

## AS Pure Mathematics 8MA0: Specimen Paper 1 Mark Scheme

Question	Scheme	Marks	AOs
<b>1 (a)</b>	$y = 2x^3 - 2x^2 - 2x + 8 \Rightarrow \frac{dy}{dx} = 6x^2 - 4x - 2$	M1 A1	1.1b 1.1b
		<b>(2)</b>	
<b>(b)</b>	Attempts $6x^2 - 4x - 2 > 0 \Rightarrow (6x + 2)(x - 1) > 0$	M1	1.1b
	$x = -\frac{1}{3}, 1$	A1	1.1b
	Chooses outside region	M1	1.1b
	$\left\{x : x < -\frac{1}{3}\right\} \cup \{x : x > 1\}$	A1	2.5
		<b>(4)</b>	
<b>(6 marks)</b>			
<b>Notes:</b>			
<p><b>(a)</b>  <b>M1:</b> Attempts to differentiate. Allow for two correct terms un-simplified  <b>A1:</b> <math>\frac{dy}{dx} = 6x^2 - 4x - 2</math></p>			
<p><b>(b)</b>  <b>M1:</b> Attempts to find the critical values of their <math>\frac{dy}{dx} &gt; 0</math> or their <math>\frac{dy}{dx} = 0</math>  <b>A1:</b> Correct critical values <math>x = -\frac{1}{3}, 1</math>  <b>M1:</b> Chooses the outside region  <b>A1:</b> <math>\left\{x : x &lt; -\frac{1}{3}\right\} \cup \{x : x &gt; 1\}</math> or <math>\left\{x : x \in \mathbb{R} \quad x &lt; -\frac{1}{3} \text{ or } x &gt; 1\right\}</math>  Accept also <math>\left\{x : x \ni -\frac{1}{3}\right\} \cup \{x : x \not\subseteq 1\}</math></p>			

Question	Scheme	Marks	AOs
2 (a)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = 6\mathbf{i} - 3\mathbf{j} - (4\mathbf{i} + 2\mathbf{j})$	M1	1.1b
	$= 2\mathbf{i} - 5\mathbf{j}$	A1	1.1b
		(2)	
1(b)	Explains that $\overrightarrow{OC}$ is parallel to $\overrightarrow{AB}$ as $8\mathbf{i} - 20\mathbf{j} = 4 \times (2\mathbf{i} - 5\mathbf{j})$	M1	1.1b
	As $\overrightarrow{OC} = 4 \times \overrightarrow{AB}$ it is parallel to it and not the same length Hence $OABC$ is a trapezium.	A1	2.4
		(2)	
<b>(4 marks)</b>			
<b>Notes:</b>			
<p>(a)</p> <p><b>M1:</b> Attempts <math>\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}</math> or equivalent. This may be implied by one correct component</p> <p><b>A1:</b> <math>2\mathbf{i} - 5\mathbf{j}</math></p> <p>(b)</p> <p><b>M1:</b> Attempts to compare vectors <math>\overrightarrow{OC}</math> and <math>\overrightarrow{AB}</math> by considering their directions</p> <p><b>A1:</b> Fully explains why <math>OABC</math> is a trapezium. (The candidate is required to state that <math>OC</math> is parallel to <math>AB</math> but not the same length as it.)</p>			

