$ $= 2e^{\frac{-\pi i}{9}}, 2e^{\frac{-7\pi i}{9}}, 2e^{\frac{-5\pi i}{9}}$	A3,2,1F	5	Allow A1 for any 2 roots not +/– each other Allow A2 for any 3 roots not +/– each other Allow A3 for all 6 correct roots Deduct A1 for each incorrect root in the interval; ignore roots outside the interval ft incorrect *r*
(ii)	 Radius 2	B1F		clearly indicated; ft incorrect *r*
	Plotting roots	B2,1	3	allow B1 for 3 correct points condone lines
(d)(i)	Sum of roots = 0 as coefficient of $z^5 = 0$	E1	1	OE
(ii)	Use of, say, $\frac{1}{2}\left(e^{\frac{\pi i}{9}} + e^{\frac{-\pi i}{9}}\right) = \cos\frac{\pi}{9}$	M1		
	$\cos\frac{3\pi}{9} = \frac{1}{2}$ used	A1		
	$\cos\frac{\pi}{9} + \cos\frac{3\pi}{9} + \cos\frac{5\pi}{9} + \cos\frac{7\pi}{9} = \frac{1}{2}$	A1	3	AG
	Total		**17**	

Imagine having a regular n-gon in your complex HOME...

AQA JAN 2013 FP2

8 (a) Express $-4+4\sqrt{3}\,\mathrm{i}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\leqslant\pi$. *(3 marks)*

(b) (i) Solve the equation $z^3=-4+4\sqrt{3}\,\mathrm{i}$, giving your answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\leqslant\pi$. *(4 marks)*

(ii) The roots of the equation $z^3=-4+4\sqrt{3}\,\mathrm{i}$ are represented by the points P, Q and R on an Argand diagram.

Find the area of the triangle PQR, giving your answer in the form $k\sqrt{3}$, where k is an integer. *(3 marks)*

(c) By considering the roots of the equation $z^3=-4+4\sqrt{3}\,\mathrm{i}$, show that

$$\cos\frac{2\pi}{9}+\cos\frac{4\pi}{9}+\cos\frac{8\pi}{9}=0$$ *(4 marks)*

MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$r = 8$	B1		
	$\tan^{-1} \pm \frac{4\sqrt{3}}{4}$ or $\pm\frac{\pi}{3}$ seen	M1		or $\frac{\pi}{6}$ marked as angle to Im axis with "vector" in second quadrant on Arg diag
	$\Rightarrow \theta = \frac{2\pi}{3}$	A1	3	$-4 + 4\sqrt{3}\mathrm{i} = 8\,\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$
(b)(i)	modulus of each root = 2	B1√		
		M1		use of De Moivre – dividing argument by 3
	$\Rightarrow \theta = -\frac{4\pi}{9}, \frac{2\pi}{9}, \frac{8\pi}{9}$	A2	4	A1 if 3 "correct" values not all in requested interval $2\,\mathrm{e}^{-\mathrm{i}\frac{4\pi}{9}},\ 2\,\mathrm{e}^{\mathrm{i}\frac{2\pi}{9}},\ 2\,\mathrm{e}^{\mathrm{i}\frac{8\pi}{9}}$
(ii)	Area = $3 \times \frac{1}{2} \times PO \times OR \times \sin\frac{2\pi}{3}$	M1		Correct expression for area of triangle PQR
	$= 3 \times \frac{1}{2} \times 2 \times 2 \times \sin\frac{2\pi}{3}$	A1		correct values of lengths in formula
	$= 3\sqrt{3}$	A1cso	3	

(c)	Sum of roots (of cubic) = 0	E1		must be stated explicitly
	Sum of 3 roots including Im terms	M1		in form $r(\cos\theta + i\sin\theta)$
	$2\left(\cos\frac{(-)4\pi}{9} + \cos\frac{2\pi}{9} + \cos\frac{8\pi}{9}\right)$	A1		isolating real terms ; correct and with "2"
	$e^{-i\frac{4\pi}{9}} = \cos\frac{4\pi}{9} - i\sin\frac{4\pi}{9}$ seen earlier			*or* $\cos\frac{-4\pi}{9} = \cos\frac{4\pi}{9}$ explicitly stated to earn final A1 mark
	$\cos\frac{2\pi}{9} + \cos\frac{4\pi}{9} + \cos\frac{8\pi}{9} = 0$	A1**cso**	4	**AG**
	Total		**14**	

AQA JAN 2007 FP2

Imagine having a regular n-gon in your complex HOME…

6 (a) Find the three roots of $z^3 = 1$, giving the non-real roots in the form $e^{i\theta}$, where $-\pi < \theta \leqslant \pi$. *(2 marks)*

