# 06-S2-Q10



**Collision**

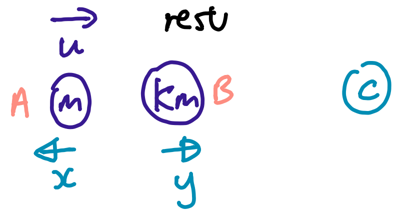
Coefficient of restitution, second time.

**10** Three particles, $A$, $B$ and $C$, of masses $m$, $km$ and $3m$ respectively, are initially at rest lying in a straight line on a smooth horizontal surface. Then $A$ is projected towards $B$ at speed $u$. After the collision, $B$ collides with $C$. The coefficient of restitution between $A$ and $B$ is $\frac{1}{2}$ and the coefficient of restitution between $B$ and $C$ is $\frac{1}{4}$.

**(i)** Find the range of values of $k$ for which $A$ and $B$ collide for a second time.

**(ii)** Given that $k = 1$ and that $B$ and $C$ are initially a distance $d$ apart, show that the time that elapses between the two collisions of $A$ and $B$ is $\dfrac{60d}{13u}$.

**[STEP II 2006 Question 10 (Pure)]**




$mu = -mx + kmy$

$e = \dfrac{x+y}{u}$

$-x + ky = u$ (1)

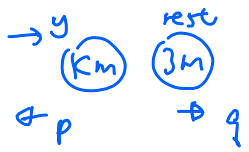
$x + y = eu$ (2)

$y(k+1) = u(e+1)$ $\qquad$ $kx + ky = keu$

$y = \dfrac{u(e+1)}{k+1}$, $\qquad$ $x(k+1) = u(ke-1)$

$x = \dfrac{u(ke-1)}{k+1}$

$y = \dfrac{\frac{3}{2}u}{k+1} = \dfrac{3u}{2(k+1)}$ $\qquad$ $x = \dfrac{(\frac{1}{2}k-1)u}{k+1}$




$kmy = -kmp + 3mq$

$\dfrac{p+q}{y} = \dfrac{1}{4}$

$p + q = \frac{1}{4}y$

$-kp + 3q = ky$

$- \quad 3p + 3q = \frac{3}{4}y$

$p(3+k) = y(\frac{3}{4} - k)$

$p = \dfrac{y(\frac{3}{4}-k)}{3+k}$

Collision occur if:

$p > x$

$\dfrac{\frac{3u}{2(k+1)}(\frac{3}{4}-k)}{3+k} > \dfrac{(\frac{1}{2}k-1)(u)}{k+1}$

$\frac{3}{2}(\frac{3}{4}-k) > (\frac{1}{2}k-1)(3+k)$

$\frac{9}{8} - \frac{3}{2}k > \frac{3}{2}k - 3 + \frac{k^2}{2} - k$

$0 > \frac{k^2}{2} + \frac{1}{2}k + \frac{3}{2}k - \frac{24}{8} - \frac{9}{8}$

$0 > \frac{k^2}{2} + \frac{4k}{2} - \frac{33}{8}$

$0 > 4k^2 + 16k - 33$

$(2k+11)(2k-3) < 0$

$\boxed{0 < k < \frac{3}{2}}$

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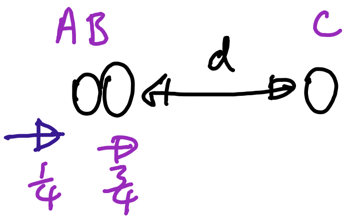


$y = \frac{\frac{3}{2}u}{k+1} = \frac{3u}{2(k+1)}$ $\quad x = \frac{(\frac{1}{2}k-1)u}{k+1}$ $\quad p = \frac{y(\frac{3}{4}-k)}{3+k}$ $\quad \boxed{S=ut}$

$y = \frac{3u}{4}$ $\quad x = \frac{-\frac{1}{2}}{2}u = -\frac{1}{4}u$ $\quad p = \frac{-\frac{1}{4}}{4}y = -\frac{1}{16}y = \frac{-3u}{64}$

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$S=ut$ for B

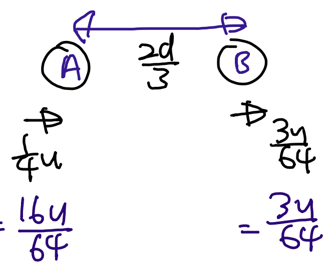
$d = \frac{3}{4}ut$

$\frac{4d}{3u} = t$

$S=ut$ for A $\quad S = \frac{1}{4}u\left(\frac{4d}{3u}\right)$

$S = \frac{d}{3}$

which means:




$S = \frac{2d}{3}$ $\quad$ relative speed $= \frac{13u}{64}$

$S = ut_2$

$\frac{2d}{3} = \frac{13u}{64}t_2$ $\quad t_2 = \frac{128d}{39u}$

$t_1 + t_2$

$\Rightarrow$

$\frac{4d}{3u} + \frac{128}{39}u$

$\frac{52+128}{39} \frac{d}{u}$

$= \frac{180}{39}\frac{d}{u}$

$= \frac{60}{13}\frac{d}{u}$ //