## Mark Scheme

Q1.

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| | $(1+ax)^{\frac{2}{3}} \approx 1+\frac{1}{2}x+kx^2$; $\mathrm{f}(x)=(4-9x)(1+ax)^{\frac{2}{3}}$, $\lvert ax\rvert<1$ | | |
| | $\left\{(1+ax)^{\frac{2}{3}} \approx 1+\left(\frac{2}{3}\right)(ax)+\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(ax)^2+\ldots=1+\frac{2}{3}ax-\frac{1}{9}a^2x^2+\ldots\right\}$ | | |
| (a) | $\frac{2}{3}a=\frac{1}{2}$ | **see notes** | M1 |
| | $a=\frac{3}{4}$ | $a=\frac{3}{4}$ or 0.75 | A1 o.e. |
| | | | **(2)** |
| (b) | $\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(a)^2$ | **Either** $\frac{(\frac{2}{3})(\frac{2}{3}-1)}{2!}(a)^2$ or $\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(a)^2$ or $\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(ax)^2$ or $\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(\text{their } a)^2$ or $-\frac{1}{9}a^2$ | M1 |
| | $\left\{k=\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}\left(\frac{3}{4}\right)^2\right\} \Rightarrow k=-\frac{1}{16}$ | $k=-\frac{1}{16}$ or $-0.0625$ | A1 |
| | | | **(2)** |
| (c) | $\left\{(4-9x)\left(1+\frac{1}{2}x-\frac{1}{16}x^2\right)\right\}$ | | |
| | $\{x^2:\}\ -\frac{1}{4}-\frac{9}{2};\ =-\frac{19}{4}$ or $-4.75$ | Either $4(\text{their } k)-\frac{9}{2}$ or $4(\text{their } k)x^2-\frac{9}{2}x^2$ | M1 |
| | | $-\frac{19}{4}$ or $-4.75$ | A1 |
| | | | **(2)** |
| | | | **6** |

| | | Question Notes |
|---|---|---|
| | **Note** | Writing down $\left\{(1+ax)^{\frac{2}{3}}\right\}=1+\left(\frac{2}{3}\right)(ax)+\frac{(\frac{2}{3})(-\frac{1}{3})}{2!}(ax)^2+\ldots$ gets (a) M1 and (b) M1 |
| (a) | **Note** | Give M1 for any of<br>• writing down $\frac{2}{3}a=\frac{1}{2}$<br>• expanding $(1+ax)^{\frac{2}{3}}$ to give $1+\left(\frac{2}{3}\right)(ax)$<br>• writing down $\frac{2}{3}ax=\frac{1}{2}$ or $\frac{2}{3}a=\frac{1}{2}x$ or $\frac{2}{3}ax=\frac{1}{2}x$ |
| | **Note** | Give M1 A1 $a=\frac{3}{4}$ from no working |
| (b) | **Note** | Give A0 for $k=-\frac{1}{16}x^2$ or $-0.0625x^2$ without reference to $k=-\frac{1}{16}$ or $-0.0625$ |
| | **Note** | Allow A1 for $k=-\frac{1}{16}x^2$ or $-0.0625x^2$ followed by $k=-\frac{1}{16}$ or $-0.0625$ |
| (c) | **Note** | Give A0 for $-\frac{19}{4}x^2$ or $-4.75x^2$ without reference to $-\frac{19}{4}$ or $-4.75$ |
| | **Note** | Allow A1 for $-\frac{19}{4}x^2$ or $-4.75x^2$ followed by $-\frac{19}{4}$ or $-4.75$ |

**Q2.**

| Question Number | Scheme | Marks |
|---|---|---|
| **2.(a)** | $2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \quad \Rightarrow 2x+1=e^5 \Rightarrow x=..$ | M1 |
| | $\Rightarrow x=\dfrac{e^5-1}{2}$ | A1 |
| | | (2) |
| **(b)** | $3^x e^{4x}=e^7 \Rightarrow \ln(3^x e^{4x})=\ln e^7$ | |
| | $\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$ | M1,M1 |
| | $x(\ln 3+4)=7 \Rightarrow x=...$ | dM1 |
| | $x=\dfrac{7}{(\ln 3+4)}$ oe | A1 |
| | | (4) **6 marks** |
| **Alt 1 2(b)** | $3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$ | |
| | $3^x=e^{7-4x} \Rightarrow x\ln 3=(7-4x)\ln e$ | M1,M1 |
| | $x(\ln 3+4)=7 \Rightarrow x=...$ | dM1 |
| | $x=\dfrac{7}{(\ln 3+4)}$ | A1 |
| | | (4) |
| **Alt 2 2(b) Using logs** | $3^x e^{4x}=e^7 \Rightarrow \log(3^x e^{4x})=\log e^7$ | |
| | $\log 3^x+\log e^{4x}=\log e^7 \Rightarrow x\log 3+4x\log e=7\log e$ | M1, M1 |
| | $x(\log 3+4\log e)=7\log e \Rightarrow x=...$ | dM1 |
| | $x=\dfrac{7\log e}{(\log 3+4\log e)}$ | A1 |
| | | (4) |
| **Alt 3 2(b) Using $\log_3$** | $3^x e^{4x}=e^7 \Rightarrow 3^x=\dfrac{e^7}{e^{4x}}$ | |
| | $3^x=e^{7-4x} \Rightarrow x=(7-4x)\log_3 e$ | M1,M1 |
| | $x(1+4\log_3 e)=7\log_3 e \Rightarrow x=...$ | dM1 |
| | $x=\dfrac{7\log_3 e}{(1+4\log_3 e)}$ | A1 |
| | | (4) |
| **Alt 4 2(b) Using $3^x=e^{x\ln 3}$** | $3^x e^{4x}=e^7 \Rightarrow e^{x\ln 3}e^{4x}=e^7$ | |
| | $\Rightarrow e^{x\ln 3+4x}=e^7, \Rightarrow x\ln 3+4x=7$ | M1,M1 |
| | $x(\ln 3+4)=7 \Rightarrow x=... \qquad x=\dfrac{7}{(\ln 3+4)}$ | dM1 A1 |
| | | (4) |

(a)

M1 Proceeds from $2\ln(2x+1)-10=0$ to $\ln(2x+1)=5$ before taking exp's to achieve $x$ in terms of $e^5$

Accept for M1 $2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \Rightarrow x=f(e^5)$

Alternatively they could use the power law before taking exp's to achieve $x$ in terms of $\sqrt{e^{10}}$

$2\ln(2x+1)=10 \Rightarrow \ln(2x+1)^2=10 \Rightarrow (2x+1)^2=e^{10} \Rightarrow x=g\left(\sqrt{e^{10}}\right)$

A1 cso. Accept $x=\frac{e^5-1}{2}$ or other exact simplified alternatives such as $x=\frac{e^5}{2}-\frac{1}{2}$. Remember to isw.