(b) Given that ω is one of the non-real roots of $z^3 = 1$, show that

$$1 + \omega + \omega^2 = 0$$

(2 marks)

(c) By using the result in part (b), or otherwise, show that:

(i) $\dfrac{\omega}{\omega + 1} = -\dfrac{1}{\omega}$; *(2 marks)*

(ii) $\dfrac{\omega^2}{\omega^2 + 1} = -\omega$; *(1 mark)*

(iii) $\left(\dfrac{\omega}{\omega + 1}\right)^k + \left(\dfrac{\omega^2}{\omega^2 + 1}\right)^k = (-1)^k\, 2\cos\frac{2}{3}k\pi$, where k is an integer. *(5 marks)*

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	$1, e^{\pm\frac{2\pi i}{3}}$	M1A1	2	M1 for any method which would lead to the correct answers Accept e^0 or e^{0i} Also accept answers written down correctly
(b)	Any correct method Shown for one root	M1 A1	2	 AG

(c)(i)	$\frac{\omega}{\omega+1}=\frac{\omega}{-\omega^2}$	M1		ie use of result in (b)
	$=-\frac{1}{\omega}$	A1	2	AG
(ii)	$\frac{\omega^2}{\omega^2+1}=-\omega$	A1	1	AG
(iii)	$\left(\frac{\omega}{\omega+1}\right)^k+\left(\frac{\omega^2}{\omega^2+1}\right)^k=\left(-\frac{1}{\omega}\right)^k+(-\omega)^k$	M1A1		
	Use of $\omega=\mathrm{e}^{\frac{2\pi i}{3}}$	m1		
	$=(-1)^k\left(\mathrm{e}^{\frac{-2k\pi i}{3}}+\mathrm{e}^{\frac{2k\pi i}{3}}\right)$	A1		
	$=(-1)^k\,2\cos\frac{2k\pi}{3}$	A1	5	AG
	Total		**12**	

Imagine having a regular n-gon in your complex HOME...

AQA JAN 2010 Q8

8 (a) (i) Show that $\omega = e^{\frac{2\pi i}{7}}$ is a root of the equation $z^7 = 1$. *(1 mark)*

(ii) Write down the five other non-real roots in terms of ω. *(2 marks)*

(b) Show that

$$1 + \omega + \omega^2 + \omega^3 + \omega^4 + \omega^5 + \omega^6 = 0$$ *(2 marks)*

(c) Show that:

(i) $\omega^2 + \omega^5 = 2\cos\frac{4\pi}{7}$; *(3 marks)*

(ii) $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} = -\frac{1}{2}$. *(4 marks)*

MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\left(e^{\frac{2\pi i}{7}}\right)^7 = e^{2\pi i} = 1$	B1	1	Or $z^7 = e^{2k\pi i}$ $z = e^{\frac{2k\pi i}{7}}$ $k = 1$
(ii)	Roots are $\omega^2, \omega^3, \omega^4, \omega^5, \omega^6$	M1A1	2	OE; M1A0 for incomplete set **SC** B1 for a set of correct roots in terms of $e^{i\theta}$
(b)	Sum of roots considered $= 0$	M1 A1	2	$\left\{ \text{or } \sum_{r=0}^{6} \omega^6 = \frac{\omega^7 - 1}{\omega - 1} = 0 \right.$
(c)(i)	$\omega^2 + \omega^5 = e^{\frac{4\pi i}{7}} + e^{\frac{10\pi i}{7}}$ $= e^{\frac{4\pi i}{7}} + e^{\frac{-4\pi i}{7}}$ $= 2\cos\frac{4\pi}{7}$	M1 A1 A1	3	 Or $\cos\frac{4\pi}{7} + i\sin\frac{4\pi}{7} + \cos\frac{4\pi}{7} - i\sin\frac{4\pi}{7}$ AG
(ii)	$\omega + \omega^6 = 2\cos\frac{2\pi}{7}$; $\omega^3 + \omega^4 = 2\cos\frac{6\pi}{7}$ Using part (b) Result	B1,B1 M1 A1	4	Allow these marks if seen earlier in the solution AG
	Total		**12**	

Imagine having a regular n-gon in your complex HOME...

AQA JAN 2013 FP2

8 (a) Express $-4+4\sqrt{3}\,\mathrm{i}$ in the form $re^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\leqslant\pi$. *(3 marks)*

(b) (i) Solve the equation $z^3=-4+4\sqrt{3}\,\mathrm{i}$, giving your answers in the form $re^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\leqslant\pi$. *(4 marks)*

(ii) The roots of the equation $z^3=-4+4\sqrt{3}\,\mathrm{i}$ are represented by the points P, Q and R on an Argand diagram.