Question	Scheme	Marks	AOs
<b>3(a)</b>	Uses or implies that $V = at + b$	B1	3.3
	Uses both $4 = 24a + b$ and $2.8 = 60a + b$ to get either $a$ or $b$	M1	3.1b
	Uses both $4 = 24a + b$ and $2.8 = 60a + b$ to get both $a$ and $b$	M1	1.1b
	$\Rightarrow V = -\frac{1}{30}t + 4.8$	A1	1.1b
		<b>(4)</b>	
<b>(b)</b>	(i) States that the initial volume is $4.8 \text{ m}^3$	B1 ft	3.4
	(ii) Attempts to solve $0 = -\frac{1}{30}t + 4.8$	M1	3.4
	States 144 minutes	A1	1.1b
		<b>(3)</b>	
<b>(c)</b>	States any logical reason <ul style="list-style-type: none"> <li>The tank will leak more quickly at the start due to the greater water pressure</li> <li>The hole will probably get larger and so will start to leak more quickly</li> <li>Sediment could cause the leak to be plugged and so the tank would not empty.</li> </ul>	B1	3.5b
		<b>(1)</b>	
<b>(8 marks)</b>			
<b>Notes:</b>			
<p><b>(a)</b>  <b>B1:</b> Uses or implies that <math>V = at + b</math>  You may award this at their final line but it must be <math>V = f(t)</math>  <b>M1:</b> Awarded for translating the problem in context and starting to solve. It is scored when both <math>4 = 24a + b</math> and <math>2.8 = 60a + b</math> are written down and the candidate proceeds to find either <math>a</math> or <math>b</math>. You may just see a line <math>\pm \frac{4 - 2.8}{60 - 24}</math>  <b>M1:</b> Uses <math>4 = 24a + b</math> and <math>2.8 = 60a + b</math> to find both <math>a</math> and <math>b</math>  <b>A1:</b> <math>V = -\frac{1}{30}t + 4.8</math> or exact equivalent. Eg <math>30V + t = 144</math></p>			
<p><b>(b)(i)</b>  <b>B1ft:</b> Follow through on their 'b'  <b>(b)(ii)</b>  <b>M1:</b> States that <math>V = 0</math> and finds <math>t</math> by attempting to solve their <math>0 = -\frac{1}{30}t + 4.8</math>  <b>A1:</b> States 144 minutes</p>			
<p><b>(c)</b>  <b>B1:</b> States any logical reason. There must be a statement and a reason that matches See scheme</p>			

Question	Scheme	Marks	AOs
<b>4(a)</b>	$(4, -3)$	B1	1.2
		<b>(1)</b>	
<b>(b)</b>	$x = 6$	B1	1.1b
		<b>(1)</b>	
<b>(c)</b>	$x \geq 4$	B1	1.1b
		<b>(1)</b>	
<b>(d)</b>	$k > 1.5$	B1	2.2a
		<b>(1)</b>	
<b>(4 marks)</b>			
<b>Notes:</b>			
See m/scheme			

Question	Scheme	Marks	AOs
5(a)	$f(-3) = (-3)^3 + 3 \times (-3)^2 - 4 \times (-3) - 12$	M1	1.1b
	$f(-3) = 0 \Rightarrow (x+3)$ is a factor $\Rightarrow$ Hence $f(x)$ is divisible by $(x+3)$ .	A1	2.4
		(2)	
(b)	$x^3 + 3x^2 - 4x - 12 = (x+3)(x^2 - 4)$	M1	1.1b
	$= (x+3)(x+2)(x-2)$	dM1 A1	1.1b 1.1b
		(3)	
(c)	$\frac{x^3 + 3x^2 - 4x - 12}{x^3 + 5x^2 + 6x} = \frac{\dots}{x(x^2 + 5x + 6)}$	M1	3.1a
	$= \frac{(x+3)(x+2)(x-2)}{x(x+3)(x+2)}$	dM1	1.1b
	$= \frac{(x-2)}{x} = 1 - \frac{2}{x}$	A1	2.1
		(3)	

**(8 marks)**

**Notes:**

(a)

**M1:** Attempts  $f(-3)$

**A1:** Achieves  $f(-3) = 0$  and explains that  $(x+3)$  is a factor and hence  $f(x)$  is divisible by  $(x+3)$ .

(b)

**M1:** Attempts to divide by  $(x+3)$  to get the quadratic factor.

By division look for the first two terms. ie  $x^2 + 0x$

$$\begin{array}{r}
 x^2 \pm 0x \dots\dots\dots \\
 x+3 \overline{) x^3 + 3x^2 - 4x - 12} \\
 \underline{x^3 + 3x^2} \phantom{- 4x - 12} \\
 \phantom{x^3 + 3x^2} - 4x - 12
 \end{array}$$