The decimal answer of 73.7 will score M1A0 unless the exact answer has also been given.

The answer $\frac{\sqrt{e^{10}}-1}{2}$ does not score this mark unless simplified. $x=\frac{\pm e^5-1}{2}$ is M1A0

(b)

M1 Takes ln's or logs of both sides and applies the addition law.

$\ln(3^x e^{4x})=\ln 3^x+\ln e^{4x}$ or $\ln(3^x e^{4x})=\ln 3^x+4x$ is evidence for the addition law

If the $e^{4x}$ was 'moved' over to the right hand side score for either $e^{7-4x}$ or the subtraction law.

$\ln\frac{e^7}{e^{4x}}=\ln e^7-\ln e^{4x}$ or $3^x e^{4x}=e^7 \Rightarrow 3^x=\frac{e^7}{e^{4x}} \Rightarrow 3^x=e^{7-4x}$ is evidence of the subtraction law

M1 Uses the power law of logs (seen at least once in a term with x as the index Eg $3^x, e^{4x}$ or $e^{7-4x}$ ).

$\ln 3^x+\ln e^{4x}=\ln e^7 \Rightarrow x\ln 3+4x\ln e=7\ln e$ is an example after the addition law

$3^x=e^{7-4x} \Rightarrow x\log 3=(7-4x)\log e$ is an example after the subtraction law.

It is possible to score M0M1 by applying the power law after an incorrect addition/subtraction law

For example $3^x e^{4x}=e^7 \Rightarrow \ln(3^x)\times\ln(e^{4x})=\ln e^7 \Rightarrow x\ln 3\times 4x\ln e=7\ln e$

dM1 This is dependent upon **both** previous M's. Collects/factorises out term in $x$ and proceeds to $x=$ .

Condone sign slips for this mark. An unsimplified answer can score this mark.

A1 If the candidate has taken ln's then they must use $\ln e=1$ and achieve $x=\frac{7}{(\ln 3+4)}$ or equivalent.

If the candidate has taken log's they must be writing log as oppose to ln and achieve

$x=\frac{7\log e}{(\log 3+4\log e)}$ or other exact equivalents such as $x=\frac{7\log e}{\log 3e^4}$.

**Q3**

| Question Number | Scheme | Marks |
|---|---|---|
| **3** | States the largest odd number and an odd number that is greater<br>E.g. odd number $n$ and $n+2$ | M1 |
| | Fully correct proof including<br>• the assumption: there exists a greatest odd number "$n$"<br>• a correct statement that their second odd number is greater than their assumed greatest odd number<br>• a minimal conclusion " this is a contradiction, hence proven"<br><br>You can ignore any spurious information e.g. $n > 0$, $n+2 > 0$ etc. | A1* |
| | | **(2)** |
| | | **(2 marks)** |

M1: For starting the proof by **stating** an odd number and a larger odd number.

Examples of an allowable start are

- **odd number** "$n$" with "$n + 2$"
- **odd number** "$n$" with "$n^2$"
- "$2k + 1$" with "$2k + 3$"
- "$2k + 1$" with "$(2k + 1)^3$"
- "$2k + 1$" with "$2k + 1 + 2k$"

Note that stating $n = 2k$ , even when accompanied by the statement that "$n$" is odd is M0

A1*: A fully correct proof using contradiction

This must consist of

1) An assumption E.g. "(Assume that) there exists a greatest odd number $n$"

"Let "$2k + 1$" be the greatest odd number"

2) A minimal statement showing their second number is greater than the first,

2) A minimal statement showing their second number is greater than the first,

E.g. If "$n$" is odd and "$n + 2$" is greater than $n$

If "$n$" is odd and $n^2 > n$

$2k+3 > 2k+1$

$2k+2k+1 > 2k+1$

Any algebra (e.g. expansions) must be correct. So $(2k+1)^2 = 4k^2 + 2k + 1$ would be A0

3) A minimal conclusion which could be

"hence there is no greatest odd number", "hence proven", or simply ✓

**Q4.**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$ | M1 |
| | $\sin x(\cos 40 + 2\sin 50) = \cos x(2\cos 50 - \sin 40)$ | |
| | $\div\cos x \Rightarrow \tan x(\cos 40 + 2\sin 50) = 2\cos 50 - \sin 40$ | M1 |
| | $\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$, (or numerical answer awrt 0.28) | A1 |
| | States or uses cos50 = sin40 and cos40 =sin50 and so $\tan x° = \frac{1}{3}\tan 40°$ * cao | A1 * |
| | | (4) |
| (b) | Deduces $\tan 2\theta = \frac{1}{3}\tan 40$ | M1 |
| | $2\theta =$ 15.6 so $\theta =$ awrt 7.8(1) One answer | A1 |
| | Also $2\theta =$ 195.6, 375.6 , 555.6 $\Rightarrow \theta = ..$ | M1 |
| | $\theta =$ awrt 7.8 , 97.8, 187.8, 277.8 All 4 answers | A1 |
| | | (4) |
| | | [8 marks ] |
| Alt 1 (a) | $2\cos x\cos 50 - 2\sin x\sin 50 = \sin x\cos 40 + \cos x\sin 40$ | M1 |
| | $2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$ | |
| | $\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$ | M1 |
| | $\tan x = \dfrac{\sin 40}{3\cos 40}$( or numerical answer awrt 0.28), $\Rightarrow \tan x = \dfrac{1}{3}\tan 40$ | A1,A1 |
| Alt 2 (a) | $2\cos(x+50) = \sin(x+40) \Rightarrow 2\sin(40-x) = \sin(x+40)$ | |
| | $2\cos x\sin 40 - 2\sin x\cos 40 = \sin x\cos 40 + \cos x\sin 40$ | M1 |
| | $\div\cos x \Rightarrow 2\sin 40 - 2\tan x\cos 40 = \tan x\cos 40 + \sin 40$ | M1 |
| | $\tan x = \dfrac{\sin 40}{3\cos 40}$( or numerical answer awrt 0.28), $\Rightarrow \tan x = \dfrac{1}{3}\tan 40$ | A1,A1 |

## Notes for Question

(a)

M1 Expand both expressions using $\cos(x+50) = \cos x \cos 50 - \sin x \sin 50$ and $\sin(x+40) = \sin x \cos 40 + \cos x \sin 40$. Condone a missing bracket on the lhs.

The terms of the expansions must be correct as these are given identities. You may condone a sign error on one of the expressions.

Allow if written separately and not in a connected equation.

M1 Divide by $\cos x$ to reach an equation in $\tan x$.

Below is an example of M1M1 with incorrect sign on left hand side

$2\cos x \cos 50 + 2\sin x \sin 50 = \sin x \cos 40 + \cos x \sin 40$

$\Rightarrow 2\cos 50 + 2\tan x \sin 50 = \tan x \cos 40 + \sin 40$

This is independent of the first mark.