Find the area of the triangle PQR, giving your answer in the form $k\sqrt{3}$, where k is an integer. *(3 marks)*

(c) By considering the roots of the equation $z^3=-4+4\sqrt{3}\,\mathrm{i}$, show that

$$\cos\frac{2\pi}{9}+\cos\frac{4\pi}{9}+\cos\frac{8\pi}{9}=0$$

(4 marks)

MFP2 (cont)

Q	Solution	Marks	Total	Comments
8(a)	$r = 8$	B1		
	$\tan^{-1} \pm \frac{4\sqrt{3}}{4}$ or $\pm\frac{\pi}{3}$ seen	M1		or $\frac{\pi}{6}$ marked as angle to Im axis with "vector" in second quadrant on Arg diag
	$\Rightarrow \theta = \frac{2\pi}{3}$	A1	3	$-4 + 4\sqrt{3}\mathrm{i} = 8\,\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$
(b)(i)	modulus of each root = 2	B1√		
		M1		use of De Moivre – dividing argument by 3
	$\Rightarrow \theta = -\frac{4\pi}{9}, \frac{2\pi}{9}, \frac{8\pi}{9}$	A2	4	A1 if 3 "correct" values not all in requested interval
				$2\,\mathrm{e}^{-\mathrm{i}\frac{4\pi}{9}},\ 2\,\mathrm{e}^{\mathrm{i}\frac{2\pi}{9}},\ 2\,\mathrm{e}^{\mathrm{i}\frac{8\pi}{9}}$
(ii)	Area = $3 \times \frac{1}{2} \times PO \times OR \times \sin\frac{2\pi}{3}$	M1		Correct expression for area of triangle PQR
	$= 3 \times \frac{1}{2} \times 2 \times 2 \times \sin\frac{2\pi}{3}$	A1		correct values of lengths in formula
	$= 3\sqrt{3}$	A1cso	3	

(c)	Sum of roots (of cubic) = 0 Sum of 3 roots including Im terms $2\left(\cos\frac{(-)4\pi}{9}+\cos\frac{2\pi}{9}+\cos\frac{8\pi}{9}\right)$ $e^{-i\frac{4\pi}{9}}=\cos\frac{4\pi}{9}-i\sin\frac{4\pi}{9}$ seen earlier $\cos\frac{2\pi}{9}+\cos\frac{4\pi}{9}+\cos\frac{8\pi}{9}=0$	E1 M1 A1 A1**cso**	 4	must be stated explicitly in form $r(\cos\theta+i\sin\theta)$ isolating real terms ; correct and with "2" *or* $\cos\frac{-4\pi}{9}=\cos\frac{4\pi}{9}$ explicitly stated to earn final A1 mark **AG**
	Total		**14**	

Imagine having a regular n-gon in your complex HOME...

MEI JUNE 2007 FP2

2 **(a)** Use de Moivre's theorem to show that $\sin 5\theta = 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$. [5]

(b) **(i)** Find the cube roots of $-2 + 2\mathrm{j}$ in the form $re^{\mathrm{j}\theta}$ where $r > 0$ and $-\pi < \theta \leqslant \pi$. [6]

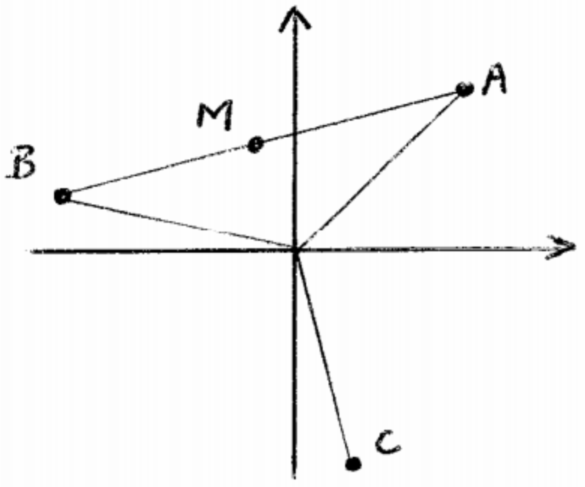
These cube roots are represented by points A, B and C in the Argand diagram, with A in the first quadrant and ABC going anticlockwise. The midpoint of AB is M, and M represents the complex number w.