By inspection look for the first and last term  $x^3 + 3x^2 - 4x - 12 = (x+3)(x^2 + \dots x \pm 4)$

**dM1:** For an attempt at factorising their  $(x^2 - 4)$ . (Need to check first and last terms)

**A1:**  $f(x) = (x+3)(x+2)(x-2)$

(c)

**M1:** Takes a common factor of  $x$  out of the denominator and writes the numerator in factors.

Alternatively rewrites to  $x^3 + 3x^2 - 4x - 12 = A(x^3 + 5x^2 + 6x) + B(x^2 + 5x + 6)$

**dM1:** Further factorises the denominator and cancels

Alternatively compares terms or otherwise to find either  $A$  or  $B$

**A1:** Shows that  $\frac{x^3 + 3x^2 - 4x - 12}{x^3 + 5x^2 + 6x} = 1 - \frac{2}{x}$  with no errors or omissions

In the alternative there must be a reference to

$$x^3 + 3x^2 - 4x - 12 \equiv 1(x^3 + 5x^2 + 6x) - 2(x^2 + 5x + 6) \text{ and hence } \frac{x^3 + 3x^2 - 4x - 12}{x^3 + 5x^2 + 6x} = 1 - \frac{2}{x}$$

Question	Scheme	Marks	AOs
<b>6(i)</b>	Tries at least one value in the interval Eg $4^2 - 4 - 1 = 11$	M1	1.1b
	States that when $n = 8$ it is FALSE and provides evidence $8^2 - 8 - 1 = 55 = (11 \times 5)$ Hence NOT PRIME	A1	2.4
		<b>(2)</b>	
<b>(ii)</b>	Knows that an odd number is of the form $2n + 1$	B1	3.1a
	Attempts to simplify $(2n + 1)^3 - (2n + 1)^2$	M1	2.1
	.....and factorise $8n^3 + 8n^2 + 2n = 2(4n^3 + 4n^2 + 1n) =$	dM1	1.1b
	with statement $2 \times ..$ is always even	A1	2.4
		<b>(4)</b>	
<b>Alt (ii)</b>	Let the odd number be ' $n$ ' and attempts $n^3 - n^2$	B1	3.1a
	Attempts to factorise $n^3 - n^2 = n^2(n - 1)$	M1	2.1
	States that $n^2$ is odd (odd $\times$ odd) and $(n - 1)$ is even (odd - 1)	dM1	1.1b
	States that the product is even ( odd $\times$ even)	A1	2.4

**(6 marks)**

**Notes: See above**

**(i)**

**M1:** Attempts any  $n^2 - n - 1$  for  $n$  in the interval. It is acceptable just to show  $8^2 - 8 - 1 = 55$   
**A1:** States that when  $n = 8$  it is FALSE and provides evidence. A comment that  $55 = 11 \times 5$  and hence not prime is required

**(ii)**

**See scheme for two examples of proof**  
 Note that Alt (i) works equally well with an odd number of the form  $2n - 1$   
 For example  $(2n - 1)^3 - (2n - 1)^2 = (2n - 1)^2 \{2n - 1 - 1\} = (2n - 1)^2 \{2n - 2\} = 2 \times (2n - 1)^2 \{n - 1\}$