A1 $\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50}$

Accept for this mark $\tan x = \text{awrt } 0.28...$ as long as M1M1 has been achieved.

A1* States or uses cos50=sin40 and cos40=sin50 leading to showing

$\tan x = \dfrac{2\cos 50 - \sin 40}{\cos 40 + 2\sin 50} = \dfrac{\sin 40}{3\cos 40} = \dfrac{1}{3}\tan 40$

This is a given answer and all steps above must be shown. The line above is acceptable.

Do not allow from $\tan x = \text{awrt } 0.28...$

(b)

M1 For linking part (a) with (b). Award for writing $\tan 2\theta = \frac{1}{3}\tan 40$

A1 Solves to find one solution of $\theta$ which is usually (awrt) 7.8

M1 Uses the correct method to find at least another value of $\theta$. It must be a full method but can be implied by any correct answer.

Accept $\theta = \dfrac{180 + their\,\alpha}{2}, (or)\dfrac{360 + their\,\alpha}{2}, (or)\dfrac{540 + their\,\alpha}{2}$

A1 Obtains all four answers awrt 1dp. $\theta = 7.8, 97.8, 187.8, 277.8$.

Ignore any extra solutions outside the range.

Withhold this mark for extras inside the range.

Condone a different variable. Accept $x = 7.8, 97.8, 187.8, 277.8$

Answers fully given in radians, loses the first A mark.
Acceptable answers in rads are awrt 0.136, 1.71, 3.28, 4.85
Mixed units can only score the first M 1

**Q5.**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) | $R=\sqrt{20}$ | B1 |
| | $\tan\alpha=\frac{4}{2}\Rightarrow\alpha=\text{awrt }1.107$ | M1A1 |
| | | (3) |
| (b)(i) | $'4+5R^2'=104$ | B1ft |
| (ii) | $3\theta-'1.107'=\frac{\pi}{2}\Rightarrow\theta=\text{awrt }0.89$ | M1A1 |
| | | (3) |
| (c )(i) | 4 | B1 |
| (ii) | $3\theta-'1.107'=2\pi\Rightarrow\theta=\text{awrt }2.46$ | M1A1 |
| | | (3) |
| | | **( 9 marks)** |

(a)

B1 Accept $R=\sqrt{20}$ or $2\sqrt{5}$ or awrt 4.47

Do not accept $R=\pm\sqrt{20}$

This could be scored in parts (b) or (c) as long as you are certain it is $R$

M1 for sight of $\tan\alpha=\pm\frac{4}{2}$, $\tan\alpha=\pm\frac{2}{4}$. Condone $\sin\alpha=4, \cos\alpha=2\Rightarrow\tan\alpha=\frac{4}{2}$

If $R$ is found first only accept $\sin\alpha=\pm\frac{4}{R}, \cos\alpha=\pm\frac{2}{R}$

A1 $\alpha=$ awrt 1.107. The degrees equivalent $63.4°$ is A0.

If a candidate does all the question in degrees they will lose just this mark.

(b)(i)

B1ft Either 104 or if $R$ was incorrect allow for the numerical value of their $'4+5R^2'$.

Allow a tolerance of 1 dp on decimal $R$'s.

(b)(ii)

M1 Using $3\theta\pm\text{their}'1.107'=\frac{\pi}{2}\Rightarrow\theta=..$

Accept $3\theta\pm\text{their}'1.107'=(2n+1)\frac{\pi}{2}\Rightarrow\theta=..$ where $n$ is an integer

Allow slips on the lhs with an extra bracket such as

$3(\theta\pm\text{their}'1.107')=\frac{\pi}{2}\Rightarrow\theta=..$

The degree equivalent is acceptable $3\theta-\text{their}'63.4°'=90°\Rightarrow\theta=$

Do not allow mixed units in this question

A1 awrt 0.89 radians or $51.1°$. Do not allow multiple solutions for this mark.

(c)(i)

B1 4

(c)(ii)

M1 Using $3\theta\pm\text{their}'1.107'=2\pi\Rightarrow\theta=...$

Accept $3\theta\pm\text{their}'1.107'=n\pi\Rightarrow\theta=..$ where $n$ is an integer, including 0

Allow slips on the lhs with an extra bracket such as

$3(\theta\pm\text{their}'1.107')=2\pi\Rightarrow\theta=..$

The degree equivalent is acceptable $3\theta-\text{their}'63.4°'=360°\Rightarrow\theta=$ but

Do not allow mixed units in this question

A1 $\theta=$ awrt 2.46 radians or $141.1°$ Do not allow multiple solutions for this mark.

**Q6.**

| | | | |
|---|---|---|---|
| **1(a)** | $\mathrm{fg}(x)=\dfrac{28}{x-2}-1$ | $\left(=\dfrac{30-x}{x-2}\right)$ | M1 |
| | Sets $\mathrm{fg}(x)=x \Rightarrow \dfrac{28}{x-2}-1=x$ | | |
| | $\Rightarrow 28=(x+1)(x-2)$ | | |
| | $\Rightarrow x^2-x-30=0$ | | M1 |
| | $\Rightarrow (x-6)(x+5)=0$ | | |
| | $\Rightarrow x=6,\ x=-5$ | | dM1 A1 |
| | | | **(4)** |
| **(b)** | $a=6$ | | B1 ft |
| | | | **(1)** |
| | | | **5 marks** |
| **Alt 1(a)** | $\mathrm{fg}(x)=x \Rightarrow \mathrm{g}(x)=\mathrm{f}^{-1}(x)$ | | M1 |
| | $\dfrac{4}{x-2}=\dfrac{x+1}{7}$ | | |
| | $\Rightarrow x^2-x-30=0$ | | M1 |
| | $\Rightarrow (x-6)(x+5)=0$ | | |
| | $\Rightarrow x=6,\ x=-5$ | | dM1 A1 |
| | | | **4 marks** |
| **S. Case** | Uses $\mathrm{gf}(x)$ instead $\mathrm{fg}(x)$ | Makes an error on $\mathrm{fg}(x)$ | |
| | $\dfrac{4}{7x-1-2}=x$ | Sets $\mathrm{fg}(x)=x \Rightarrow \dfrac{7\times 4}{7\times(x-2)}-1=x$ | M0 |
| | $\Rightarrow 7x^2-3x-4=0$ | $\Rightarrow x^2-x-6=0$ | M1 |
| | $\Rightarrow (7x+4)(x-1)=0$ | $\Rightarrow (x+2)(x-3)=0$ | |
| | $\Rightarrow x=-\dfrac{4}{7},\quad x=1$ | $\Rightarrow x=-2,\quad x=3$ | dM1 A0 |
| | | | **2 out of 4 marks** |

(a)

M1 Sets or implies that $\mathrm{fg}(x)=\dfrac{28}{x-2}-1$ Eg accept $\mathrm{fg}(x)=7\left(\dfrac{4}{x-2}\right)-1$ followed by $\mathrm{fg}(x)=\dfrac{7\times 4}{x-2}-1$

Alternatively sets $\mathrm{g}(x)=\mathrm{f}^{-1}(x)$ where $\mathrm{f}^{-1}(x)=\dfrac{x\pm 1}{7}$

Note that $\mathrm{fg}(x)=7\left(\dfrac{4}{x-2}\right)-1=\dfrac{28}{7(x-2)}-1$ is M0

M1 Sets up a 3TQ (= 0) from an attempt at $\mathrm{fg}(x)=x$ or $\mathrm{g}(x)=\mathrm{f}^{-1}(x)$

dM1 Method of solving 3TQ (= 0) to find at least one value for $x$. See "General Priciples for Core Mathematics" on page 3 for the award of the mark for solving quadratic equations
This is dependent upon the previous M. You may just see the answers following the 3TQ.