(ii) Draw an Argand diagram, showing the points A, B, C and M. [2]

(iii) Find the modulus and argument of w. [2]

(iv) Find w^6 in the form $a + b\mathrm{j}$. [3]

2 (a)	$(\cos\theta + \mathrm{j}\sin\theta)^5$ $= c^5 + 5\mathrm{j}c^4 s - 10c^3 s^2 - 10\mathrm{j}c^2 s^3 + 5cs^4 + \mathrm{j}s^5$ Equating imaginary parts $\sin 5\theta = 5c^4 s - 10c^2 s^3 + s^5$ $= 5(1-s^2)^2 s - 10(1-s^2)s^3 + s^5$ $= 5s - 10s^3 + 5s^5 - 10s^3 + 10s^5 + s^5$ $= 5\sin\theta - 20\sin^3\theta + 16\sin^5\theta$	M1 M1 A1 M1 A1 ag **5**	
(b)(i)	$\lvert -2+2\mathrm{j} \rvert = \sqrt{8}$, $\arg(-2+2\mathrm{j}) = \frac{3}{4}\pi$ $r = \sqrt{2}$ $\theta = \frac{1}{4}\pi$ $\theta = \frac{11}{12}\pi$, $-\frac{5}{12}\pi$	B1B1 B1 ft B1 ft M1 A1 **6**	Accept 2.8; 2.4, 135° (Implies B1 for $\sqrt{8}$) One correct (Implies B1 for $\frac{3}{4}\pi$) Adding or subtracting $\frac{2}{3}\pi$ Accept $\theta = \frac{1}{4}\pi + \frac{2}{3}k\pi$, $k = 0, 1, -1$

(ii)		B2 **2**	Give B1 for two of B, C, M in the correct quadrants Give B1 ft for all four points in the correct quadrants
(iii)	$\lvert w \rvert = \frac{1}{2}\sqrt{2}$ $\arg w = \frac{1}{2}(\frac{1}{4}\pi + \frac{11}{12}\pi) = \frac{7}{12}\pi$	B1 ft B1 **2**	Accept 0.71 Accept 1.8
(iv)	$\lvert w^6 \rvert = (\frac{1}{2}\sqrt{2})^6 = \frac{1}{8}$ $\arg(w^6) = 6 \times \frac{7}{12}\pi = \frac{7}{2}\pi$ $w^6 = \frac{1}{8}(\cos\frac{7}{2}\pi + \mathrm{j}\sin\frac{7}{2}\pi)$ $= -\frac{1}{8}\mathrm{j}$	M1 A1 ft A1 **3**	Obtaining either modulus or argument Both correct (ft) Allow from $\arg w = \frac{1}{4}\pi$ etc
			SR If B, C interchanged on diagram (ii) B1 (iii) B1 B1 for $-\frac{1}{12}\pi$ (iv) M1A1A1

Imagine having a regular n-gon in your complex HOME...

MEI JAN 2009 FP2

2 **(i)** Write down the modulus and argument of the complex number $e^{j\pi/3}$. **[2]**

(ii) The triangle OAB in an Argand diagram is equilateral. O is the origin; A corresponds to the complex number $a = \sqrt{2}(1 + j)$; B corresponds to the complex number b.

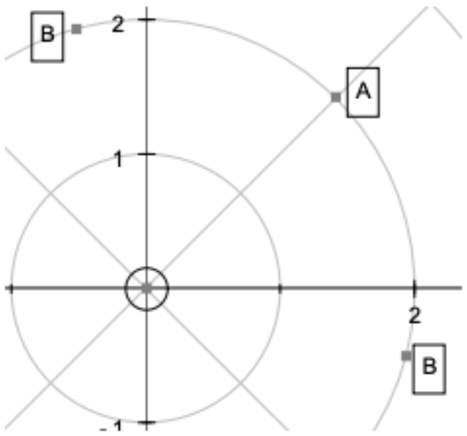
Show A and the two possible positions for B in a sketch. Express a in the form $re^{j\theta}$. Find the two possibilities for b in the form $re^{j\theta}$. **[5]**

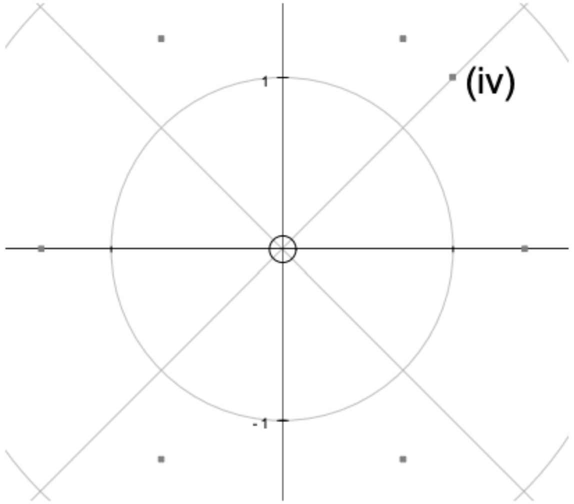
(iii) Given that $z_1 = \sqrt{2}e^{j\pi/3}$, show that $z_1^6 = 8$. Write down, in the form $re^{j\theta}$, the other five complex numbers z such that $z^6 = 8$. Sketch all six complex numbers in a new Argand diagram. **[6]**

Let $w = z_1 e^{-j\pi/12}$.