Question	Scheme	Marks	AOs
7 (a)	$\left(1 + \frac{3}{x}\right)^2 = 1 + \frac{6}{x} + \frac{9}{x^2}$	M1 A1	1.1b 1.1b
		(2)	
(b)	$\left(1 + \frac{3}{4}x\right)^6 = 1 + 6 \times \left(\frac{3}{4}x\right) + \dots$	B1	1.1b
	$\left(1 + \frac{3}{4}x\right)^6 = 1 + 6 \times \left(\frac{3}{4}x\right) + \frac{6 \times 5}{2} \times \left(\frac{3}{4}x\right)^2 + \frac{6 \times 5 \times 4}{3 \times 2} \times \left(\frac{3}{4}x\right)^3 + \dots$	M1 A1	1.1b 1.1b
	$= 1 + \frac{9}{2}x + \frac{135}{16}x^2 + \frac{135}{16}x^3 + \dots$	A1	1.1b
		(4)	
(c)	$\left(1 + \frac{3}{x}\right)^2 \left(1 + \frac{3}{4}x\right)^6 = \left(1 + \frac{6}{x} + \frac{9}{x^2}\right) \left(1 + \frac{9}{2}x + \frac{135}{16}x^2 + \frac{135}{16}x^3 + \dots\right)$		
	Coefficient of $x = \frac{9}{2} + 6 \times \frac{135}{16} + 9 \times \frac{135}{16} = \frac{2097}{16}$	M1 A1	2.1 1.1b
		(2)	
<b>(8 marks)</b>			
<b>Notes:</b>			
(a)	<p><b>M1:</b> Attempts <math>\left(1 + \frac{3}{x}\right)^2 = A + \frac{B}{x} + \frac{C}{x^2}</math></p> <p><b>A1:</b> <math>\left(1 + \frac{3}{x}\right)^2 = 1 + \frac{6}{x} + \frac{9}{x^2}</math></p>		
(b)	<p><b>B1:</b> First two terms correct, may be un-simplified</p> <p><b>M1:</b> Attempts the binomial expansion. Implied by the correct coefficient and power of <math>x</math> seen at least once in term 3 or 4</p> <p><b>A1:</b> Binomial expansion correct and un-simplified</p> <p><b>A1:</b> Binomial expansion correct and simplified.</p>		
(c)	<p><b>M1:</b> Combines all relevant terms for their <math>\left(1 + \frac{A}{x} + \frac{B}{x^2}\right) \left(1 + Cx + Dx^2 + Ex^3 + \dots\right)</math> to find the coefficient of <math>x</math>.</p> <p><b>A1:</b> Fully correct</p>		

Question	Scheme	Marks	AOs
8(a)	(i) $\int_1^a \sqrt{8x} \, dx = \sqrt{8} \times \int_1^a \sqrt{x} \, dx = 10\sqrt{8} = 20\sqrt{2}$	M1 A1	2.2a 1.1b
	(ii) $\int_0^a \sqrt{x} \, dx = \int_0^1 \sqrt{x} \, dx + \int_1^a \sqrt{x} \, dx = \left[ \frac{2}{3}x^{\frac{3}{2}} \right]_0^1 + 10 = \frac{32}{3}$	M1 A1	2.1 1.1b
		(4)	
(b)	$R = \int_1^a \sqrt{x} \, dx = \left[ \frac{2}{3}x^{\frac{3}{2}} \right]_1^a$	M1 A1	1.1b 1.1b
	$\frac{2}{3}a^{\frac{3}{2}} - \frac{2}{3} = 10 \Rightarrow a^{\frac{3}{2}} = 16 \Rightarrow a = 16^{\frac{2}{3}}$	dM1	3.1a
	$\Rightarrow a = 2^{4 \times \frac{2}{3}} = 2^{\frac{8}{3}}$	A1	2.1
		(4)	

(8 marks)

**Notes:**

(a)(i)

**M1:** For deducing that  $\int_1^a \sqrt{8x} \, dx = \sqrt{8} \times \int_1^a \sqrt{x} \, dx$  attempting to multiply  $\int_1^a \sqrt{x} \, dx$  by  $\sqrt{8}$

**A1:**  $20\sqrt{2}$  or exact equivalent

(a)(ii)

**M1:** For identifying and attempting to use  $\int_0^a \sqrt{x} \, dx = \int_0^1 \sqrt{x} \, dx + \int_1^a \sqrt{x} \, dx$