A1 Both $x=6$ and $x=-5$

(b)

B1ft For $a=6$ but you may follow through on the largest solution from part (a) provided more than one answer was found in (a). Accept 6, $a=6$ and even $x=6$
Do not award marks for part (a) for work in part (b).

**Q7.**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | $\frac{dA}{dt}=1.5$ | | B1 |
| | $A=\pi r^2 \Rightarrow \frac{dA}{dr}=2\pi r$ | | B1 |
| | When $A=2$ | | |
| | $2=\pi r^2 \Rightarrow r=\sqrt{\frac{2}{\pi}}\ \ (=0.797\,884\ ...\ )$ | | M1 |
| | $\frac{dA}{dt}=\frac{dA}{dr}\times\frac{dr}{dt}$ | | |
| | $1.5=2\pi r\frac{dr}{dt}$ | | M1 |
| | $\frac{dr}{dt}=\frac{1.5}{2\pi\sqrt{\frac{2}{\pi}}}\approx 0.299$ | awrt 0.299 | A1 |
| | | | **[5]** |

**Q8.**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | $\frac{d}{dx}\left(2^x\right)=\ln 2.2^x$ | | B1 |
| | $\ln 2.2^x+2y\frac{dy}{dx}=2y+2x\frac{dy}{dx}$ | | M1 A1= A1 |
| | Substituting $(3,2)$ | | |
| | $8\ln 2+4\frac{dy}{dx}=4+6\frac{dy}{dx}$ | | M1 |
| | $\frac{dy}{dx}=4\ln 2-2$ | Accept exact equivalents | M1 A1 (7) |
| | | | [7] |

**Q9.**

| Question | Scheme | Marks |
|---|---|---|
| (i) | $y = e^{3x}\cos 4x \Rightarrow \left(\frac{dy}{dx}\right) = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$ | M1A1 |
| | Sets $\cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x = 0 \Rightarrow 3\cos 4x - 4\sin 4x = 0$ | M1 |
| | $\Rightarrow x = \frac{1}{4}\arctan\frac{3}{4}$ | M1 |
| | $\Rightarrow x = \text{awrt } 0.9463$ 4dp | A1 |
| | | (5) |
| (ii) | $x = \sin^2 2y \Rightarrow \frac{dx}{dy} = 2\sin 2y \times 2\cos 2y$ | M1A1 |
| | Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression | M1 |
| | $\frac{dx}{dy} = 2\sin 4y \Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$ | M1A1 |
| | | (5) (10 marks) |
| (ii) Alt I | $x = \sin^2 2y \Rightarrow x = \frac{1}{2} - \frac{1}{2}\cos 4y$ | 2nd M1 |
| | $\frac{dx}{dy} = 2\sin 4y$ | 1st M1 A1 |
| | $\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$ | M1A1 |
| | | (5) |
| (ii) Alt II | $x^{\frac{1}{2}} = \sin 2y \Rightarrow \frac{1}{2}x^{-\frac{1}{2}} = 2\cos 2y\frac{dy}{dx}$ | M1A1 |
| | Uses $x^{\frac{1}{2}} = \sin 2y$ AND $\sin 4y = 2\sin 2y\cos 2y$ in their expression | M1 |
| | $\frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$ | M1A1 |
| | | (5) |
| (ii) Alt III | $x^{\frac{1}{2}} = \sin 2y \Rightarrow 2y = \text{invsin}\, x^{\frac{1}{2}} \Rightarrow 2\frac{dy}{dx} = \frac{1}{\sqrt{1-x}} \times \frac{1}{2}x^{-\frac{1}{2}}$ | M1A1 |
| | Uses $x^{\frac{1}{2}} = \sin 2y$, $\sqrt{1-x} = \cos 2y$ and $\sin 4y = 2\sin 2y\cos 2y$ in their expression | M1 |
| | $\Rightarrow \frac{dy}{dx} = \frac{1}{2\sin 4y} = \frac{1}{2}\text{cosec}4y$ | M1A1 |
| | | (5) |

(i)

M1 Uses the product rule $uv'+vu'$ to achieve $\left(\frac{dy}{dx}\right)=Ae^{3x}\cos 4x \pm Be^{3x}\sin 4x \quad A,B\neq 0$

The product rule if stated must be correct

A1 Correct (unsimplified) $\frac{dy}{dx}=\cos 4x\times 3e^{3x}+e^{3x}\times -4\sin 4x$

M1 Sets/implies their $\frac{dy}{dx}=0$ factorises/cancels)by $e^{3x}$ to form a trig equation in just $\sin 4x$ and $\cos 4x$

M1 Uses the identity $\frac{\sin 4x}{\cos 4x}\equiv \tan 4x$, moves from $\tan 4x=C, C\neq 0$ using correct order of operations to $x=...$ Accept $x=$ awrt 0.16 (radians) $x=$ awrt 9.22 (degrees) for this mark.

If a candidate elects to pursue a more difficult method using $R\cos(\theta+\alpha)$, for example, the minimum expectation will be that they get (1) the identity correct, and (2) the values of $R$ and $\alpha$ correct to 2dp. So for the correct equation you would only accept $5\cos(4x+\text{awrt } 0.93)$ or $5\sin(4x-\text{awrt } 0.64)$ before using the correct order of operations to $x=...$

Similarly candidates who square $3\cos 4x-4\sin 4x=0$ then use a Pythagorean identity should proceed from either $\sin 4x=\frac{3}{5}$ or $\cos 4x=\frac{4}{5}$ before using the correct order of operations …

A1 $\Rightarrow x=$ awrt 0.9463.

Ignore any answers outside the domain. Withhold mark for additional answers inside the domain

(ii)

M1 Uses chain rule (or product rule) to achieve $\pm P\sin 2y\cos 2y$ as a derivative.