(iv) Find w in the form $x + jy$, and mark this complex number on your Argand diagram. **[3]**

(v) Find w^6, expressing your answer in as simple a form as possible. **[2]**

2 (i)	Modulus = 1 Argument = $\frac{\pi}{3}$	B1 B1 **2**	Must be separate Accept 60°, 1.05^c
(ii)	 $a = 2e^{\frac{j\pi}{4}}$ $\arg b = \frac{\pi}{4} \pm \frac{\pi}{3}$ $b = 2e^{-\frac{j\pi}{12}}, 2e^{\frac{7j\pi}{12}}$	G2,1,0 B1 M1 A1ft **5**	G2: A in first quadrant, argument $\approx \frac{\pi}{4}$ B in second quadrant, same mod B′ in fourth quadrant, same mod Symmetry G1: 3 points and at least 2 of above, or B, B′ on axes, or BOB′ straight line, or BOB′ reflex Must be in required form (accept $r = 2$, $\theta = \pi/4$) Rotate by adding (or subtracting) $\pi/3$ to (or from) argument. Must be $\pi/3$ Both. Ft value of r for a. Must be in required form, but don't penalise twice

(iii)	$z_1^{\,6} = \left(\sqrt{2}e^{\frac{j\pi}{3}}\right)^6 = \left(\sqrt{2}\right)^6 e^{2j\pi}$ $= 8$ Others are $re^{j\theta}$ where $r = \sqrt{2}$ and $\theta = -\frac{2\pi}{3}, -\frac{\pi}{3}, 0, \frac{2\pi}{3}, \pi$  	M1 A1 (ag) M1 A1 G1 G1 **6**	$\left(\sqrt{2}\right)^6 = 8$ or $\frac{\pi}{3} \times 6 = 2\pi$ seen www "Add" $\frac{\pi}{3}$ to argument more than once Correct constant r and five values of θ. Accept θ in [0, 2π] or in degrees 6 points on vertices of regular hexagon Correctly positioned (2 roots on real axis). Ignore scales SC1 if G0 and 5 points correctly plotted
(iv)	$w = z_1 e^{-\frac{j\pi}{12}} = \sqrt{2}e^{\frac{j\pi}{3}} e^{-\frac{j\pi}{12}} = \sqrt{2}e^{\frac{j\pi}{4}}$ $= \sqrt{2}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right)$ $= 1 + j$	M1 A1 G1 **3**	$\arg w = \frac{\pi}{3} - \frac{\pi}{12}$ Or B2 Same modulus as z_1
(v)	$w^6 = \left(\sqrt{2}e^{\frac{j\pi}{4}}\right)^6 = 8e^{\frac{3j\pi}{2}}$ $= -8j$	M1 A1 **2**	Or $z_1^{\,6} e^{-\frac{j\pi}{2}} = 8\,e^{-\frac{j\pi}{2}}$ cao. Evaluated **18**

Imagine having a regular n-gon in your complex HOME...

MEI JUNE 2010 FP2

2 **(a)** Given that $z = \cos\theta + \mathrm{j}\sin\theta$, express $z^n + \frac{1}{z^n}$ and $z^n - \frac{1}{z^n}$ in simplified trigonometric form.

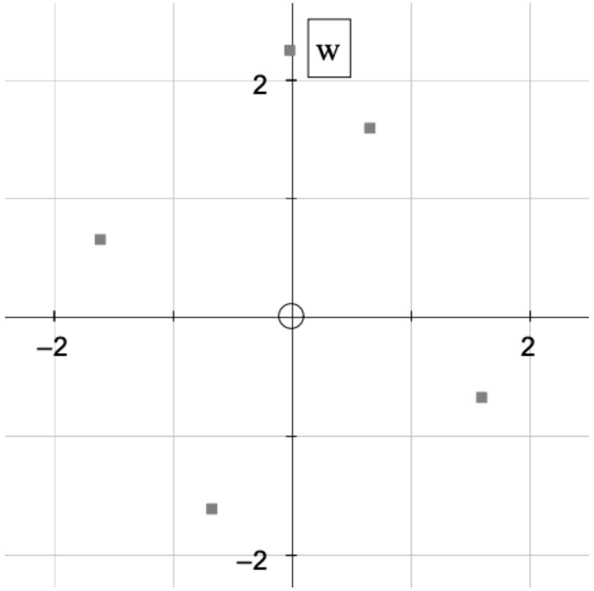
Hence find the constants A, B, C in the identity

$$\sin^5\theta \equiv A\sin\theta + B\sin 3\theta + C\sin 5\theta.$$ **[5]**

(b) **(i)** Find the 4th roots of $-9\mathrm{j}$ in the form $r\mathrm{e}^{\mathrm{j}\theta}$, where $r > 0$ and $0 < \theta < 2\pi$. Illustrate the roots on an Argand diagram. **[6]**