**A1:** For  $\frac{32}{3}$  or exact equivalent

(b)

**M1:** Attempts to integrate,  $x^{\frac{1}{2}} \rightarrow x^{\frac{3}{2}}$

**A1:**  $\int_1^a \sqrt{x} \, dx = \left[ \frac{2}{3}x^{\frac{3}{2}} \right]_1^a$

**dM1:** For a whole strategy to find  $a$ . In the scheme it is awarded for setting  $\left[ \dots x^{\frac{3}{2}} \right]_1^a = 10$ , using both limits and proceeding using correct index work to find  $a$ . Alternatively a candidate could assume

$a = 2^k$ . In such a case it is awarded for setting  $\left[ \dots x^{\frac{3}{2}} \right]_1^{2^k} = 10$ , using both limits and proceeding using correct index work to  $k = \dots$

**A1:**  $a = 2^{4 \times \frac{2}{3}} = 2^{\frac{8}{3}}$

In the alternative case, a further statement must be seen following  $k = \frac{8}{3}$  Hence True

Question	Scheme	Marks	AOs
9	$2\log_4(2-x) - \log_4(x+5) = 1$		
	Uses the power law $\log_4(2-x)^2 - \log_4(x+5) = 1$	M1	1.1b
	Uses the subtraction law $\log_4 \frac{(2-x)^2}{(x+5)} = 1$	M1	1.1b
	$\frac{(2-x)^2}{(x+5)} = 4 \rightarrow 3\text{TQ in } x$	dM1	3.1a
	$x^2 - 8x - 16 = 0$	A1	1.1b
	$(x-4)^2 = 32 \Rightarrow x =$	M1	1.1b
	$x = 4 - 4\sqrt{2}$ oe only	A1	2.3
		(6)	
<b>(6 marks)</b>			
<b>Notes:</b>			
<p><b>M1:</b> Uses the power law of logs <math>2\log_4(2-x) = \log_4(2-x)^2</math></p> <p><b>M1:</b> Uses the subtraction law of logs following the above <math>\log_4(2-x)^2 - \log_4(x+5) = \log_4 \frac{(2-x)^2}{(x+5)}</math></p> <p>Alternatively uses the addition law following use of <math>1 = \log_4 4</math> That is <math>1 + \log_4(x+5) = \log_4 4(x+5)</math></p> <p><b>dM1:</b> This can be awarded for the overall strategy leading to a 3TQ in <math>x</math>. It is dependent upon the correct use of both previous M's and for undoing the logs to reach a 3TQ equation in <math>x</math></p> <p><b>A1:</b> For a correct equation in <math>x</math></p> <p><b>M1:</b> For the correct method of solving their 3TQ = 0</p> <p><b>A1:</b> <math>x = 4 - 4\sqrt{2}</math> or exact equivalent only. (For example accept <math>x = 4 - \sqrt{32}</math>)</p>			

Question	Scheme	Marks	AOs
10(a)	Attempts to find the radius $\sqrt{(2-2)^2 + (5-3)^2}$ or radius <sup>2</sup>	M1	1.1b
	Attempts $(x-2)^2 + (y-5)^2 = r^2$	M1	1.1b
	Correct equation $(x-2)^2 + (y-5)^2 = 20$	A1	1.1b
		<b>(3)</b>	
(b)	Gradient of radius $OP$ where $O$ is the centre of $C = \frac{5-3}{2-2} = \left(\frac{1}{2}\right)$	M1	1.1b
	Equation of $l$ is $-2 = \frac{y-3}{x+2}$	dM1	3.1a
	Any correct form $y = -2x - 1$	A1	1.1b
	Method of finding $k$ Substitute $x = 2$ into their $y = -2x - 1$	M1	2.1
	$k = -5$	A1	1.1b
		<b>(5)</b>	

**(8 marks)**

**Notes:**

(a)