There is no need for lhs to be seen/ correct

If the product rule is used look for $\cancel{\frac{dy}{dx}}=\pm A\sin 2y\cos 2y\pm B\sin 2y\cos 2y,$

A1 Both lhs and rhs correct (unsimplified) . $\frac{dx}{dy}=2\sin 2y\times 2\cos 2y=(4\sin 2y\cos 2y)$ or

$1=2\sin 2y\times 2\cos 2y\frac{dy}{dx}$

M1 Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression.

You may just see a statement such as $4\sin 2y\cos 2y = 2\sin 4y$ which is fine.

Candidates who write $\cancel{\frac{dy}{dx}} = A\sin 2x\cos 2x$ can score this for $\cancel{\frac{dy}{dx}} = \frac{A}{2}\sin 4x$

M1 Uses $\frac{dy}{dx} = 1 \big/ \frac{dx}{dy}$ **for their expression in** $y$. Concentrate on the trig identity rather than the coefficient in awarding this. Eg $\frac{dx}{dy} = 2\sin 4y \Rightarrow \frac{dy}{dx} = 2\operatorname{cosec} 4y$ is condoned for the M1

If $\frac{dx}{dy} = a+b$ do not allow $\frac{dy}{dx} = \frac{1}{a}+\frac{1}{b}$

A1 $\frac{dy}{dx} = \frac{1}{2}\operatorname{cosec} 4y$ If a candidate then proceeds to write down incorrect values of $p$ and $q$ then do not withhold the mark.

NB: See the three alternatives which may be less common but mark in exactly the same way. If you are uncertain as to how to mark these please consult your team leader.

In **Alt I** the second M is for writing $x = \sin^2 2y \Rightarrow x = \pm\frac{1}{2}\pm\frac{1}{2}\cos 4y$ from $\cos 4y = \pm 1 \pm 2\sin^2 2y$

In **Alt II** the first M is for writing $x^{\frac{1}{2}} = \sin 2y$ and differentiating both sides to $Px^{-\frac{1}{2}} = Q\cos 2y\cancel{\frac{dy}{dx}}$ oe

In **Alt 111** the first M is for writing $2y = \text{invsin}\left(x^{0.5}\right)$ oe and differentiating to $M\cancel{\frac{dy}{dx}} = N\frac{1}{\sqrt{1-\left(x^{0.5}\right)^2}}\times x^{-0.5}$

**Q10.**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| | $\int_0^4 \frac{1}{2+\sqrt{(2x+1)}}\,dx\ ,\ u = 2+\sqrt{(2x+1)}$ | | |
| | $\frac{du}{dx} = (2x+1)^{-\frac{1}{2}}$ or $\frac{dx}{du} = u-2$ | Either $\frac{du}{dx} = \pm K(2x+1)^{-\frac{1}{2}}$ or $\frac{dx}{du} = \pm\lambda(u-2)$ | M1 |
| | | Either $\frac{du}{dx} = (2x+1)^{-\frac{1}{2}}$ or $\frac{dx}{du} = (u-2)$ | A1 |
| | $\left\{\int \frac{1}{2+\sqrt{(2x+1)}}\,dx\right\} = \int \frac{1}{u}(u-2)\,du$ | Correct substitution (Ignore integral sign and $du$). | A1 |
| | $= \int\left(1-\frac{2}{u}\right)du$ | An attempt to divide each term by $u$. | dM1 |
| | $= u - 2\ln u$ | $\pm Au \pm B\ln u$ | ddM1 |
| | | $u - 2\ln u$ | A1 ft |
| | $\left\{\text{So } [u-2\ln u]_3^5\right\} = (5-2\ln 5)-(3-2\ln 3)$ | Applies limits of 5 and 3 in $u$ or 4 and 0 in $x$ in their integrated function and subtracts the correct way round. | M1 |
| | $= 2+2\ln\left(\frac{3}{5}\right)$ | $2+2\ln\left(\frac{3}{5}\right)$ | A1 cao cso |
| | | | [8] 8 |

**Notes for Question**

**M1:** Also allow $du = \pm\lambda\frac{1}{(u-2)}dx$ or $(u-2)du = \pm\lambda\,dx$

**Note:** The expressions must contain $du$ and $dx$. They can be simplified or un-simplified.

**A1:** Also allow $du = \frac{1}{(u-2)}dx$ or $(u-2)du = \pm\lambda\,dx$

**Note:** The expressions must contain $du$ and $dx$. They can be simplified or un-simplified.

**A1:** $\int \frac{1}{u}(u-2)\,du$. (Ignore integral sign and $du$).

**dM1:** An attempt to divide each term by $u$.
**Note** that this mark is dependent on the previous M1 mark being awarded.
**Note** that this mark can be implied by later working.

**ddM1:** $\pm Au \pm B\ln u$, $A \neq 0, B \neq 0$
**Note** that this mark is dependent on the two previous M1 marks being awarded.

**A1ft:** $u - 2\ln u$ or $\pm Au \pm B\ln u$ being correctly followed through, $A \neq 0, B \neq 0$

**M1:** Applies limits of 5 and 3 in $u$ or 4 and 0 in $x$ in their integrated function and subtracts the correct way round.

**A1:** cso and cao. $2+2\ln\left(\frac{3}{5}\right)$ or $2+2\ln(0.6)$, $\left(= A+2\ln B, \text{ so } A=2, B=\frac{3}{5}\right)$

**Note:** $2-2\ln\left(\frac{3}{5}\right)$ is A0.

| Notes for Question Continued | |
|---|---|
| ctd | Note: $\int \frac{1}{u}(u-2)\,du = u-2\ln u$ with no working is 2nd M1, 3rd M1, 3rd A1.<br>but Note: $\int \frac{1}{u}(u-2)\,du = (u-2)\ln u$ with no working is 2nd M0, 3rd M0, 3rd A0. |

**Q11.**

| Question Number | Scheme | Marks |
|---|---|---|
| (a) (i) | Uses $\cos ADC = \dfrac{7^2+9^2-3^2}{2\times 7\times 9}$ | M1 |
| | So $ADC = 0.283$ | A1 |
| (ii) | Uses $\cos ACD = \dfrac{3^2+9^2-7^2}{2\times 3\times 9}$ or uses $\dfrac{\sin ACD}{7} = \dfrac{\sin".207"}{3}$ | M1 |
| | so $ACD = 0.709$ | A1 |
| | | **(4)** |
| (b) | Finds angle $ADB$ or angle $ACB$ by doubling their $ADC$ or $ACD$ and uses $s = r\theta$ | M1 |
| | Finds both and adds (can follow failure to double angle so can earn M0M1) | M1 |
| | to give 8.2 (cm) (allow AWRT) | A1 |
| | | 3) |
| (c) | Finds angle $ADB$ or angle $ACB$ and uses $\frac{1}{2}r^2(\theta - \sin\theta)$ for segment, or uses $\frac{1}{2}r^2\theta$ for sector **and** $\frac{1}{2}r^2\sin\theta$ for triangle and doubles at some point, with $r = 3$ or $r = 7$ | M1 |
| | Complete and correct method to establish required area (there are a few alternatives see notes below) | M1 |
| | Obtains correct expressions $\frac{1}{2}7^2(0.565 - \sin 0.565)$ and $\frac{1}{2}3^2(1.42 - \sin 1.42)$ or awrt 0.7, **and** 1.933 and is using correct method to combine them | A1 |
| | Awrt 2.6 or 2.7 | A1 |
| | | **(4)** |
| | | (11 marks) |

**Notes**

NB If there is a misread and they use $AB$ = 9cm it leads to an impossibility. Please send to review.