(ii) Let the points representing these roots, taken in order of increasing θ, be P, Q, R, S. The mid-points of the sides of PQRS represent the 4th roots of a complex number w. Find the modulus and argument of w. Mark the point representing w on your Argand diagram. **[5]**

2 (a)	$z^n + \frac{1}{z^n} = 2\cos n\theta$, $z^n - \frac{1}{z^n} = 2j\sin n\theta$	B1	Both
	$\left(z - \frac{1}{z}\right)^5 = z^5 - 5z^3 + 10z - \frac{10}{z} + \frac{5}{z^3} - \frac{1}{z^5}$	M1	Expanding $\left(z - \frac{1}{z}\right)^5$
	$= z^5 - \frac{1}{z^5} - 5\left(z^3 - \frac{1}{z^3}\right) + 10\left(z - \frac{1}{z}\right)$		
	$\Rightarrow$ $32j\sin^5\theta = 2j\sin 5\theta - 10j\sin 3\theta + 20j\sin\theta$	M1 A1	Introducing sines (and possibly cosines) of multiple angles RHS
	$\Rightarrow$ $\sin^5\theta = \frac{1}{16}\sin 5\theta - \frac{5}{16}\sin 3\theta + \frac{5}{8}\sin\theta$	A1ft	Division by 32(j)
	$A = \frac{5}{8}$, $B = -\frac{5}{16}$, $C = \frac{1}{16}$	**5**	

(b)(i)	4^{th} roots of $-9j = 9e^{\frac{3}{2}\pi j}$ are $re^{j\theta}$ where		
	$r = \sqrt{3}$	B1	Accept $9^{\frac{1}{4}}$
	$\theta = \frac{3\pi}{8}$	B1	
	$\Rightarrow \quad \theta = \frac{3\pi}{8} + \frac{2k\pi}{4}$	M1	Implied by at least two correct (ft) further values
	$\Rightarrow \quad \theta = \frac{7\pi}{8}, \frac{11\pi}{8}, \frac{15\pi}{8}$	A1	Or stating $k = (0), 1, 2, 3$ Allow arguments in range $-\pi \le \theta \le \pi$
		M1 A1 **6**	Points at vertices of a square centre O or 3 correct points (ft) or 1 point in each quadrant
(ii)	Mid-point of SP has argument $\frac{\pi}{8}$	B1	
	and modulus of $\sqrt{\frac{3}{2}}$	B1	
	Argument of $w = 4 \times \frac{\pi}{8} = \frac{\pi}{2}$		
	and modulus $= \left(\sqrt{\frac{3}{2}}\right)^4 = \frac{9}{4}$	M1 A1 G1 **5**	Multiplying argument by 4 and modulus raised to power of 4 Both correct w plotted on imag. axis above level of P **16**

MEI JUNE 2011 FP2

Imagine having a regular n-gon in your complex HOME…

2 **(a)** Use de Moivre's theorem to find expressions for $\sin 5\theta$ and $\cos 5\theta$ in terms of $\sin\theta$ and $\cos\theta$.

Hence show that, if $t = \tan\theta$, then

$$\tan 5\theta = \frac{t(t^4 - 10t^2 + 5)}{5t^4 - 10t^2 + 1}.$$ **[6]**

(b) **(i)** Find the 5th roots of $-4\sqrt{2}$ in the form $re^{j\theta}$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. **[4]**

These 5th roots are represented in the Argand diagram, in order of increasing θ, by the points A, B, C, D, E.