**M1:** As scheme or states form of circle is  $(x-2)^2 + (y-5)^2 = r^2$

**M1:** As scheme or substitutes  $(-2, 3)$  into  $(x-2)^2 + (y-5)^2 = r^2$

**A1:** For a correct equation

If students use  $x^2 + y^2 + 2fx + 2gy + c = 0$  **M1:**  $f = 2, g = 5$  **M1:** substitutes  $(2, 5)$  to find value of  $c$

**A1:**  $x^2 + y^2 - 4x - 10y + 9 = 0$

(b)

**M1:** Attempts to find the gradient of  $OP$  where  $O$  is the centre of  $C$

**dM1:** For a complete strategy of finding the equation of  $l$  using the perpendicular gradient to  $OP$  and the point  $(-2, 3)$ .

**A1:** Any correct form of  $l$  Eg  $y = -2x - 1$

**M1:** Scored for the key step of finding  $k$ . In this method they are required to substitute  $(2, k)$  in their  $y = -2x - 1$  and solve for  $k$ .

**A1:**  $k = -5$

**Alt using Pythagoras' theorem**

**M1:** Attempts Pythagoras to find both  $PQ$  and  $OQ$  in terms of  $k$  (where  $O$  is centre of  $C$ )

**dM1:** For the complete strategy of using Pythagoras theorem on triangle  $POQ$  to form an equation in  $k$

**A1:** A correct equation in  $k$  Eg.  $20 + (k-3)^2 + 16 = (k-5)^2$

**M1:** Scored for a correct attempt to solve their quadratic to find  $k$ .

**A1:**  $k = -5$

Question	Scheme	Marks	AOs
<b>11(i)</b>	$(2\theta + 10^\circ) = \arcsin(-0.6)$	M1	1.1b
	$(2\theta + 10^\circ) = -143.13^\circ, -36.87^\circ, 216.87^\circ, 323.13^\circ$ (Any two)	A1	1.1b
	Correct order to find $\theta = \dots$	dM1	1.1b
	Two of $\theta = -76.6^\circ, -23.4^\circ, 103.4^\circ, 156.6^\circ$ .	A1	1.1b
	$\theta = -76.6^\circ, -23.4^\circ, 103.4^\circ, 156.6^\circ$ , only	A1	2.1
		<b>(5)</b>	
<b>(ii)</b>	(a) Explains that the student has not considered the negative value of $x (= -29.0^\circ)$ when solving $\cos x = \frac{7}{8}$	B1	2.3
	Explains that the student should check if any solutions of $\sin x = 0$ (the cancelled term) are solutions of the given equation. $x = 0^\circ$ should have been included as a solution	B1	2.3
	(b) Attempts to solve $4\alpha + 199^\circ = (360 - 29.0)^\circ$	M1	2.2a
	$\alpha = 33.0^\circ$	A1	1.1b
		<b>(4)</b>	
<b>(9 marks)</b>			
<b>Notes:</b>			
<p>(i)</p> <p><b>M1:</b> Attempts <math>\arcsin(-0.6)</math> implied by any correct answer</p> <p><b>A1:</b> Any two of <math>-143.13^\circ, -36.87^\circ, 216.87^\circ, 323.13^\circ</math></p> <p><b>dM1:</b> Correct method to find any value of <math>\theta</math></p> <p><b>A1:</b> Any two of <math>\theta = -76.6^\circ, -23.4^\circ, 103.4^\circ, 156.6^\circ</math>.</p> <p><b>A1:</b> A full solution leading to all four answers and no extras  <math>\theta = -76.6^\circ, -23.4^\circ, 103.4^\circ, 156.6^\circ</math>, only</p>			
<p>(ii)(a)</p> <p><b>B1:</b> See scheme</p> <p><b>B1:</b> See scheme</p> <p>(ii)(b)</p> <p><b>M1:</b> For deducing the smallest positive solution occurs when <math>4\alpha + 199^\circ = (360 - 29.0)^\circ</math></p> <p><b>A1:</b> <math>\alpha = 33^\circ</math></p>			