(a) (i) (Mark parts (i) and (ii) together. Some find the answer to part (ii) first, then may use sine rule in part (i))
M1: Uses cosine rule correctly to obtain required angle
A1: allow awrt 0.283 The answer in degrees is 16.2 and gets M1A0
(If they double this answer then do not isw here so 0.283 followed by 0.565 is M1A0)
(a) (ii)
M1: Uses cosine rule or sine rule correctly
A1: allow awrt 0.709 The answer in degrees is 40.6 (**only penalise the first time for answers in degrees**)

(b)
M1 Doubles one of the angles and uses formula for arc length (These are 3.962 and 4.254)
M1 for adding two appropriate arc lengths
A1 for awrt 8.2 (do not need to see units)

(c)
M1: Uses formula for segment with an appropriate angle, or uses at least one area of sector and corresponding area of triangle, or finds area of kite together with at least one area of sector
M1: Adds two segment areas, or two sectors and subtracts two triangles which form the kite.(They might even use four triangles to form the kite but this results in a long method)
A1: For **two** correct expressions added or for both awrt 0.73 and 1.9 added

OR $\frac{1}{2}7^2 \times 0.565 + \frac{1}{2}3^2 \times 1.42 - \frac{1}{2}7^2 \times \sin 0.565 - \frac{1}{2}3^2 \times \sin 1.42$ (Needs both M marks)

A1: allow awrt 2.6 or 2.7 (do not need to see units)
(There are a number of ways of obtaining this area)

**Q12.**

| Question | Scheme | Marks |
|---|---|---|
| (a) | $2\cot 2x + \tan x \equiv \dfrac{2}{\tan 2x} + \tan x$ | B1 |
| | $\equiv \dfrac{(1-\tan^2 x)}{\tan x} + \dfrac{\tan^2 x}{\tan x}$ | M1 |
| | $\equiv \dfrac{1}{\tan x}$ | M1 |
| | $\equiv \cot x$ | A1* |
| | | (4) |
| (b) | $6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2 \Rightarrow 3\cot x = \operatorname{cosec}^2 x - 2$ | |
| | $\Rightarrow 3\cot x = 1 + \cot^2 x - 2$ | M1 |
| | $\Rightarrow 0 = \cot^2 x - 3\cot x - 1$ | A1 |
| | $\Rightarrow \cot x = \dfrac{3 \pm \sqrt{13}}{2}$ | M1 |
| | $\Rightarrow \tan x = \dfrac{2}{3 \pm \sqrt{13}} \Rightarrow x = ..$ | M1 |
| | $\Rightarrow x = 0.294, -2.848, -1.277, 1.865$ | A2,1,0 |
| | | (6) |
| | | (10 marks) |
| (a)alt 1 | $2\cot 2x + \tan x \equiv \dfrac{2\cos 2x}{\sin 2x} + \tan x$ | B1 |
| | $\equiv 2\dfrac{\cos^2 x - \sin^2 x}{2\sin x \cos x} + \dfrac{\sin x}{\cos x}$ | M1 |
| | $\equiv \dfrac{\cos^2 x - \sin^2 x}{\sin x \cos x} + \dfrac{\sin^2 x}{\sin x \cos x} \equiv \dfrac{\cos^2 x}{\sin x \cos x}$ | M1 |
| | $\equiv \dfrac{\cos x}{\sin x}$ | |
| | $\equiv \cot x$ | A1* |
| (a)alt 2 | $2\cot 2x + \tan x \equiv 2\dfrac{(1-\tan^2 x)}{2\tan x} + \tan x$ | B1M1 |
| | $\equiv \dfrac{2}{2\tan x} - \dfrac{2\tan^2 x}{2\tan x} + \tan x$ or $\dfrac{(1-\tan^2 x) + \tan^2 x}{\tan x}$ | |
| | $\equiv \dfrac{2}{2\tan x} = \cot x$ | M1A1* |

| | | |
|---|---|---|
| Alt (b) | $6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2 \Rightarrow \dfrac{3\cos x}{\sin x} = \dfrac{1}{\sin^2 x} - 2$ | |
| | $\left(\times \sin^2 x\right) \Rightarrow 3\sin x \cos x = 1 - 2\sin^2 x$ | M1 |
| | $\Rightarrow \dfrac{3}{2}\sin 2x = \cos 2x$ | M1A1 |
| | $\Rightarrow \tan 2x = \dfrac{2}{3} \Rightarrow x = ..$ | M1 |
| | $\Rightarrow x = 0.294, -2.848, -1.277, 1.865$ | A2,1,0 |
| | | (6) |

(a)

B1 States or uses the identity $2\cot 2x = \dfrac{2}{\tan 2x}$ or alternatively $2\cot 2x = \dfrac{2\cos 2x}{\sin 2x}$

This may be implied by $2\cot 2x = \dfrac{1-\tan^2 x}{\tan x}$. Note $2\cot 2x = \dfrac{1}{2\tan 2x}$ is B0

M1 Uses the correct double angle identity $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

Alternatively uses $\sin 2x = 2\sin x\cos x$, $\cos 2x = \cos^2 x - \sin^2 x$ oe and $\tan x = \dfrac{\sin x}{\cos x}$

M1 Writes their two terms with a single common denominator and simplifies to a form $\dfrac{ab}{cd}$.

For this to be scored the expression must be in either $\sin x$ and $\cos x$ or just $\tan x$.

In alternative 2 it is for splitting the complex fraction into parts and simplifying to a form $\dfrac{ab}{cd}$.

You are awarding this for a correct method to proceed to terms like $\dfrac{\cos^2 x}{\sin x\cos x}$, $\dfrac{2\cos^3 x}{2\sin x\cos^2 x}$, $\dfrac{2}{2\tan x}$

A1* cso. For proceeding to the correct answer. This is a given answer and all aspects must be correct including the consistent use of variables. If the candidate approaches from both sides there must be a conclusion for this mark to be awarded. Occasionally you may see a candidate attempting to prove $\cot x - \tan x \equiv 2\cot 2x$. This is fine but again there needs to be a conclusion for the A1*
If you are unsure of how some items should be marked then please use review

(b)

M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)

WITH an attempt at using $\text{cosec}^2 x = \pm 1 \pm \cot^2 x$ to produce a quadratic equation in just $\cot x / \tan x$

A1 $\cot^2 x - 3\cot x - 1 = 0$ The $= 0$ may be implied by subsequent working

Alternatively accept $\tan^2 x + 3\tan x - 1 = 0$

M1 Solves a 3TQ=0 in $\cot x$ (or tan) using the formula or any suitable method for their quadratic to find at least one solution. Accept answers written down from a calculator. You may have to check these from an incorrect quadratic. FYI answers are $\cot x =$ awrt $3.30$, $-0.30$

Be aware that $\cot x = \dfrac{3 \pm \sqrt{13}}{2} \Rightarrow \tan x = \dfrac{-3 \pm \sqrt{13}}{2}$

M1 For $\tan x = \dfrac{1}{\cot x}$ and using arctan producing at least one answer for $x$ in degrees or radians.