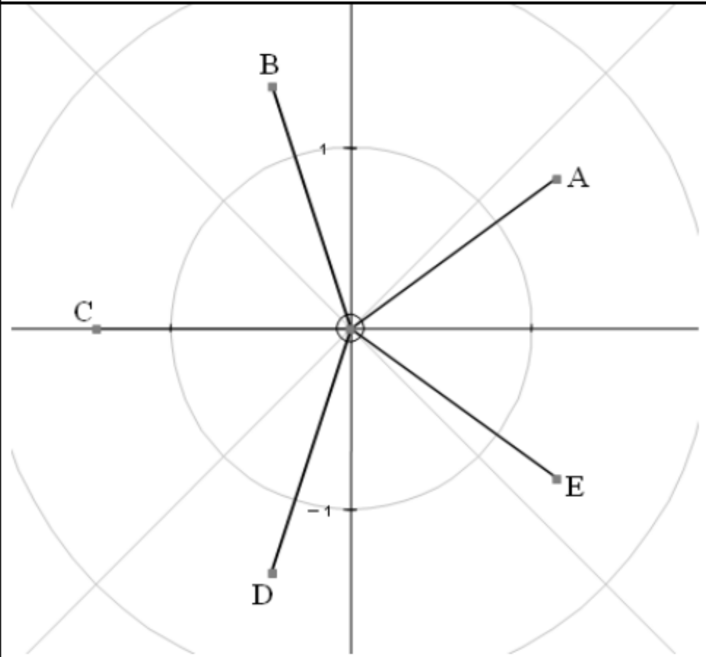
(ii) Draw the Argand diagram, making clear which point is which. **[2]**

The mid-point of AB is the point P which represents the complex number w.

(iii) Find, in exact form, the modulus and argument of w. **[3]**

(iv) w is an nth root of a real number a, where n is a positive integer. State the least possible value of n and find the corresponding value of a. **[3]**

2 (a)	$\cos 5\theta + j\sin 5\theta = (\cos\theta + j\sin\theta)^5$ $= c^5 + 5c^4js - 10c^3s^2 - 10c^2js^3 + 5cs^4 + js^5$	M1 M1	Expanding Separating real and imaginary parts. Dependent on first M1
	$\Rightarrow \cos 5\theta = c^5 - 10c^3s^2 + 5cs^4$ $\sin 5\theta = 5c^4s - 10c^2s^3 + s^5$	A1 A1	Alternative: $16c^5 - 20c^3 + 5c$ Alternative: $16s^5 - 20s^3 + 5s$
	$\Rightarrow \tan 5\theta = \dfrac{5c^4s - 10c^2s^3 + s^5}{c^5 - 10c^3s^2 + 5cs^4}$ $= \dfrac{5t - 10t^3 + t^5}{1 - 10t^2 + 5t^4}$	M1	Using $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ and simplifying
	$= \dfrac{t\left(t^4 - 10t^2 + 5\right)}{5t^4 - 10t^2 + 1}$	A1 (ag) **6**	
(b)(i)	$\arg(-4\sqrt{2}) = \pi$ $\Rightarrow$ fifth roots have $r = \sqrt{2}$	B1	
	and $\theta = \dfrac{\pi}{5}$	B1	No credit for arguments in degrees
	$\Rightarrow z = \sqrt{2}e^{\frac{1}{5}j\pi}, \sqrt{2}e^{\frac{3}{5}j\pi}, \sqrt{2}e^{j\pi}, \sqrt{2}e^{\frac{7}{5}j\pi}, \sqrt{2}e^{\frac{9}{5}j\pi}$	M1 A1 **4**	Adding (or subtracting) $\dfrac{2\pi}{5}$ All correct. Allow $-\pi \le \theta < \pi$

(ii)		G1 G1 **2**	Points at vertices of "regular" pentagon, with one on negative real axis Points correctly labelled
(iii)	$\arg(w)=\frac{1}{2}\left(\frac{\pi}{5}+\frac{3\pi}{5}\right)=\frac{2\pi}{5}$ $\lvert w\rvert=\sqrt{2}\cos\frac{\pi}{5}$	B1 M1 A1ft **3**	 Attempting to find length F.t. (positive) r from (i)
(iv)	$w=\sqrt{2}\cos\frac{\pi}{5}\,e^{\frac{2}{5}\pi j}\Rightarrow w^n=\left(\sqrt{2}\cos\frac{\pi}{5}\right)^n e^{\frac{2}{5}\pi nj}$ which is real if $\sin\frac{2\pi n}{5}=0\Rightarrow n=5$ $\lvert w^5\rvert=\left(\sqrt{2}\cos\frac{\pi}{5}\right)^5$ $\Rightarrow a=2^{\frac{5}{2}}\cos^5\frac{\pi}{5}$	 B1 M1 A1 **3**	 Attempting the nth power of his modulus in (iii), or attempting the modulus of the nth power here Accept 1.96 or better **18**

MEI JUNE 2012 FP2

Imagine having a regular n-gon in your complex HOME...