Question	Scheme	Marks	AOs	
12(a)	Sets $3x - 2\sqrt{x} = 8x - 16$	B1	1.1a	
	$2\sqrt{x} = 16 - 5x$ $4x = (16 - 5x)^2 \Rightarrow x = ..$	$5x + 2\sqrt{x} - 16 = 0$ $\Rightarrow (5\sqrt{x} \pm 8)(\sqrt{x} \pm 2) = 0$	M1	3.1a
	$25x^2 - 164x + 256 = 0$	$(5\sqrt{x} - 8)(\sqrt{x} + 2) = 0$	A1	1.1b
	$(25x - 64)(x - 4) = 0 \Rightarrow x = ..$	$\sqrt{x} = \frac{8}{5}, (-2) \Rightarrow x = ..$	M1	1.1b
	$x = \frac{64}{25}$ only		A1	2.3
			(5)	
(b)	Attempts to solve $3x - 2\sqrt{x} = 0$	M1	2.1	
	Correct solution $x = \frac{4}{9}$	A1	1.1b	
	$y \geq 3x - 2\sqrt{x}, y > 8x - 16 \quad x \leq \frac{4}{9}$	B1ft	1.1b	
		(3)		
<b>(8 marks)</b>				

**Notes:**

(a)

**B1:** Sets the equations equal to each other and achieves a correct equation

**M1:** Awarded for the key step in solving the problem. This can be awarded via two routes. Both routes must lead to a value for  $x$ .

- Making the  $\sqrt{x}$  term the subject and squaring both sides (not each term)
- Recognising that this is a quadratic in  $\sqrt{x}$  and attempting to factorise  
 $\Rightarrow (5\sqrt{x} \pm 8)(\sqrt{x} \pm 2) = 0$

**A1:** A correct intermediate line  $25x^2 - 164x + 256 = 0$  or  $(5\sqrt{x} - 8)(\sqrt{x} + 2) = 0$

**M1:** A correct method to find at least one value for  $x$ . Way One it is for factorising (usual rules), Way Two it is squaring at least one result of their  $\sqrt{x}$

**A1:** Realises that  $x = \frac{64}{25}$  is the only solution  $x = \frac{64}{25}, 4$  is A0

(b) **M1:** Attempts to solve  $3x - 2\sqrt{x} = 0$  For example

Allow  $3x = 2\sqrt{x} \Rightarrow 9x^2 = 4x \Rightarrow x = ...$

Allow  $3x = 2\sqrt{x} \Rightarrow x^{\frac{1}{2}} = \frac{2}{3} \Rightarrow x = ...$

**A1:** Correct solution to  $3x - 2\sqrt{x} = 0 \Rightarrow x = \frac{4}{9}$

**B1:** For a **consistent** solution defining  $R$  using either convention

Either  $y \geq 3x - 2\sqrt{x}, y > 8x - 16 \quad x \leq \frac{4}{9}$  Or  $y < 3x - 2\sqrt{x}, y \leq 8x - 16 \quad x > \frac{4}{9}$