You may have to check these with your calculator.

A1 Two of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3dp) in radians or degrees.

In degrees the answers you would accept are (awrt 2dp) $x = 16.8°, 106.8°, -73.2°, -163.2°$

A1 All four of $x = 0.294, -2.848, -1.277, 1.865$ (awrt 3 dp) with no extra solutions in the range $-\pi$ ,, **x** x $\pi$

See main scheme for Alt to (b) using Double Angle formulae still entered M A M M A A in epen

1st M1 For using part (a) and writing $6\cot 2x + 3\tan x$ as $k\cot x$, $k \neq 0$ in their equation (or equivalent)

then using $\cot x = \dfrac{\cos x}{\sin x}$, $\text{cosec}^2 x = \dfrac{1}{\sin^2 x}$ and $\times \sin^2 x$ to form an equation sin and cos

1st A1 For $\dfrac{3}{2}\sin 2x = \cos 2x$ or equivalent. **Attached to the next M**

2nd M1 For using both correct double angle formula

3rd M1 For moving from $\tan 2x = C$ to $x$=..using the correct order of operations.

**Q.13**

| Question Number | Scheme | Marks |
|---|---|---|
| **2(a)** | $\overrightarrow{BA}.\overrightarrow{BC} = -6\times 2 + 2\times 5 - 3\times 8 = (-26)$ | M1 |
| | Uses $\overrightarrow{BA}.\overrightarrow{BC} = \left\|\overrightarrow{BA}\right\|\left\|\overrightarrow{BC}\right\|\cos\theta \Rightarrow -26 = \sqrt{49}\times\sqrt{93}\cos\theta \Rightarrow \theta = ...$ | **d**M1 |
| | $\theta = 112.65°$ | A1 |
| | | **(3)** |
| **(b)** | Attempts to use $\left\|\overrightarrow{BA}\right\|\left\|\overrightarrow{BC}\right\|\sin\theta$ with their $\theta$ | M1 |
| | Area = awrt 62.3 | A1 |
| | | **(2)** |
| | | **(5 marks)** |

(a)

M1: Attempts the scalar product of $\pm\overrightarrow{AB}.\pm\overrightarrow{BC}$ condone slips as long as the intention is clear

**Or** attempts the vector product $\pm\overrightarrow{AB}\times\pm\overrightarrow{BC}$ **(see alternative 1)**

**Or attempts vector AC (see alternative 2)**

**d**M1: Attempts to use $\pm\overrightarrow{AB}.\overrightarrow{BC} = \left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right|\cos\theta$ AND proceeds to a value for $\theta$

Expect to see at least one correct attempted calculation for a modulus.

For example $\sqrt{2^2+5^2+8^2}\left(=\sqrt{93}\right)$ or $\sqrt{6^2+2^2+3^2}\,(=7)$

**Note that we condone poor notation such as:** $\cos\theta = \frac{26}{7\sqrt{93}} = 67.35°$ **Depends on the first mark.**

**<u>Must be an attempt to find the correct angle.</u>**

A1: $\theta = \text{awrt } 112.65°$ Versions finishing with $\theta = \text{awrt } 67.35°$ will normally score M1 dM1 A0

Angles given in radians also score A0 (NB $\theta = 1.9661...$ or acute $1.1754...$)

Allow e.g. $\theta = 67.35° \Rightarrow \theta = 180 - 67.35° = 112.65$ and allow $\cos\theta = \frac{26}{7\sqrt{93}} \Rightarrow \theta = 112.65$

**<u>1. Alternative using the vector product:</u>**

M1: Attempts the vector product $\pm\overrightarrow{AB}\times\pm\overrightarrow{BC} = \pm\begin{pmatrix}6\\-2\\3\end{pmatrix}\times\pm\begin{pmatrix}2\\5\\8\end{pmatrix} = \pm\begin{pmatrix}-31\\-42\\34\end{pmatrix}$ condone slips as long as the intention is clear

**d**M1: Attempts to use $\pm\overrightarrow{AB}\times\overrightarrow{BC} = \left|\overrightarrow{AB}\right|\left|\overrightarrow{BC}\right|\sin\theta$ AND proceeds to a value for $\theta$

Expect to see at least one correct attempted calculation for a modulus on rhs and attempt at the modulus of the vector product

For example $\sqrt{2^2+5^2+8^2}$ or $\sqrt{6^2+2^2+3^2}$ and $\sqrt{31^2+42^2+34^2}\left(=\sqrt{3881}\right)$

For example $\sqrt{2^2+5^2+8^2}$ or $\sqrt{6^2+2^2+3^2}$ and $\sqrt{31^2+42^2+34^2}\left(=\sqrt{3881}\right)$

**Note that we condone poor notation such as:** $\sin\theta = \frac{\sqrt{3881}}{7\sqrt{93}} = 67.35°$ **Depends on the first mark.**

**<u>Must be an attempt to find the correct angle.</u>**

A1: $\theta = \text{awrt } 112.65°$ Versions finishing with $\theta = \text{awrt } 67.35°$ will normally score M1 dM1 A0

**2. Alternative using cosine rule:**

M1: Attempts $\pm\overrightarrow{AC} = \pm\left(\overrightarrow{AB} + \overrightarrow{BC}\right) = \pm(8\mathbf{i} + 3\mathbf{j} + 11\mathbf{k})$ condone slips and poor notation as long as the intention is clear e.g. allow $\begin{pmatrix} 8\mathbf{i} \\ 3\mathbf{j} \\ 11\mathbf{k} \end{pmatrix}$