2 **(a)** **(i)** Show that

$$1 + e^{j2\theta} = 2\cos\theta(\cos\theta + j\sin\theta).$$ **[2]**

(ii) The series C and S are defined as follows.

$$C = 1 + \binom{n}{1}\cos 2\theta + \binom{n}{2}\cos 4\theta + \ldots + \cos 2n\theta$$

$$S = \binom{n}{1}\sin 2\theta + \binom{n}{2}\sin 4\theta + \ldots + \sin 2n\theta$$

By considering $C + jS$, show that

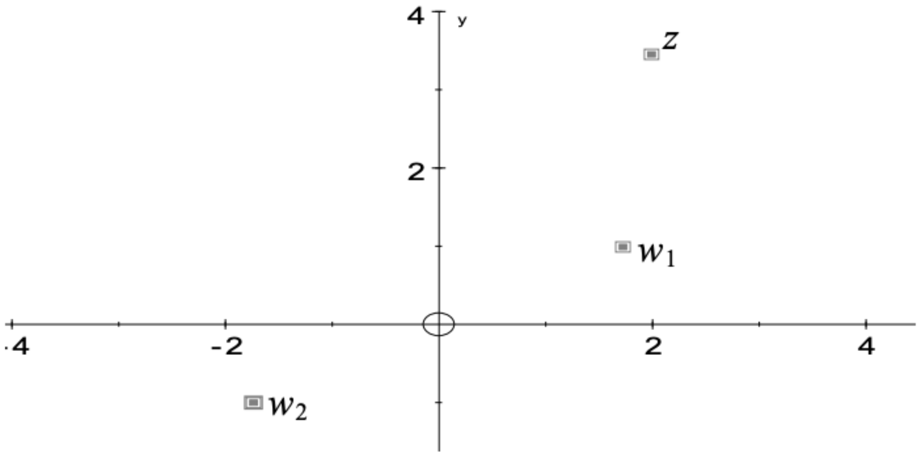
$$C = 2^n \cos^n\theta \cos n\theta,$$

and find a corresponding expression for S. **[7]**

(b) **(i)** Express $e^{j2\pi/3}$ in the form $x + jy$, where the real numbers x and y should be given exactly. **[1]**

(ii) An equilateral triangle in the Argand diagram has its centre at the origin. One vertex of the triangle is at the point representing $2 + 4j$. Obtain the complex numbers representing the other two vertices, giving your answers in the form $x + jy$, where the real numbers x and y should be given exactly. **[6]**

(iii) Show that the length of a side of the triangle is $2\sqrt{15}$. **[2]**

Question			Answer	Marks	Guidance	
2	(a)	(iii)	$\cos^4\theta = \frac{3}{8} + \frac{1}{2}\left(2\cos^2\theta - 1\right) + \frac{1}{8}\cos 4\theta$	M1	Using (ii), obtaining cos 4θ and expressing cos 2θ in terms of $\cos^2\theta$	Condone cos $2\theta = \pm 1 \pm 2\cos^2\theta$
			$\Rightarrow \cos^4\theta = \cos^2\theta - \frac{1}{8} + \frac{1}{8}\cos 4\theta$			
			$\Rightarrow \cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$	A1 [2]	c.a.o.	
2	(b)	(i)	$z = 4e^{\frac{j\pi}{3}}$ and $w^2 = z$: let $w = re^{j\theta} \Rightarrow w^2 = r^2 e^{2j\theta}$			
			$\Rightarrow r^2 = 4 \Rightarrow r = 2$	B1		Condone $r = \pm 2$
			and $\theta = \frac{\pi}{6}, \frac{7\pi}{6}$	B1B1	Or $-\frac{5\pi}{6}$	Award B2 for $\pi\left(k + \frac{1}{6}\right)$
						Ignore annotations and scales
				B1	Roots with approx. equal moduli and approx. correct argument	$\leq \pi/4$
				B1	Dependent on first B1 z in correct position	Modulus and argument bigger
				[5]		

2	(b)	(ii)	$z=4e^{\frac{j\pi}{3}} \Rightarrow z^n=4^n e^{\frac{j\pi n}{3}}$ so real if $\frac{\pi n}{3}=\pi \Rightarrow n=3$ Imaginary if $\frac{\pi n}{3}=\frac{\pi}{2}+k\pi \Rightarrow n=\frac{3}{2}+3k$ which is not an integer for any k $w_1=2e^{\frac{j\pi}{6}} \Rightarrow w_1{}^3=8e^{\frac{j\pi}{2}}=8j$ $w_2=2e^{\frac{7j\pi}{6}} \Rightarrow w_2{}^3=8e^{\frac{7j\pi}{2}}=-8j$	B1 M1 A1(ag) M1 A1 **[5]**	 $\cos\frac{\pi n}{3}=0$ or $\frac{\pi n}{3}=\frac{\pi}{2}$... An argument which covers the positive and negative im. axis Attempting their w^3 in any form 8j, −8j	Ignore other larger values Must deal with mod and arg