Question	Scheme	Marks	AOs
13(a)	0.2 m <sup>2</sup>	B1	3.4
		(1)	
(b)	$A = 0.2e^{0.3t}$ Rate of change = gradient = $\frac{dA}{dt} = 0.06e^{0.3t}$	M1	3.1b
	At $t = 5 \Rightarrow$ Rate of Growth is $0.06e^{1.5} = 0.269$ m <sup>2</sup> /day	A1	1.1b
		(2)	
(c)	$100 = 0.2e^{0.3t} \Rightarrow e^{0.3t} = 500$	M1 A1	3.1a 1.1b
	$\Rightarrow t = \frac{\ln(500)}{0.3} = 20.7$ days      20 days 17 hours	M1 A1	1.1b 3.2a
		(4)	
(d)	The model given suggests that the pond is fully covered after 20 days 17 hours. Observed data is inconsistent with this as the pond is only 90% covered by the end of one month (28/29/30/31 days). Hence the model is not accurate	B1	3.5a
		(1)	
<b>(8 marks)</b>			
<b>Notes:</b>			
<p>(a)  <b>B1:</b> 0.2 m<sup>2</sup> oe</p> <p>(b)  <b>M1:</b> Links rate of change to gradient and differentiates <math>0.2e^{0.3t} \rightarrow ke^{0.3t}</math>  <b>A1:</b> Correct answer 0.269 m<sup>2</sup>/day</p> <p>(c)  <b>M1:</b> Substitutes <math>A = 100</math> and proceeds to <math>e^{0.3t} = k</math>  <b>A1:</b> <math>e^{0.3t} = 500</math>  <b>M1:</b> Correct method when proceeding from <math>e^{0.3t} = k \Rightarrow t = ..</math>  <b>A1:</b> 20 days 17 hours</p> <p>(d)  <b>B1:</b> Valid conclusion following through on their answer to (c).</p>			

Question	Scheme	Marks	AOs
14	$y = (x-2)^2(x+3) = (x^2 - 4x + 4)(x+3) = x^3 - 1x^2 - 8x + 12$	B1	1.1b
	An attempt to find $x$ coordinate of the maximum point. To score this you must see either <ul style="list-style-type: none"> <li>an attempt to expand <math>(x-2)^2(x+3)</math>, an attempt to differentiate the result, followed by an attempt at solving <math>\frac{dy}{dx} = 0</math></li> <li>an attempt to differentiate <math>(x-2)^2(x+3)</math> by the product rule followed by an attempt at solving <math>\frac{dy}{dx} = 0</math></li> </ul>	M1	3.1a
	$y = x^3 - 1x^2 - 8x + 12 \Rightarrow \frac{dy}{dx} = 3x^2 - 2x - 8$	M1	1.1b
	Maximum point occurs when $\frac{dy}{dx} = 0 \Rightarrow (x-2)(3x+4) = 0$ $\Rightarrow x = -\frac{4}{3}$	M1 A1	1.1b 1.1b
	An attempt to find the area under $y = (x-2)^2(x+3)$ between two values. To score this you must see an attempt to expand $(x-2)^2(x+3)$ followed by an attempt at using two limits	M1	3.1a
	Area = $\int (x^3 - 1x^2 - 8x + 12) dx = \left[ \frac{x^4}{4} - \frac{x^3}{3} - 4x^2 + 12x \right]$	M1	1.1b
	Uses a top limit of 2 and a bottom limit of their $x = -\frac{4}{3} R = \left[ \frac{x^4}{4} - \frac{x^3}{3} - 4x^2 + 12x \right]_{-\frac{4}{3}}^2$	M1	2.2a
	Uses = $\frac{28}{3} - \frac{1744}{81} = \frac{2500}{81}$	A1	2.1
		(9)	

**(9 marks)**

**Notes:**

**B1:** Expands  $(x-2)^2(x+3)$  to  $x^3 - 1x^2 - 8x + 12$  seen at some point in their solution. It may appear just on their integral for example.

**M1:** This is a problem solving mark for knowing the method of finding the maximum point. You should expect to see the key points used (i) differentiation (ii) solution of their  $\frac{dy}{dx} = 0$

**M1:** For correctly differentiating their cubic with at least two terms correct (for their cubic).

**M1:** For setting their  $\frac{dy}{dx} = 0$  and solves using a correct method (including calculator methods)

**A1:**  $\Rightarrow x = -\frac{4}{3}$

**M1:** This is a problem solving mark for knowing how integration is used to find the area underneath a curve between two points.

**M1:** For correctly integrating their cubic with at least two correct terms (for their cubic).

**M1:** For deducing the top limit is 2, the bottom limit is their  $x = -\frac{4}{3}$  and applying these correctly within their integration.

**A1:** Shows above steps clearly and proceeds to  $R = \frac{2500}{81}$