**d**M1: Attempts to use $AC^2 = AB^2 + BC^2 - 2AB.BC\cos\theta$ AND proceeds to a value for $\theta$

**Must be an attempt to find the correct angle.**

A1: $\theta =$ awrt $112.65°$

**Q14**

| Question Number | Scheme | Marks |
|---|---|---|
| **7.** | $\frac{dh}{dt} = k\sqrt{(h-9)}$, $9 < h \le 200$; $h = 130$, $\frac{dh}{dt} = -1.1$ | |
| (a) | $-1.1 = k\sqrt{(130-9)} \Rightarrow k = ...$ | M1 |
| | so, $k = -\frac{1}{10}$ or $-0.1$ | A1 |
| | | **[2]** |
| (b) | $\int \frac{dh}{\sqrt{(h-9)}} = \int k\,dt$ | B1 |
| | $\int (h-9)^{-\frac{1}{2}}\,dh = \int k\,dt$ | |
| | $\frac{(h-9)^{\frac{1}{2}}}{(\frac{1}{2})} = kt\ (+c)$ | M1 A1 |
| | $\{t = 0, h = 200 \Rightarrow\}$ $2\sqrt{(200-9)} = k(0) + c$ | M1 |
| | $\Rightarrow c = 2\sqrt{191} \Rightarrow 2(h-9)^{\frac{1}{2}} = -0.1t + 2\sqrt{191}$<br>$\{h = 50 \Rightarrow\}$ $2\sqrt{(50-9)} = -0.1t + 2\sqrt{191}$<br>$t = ...$ | dM1 |
| | $t = 20\sqrt{191} - 20\sqrt{41}$<br>or $t = 148.3430145... = 148$ (minutes) (nearest minute) | A1 cso |
| | | **[6]** |
| | | **8** |

**(these snippings have been taken from emporium, I know they are a bit glitchy, use for notes)**

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| 7. | $\frac{dh}{dt} = k\sqrt{(h-9)}$, $9 < h \le 200$; $h = 130$, $\frac{dh}{dt} = -1.1$ | | |
| (a) | $-1.1 = k\sqrt{(130-9)}$ □ \| $k = ...$ | Substitutes $h = 130$ and either $\frac{dh}{dt} = -1.1$ or $\frac{dh}{dt} = 1.1$ into the printed equation and rearranges to give $k = ...$ | M1 |
| | so, $k = -\frac{1}{10}$ or $-0.1$ | $k = -\frac{1}{10}$ or $-0.1$ | A1 |
| | | | [2] |
| (b) Way 1 | $\int \frac{dh}{\sqrt{(h-9)}} = \int k\,dt$ | Separates the variables correctly. $dh$ and $dt$ should not be in the wrong positions, although this mark can be implied by later working. Ignore the integral signs. | B1 |
| | $\int (h-9)^{-\frac{1}{2}}\,dh = \int k\,dt$ | | |
| | $\frac{(h-9)^{\frac{1}{2}}}{(\frac{1}{2})} = kt\,(+c)$ | Integrates $\frac{\pm\lambda}{\sqrt{(h-9)}}$ to give $\pm\mu\sqrt{(h-9)}$; $\lambda, \mu$ □ 0 | M1 |
| | | $\frac{(h-9)^{\frac{1}{2}}}{(\frac{1}{2})} = kt$ or $\frac{(h-9)^{\frac{1}{2}}}{(\frac{1}{2})} = (\text{their } k)t$, with/without $+c$, or equivalent, which can be un-simplified or simplified. | A1 |
| | $\{t = 0, h = 200$ □ $\}$ $2\sqrt{(200-9)} = k(0) + c$ | Some evidence of applying both $t = 0$ and $h = 200$ to changed equation containing a constant of integration, e.g. $c$ or $A$ | M1 |
| | □ $c = 2\sqrt{191}$ □ $2(h-9)^{\frac{1}{2}} = -0.1t + 2\sqrt{191}$<br>$\{h = 50 \Rightarrow\}$ $2\sqrt{(50-9)} = -0.1t + 2\sqrt{191}$<br>$t = ...$ | **dependent on the previous M mark**<br>Applies $h = 50$ and their value of $c$ to their changed equation and rearranges to find the value of $t = ...$ | dM1 |
| | $t = 20\sqrt{191} - 20\sqrt{41}$<br>or $t = 148.3430145... = 148$ (minutes) (nearest minute) | $t = 20\sqrt{191} - 20\sqrt{41}$ isw<br>or awrt 148 | A1 cso |
| | | | [6] |

| Question Number | Scheme | Notes | Marks |
|---|---|---|---|
| (b) Way 2 | $\int_{200}^{50} \frac{dh}{\sqrt{(h-9)}} = \int_0^T k\,dt$ | Separates the variables correctly. $dh$ and $dt$ should not be in the wrong positions, although this mark can be implied by later working. Integral signs and limits not necessary. | B1 |
| | $\int_{200}^{50} (h-9)^{-\frac{1}{2}}\,dh = \int_0^T k\,dt$ | | |
| | $\left[\frac{(h-9)^{\frac{1}{2}}}{(\frac{1}{2})}\right]_{200}^{50} = [kt]_0^T$ | Integrates $\frac{\pm\lambda}{\sqrt{(h-9)}}$ to give $\pm\mu\sqrt{(h-9)}$; $\lambda, \mu$ □ 0 | M1 |
| | | $\frac{(h-9)^{\frac{1}{2}}}{(\frac{1}{2})} = kt$ or $\frac{(h-9)^{\frac{1}{2}}}{(\frac{1}{2})} = (\text{their } k)t$, with/without limits, or equivalent, which can be un-simplified or simplified. | A1 |
| | $2\sqrt{41} - 2\sqrt{191} = kt$ or $kT$ | Attempts to apply limits of $h = 200$, $h = 50$ and (can be implied) $t = 0$ to their changed equation | M1 |
| | $t = \frac{2\sqrt{41} - 2\sqrt{191}}{-0.1}$ | **dependent on the previous M mark**<br>Then rearranges to find the value of $t = ...$ | dM1 |
| | $t = 20\sqrt{191} - 20\sqrt{41}$<br>or $t = 148.3430145... = 148$ (minutes) (nearest minute) | $t = 20\sqrt{191} - 20\sqrt{41}$ **or** awrt 148<br>**or** 2 hours and awrt 28 minutes | A1 cso |
| | | | [6] |
| | | | 8 |

| | | |
|---|---|---|
| | **Question 7 Notes** | |
| 7. (b) | **Note** | Allow first B1 for writing $\frac{\mathrm{d}t}{\mathrm{d}h} = \frac{1}{k\sqrt{(h-9)}}$ or $\frac{\mathrm{d}t}{\mathrm{d}h} = \frac{1}{(\text{their } k)\sqrt{(h-9)}}$ or equivalent |
| | **Note** | $\frac{\mathrm{d}t}{\mathrm{d}h} = \frac{1}{k\sqrt{(h-9)}}$ leading to $t = \frac{2}{k}\sqrt{(h-9)}$ $(+\,c)$ with/without $+\,c$ is B1M1A1 |
| | **Note** | After finding $k = 0.1$ in part (a), it is only possible to gain full marks in part (b) by **initially writing** $\frac{\mathrm{d}h}{\mathrm{d}t} = -\,k\sqrt{(h-9)}$ **or** $\int \frac{\mathrm{d}h}{\sqrt{(h-9)}} = \int -k\,\mathrm{d}t$ **or** $\frac{\mathrm{d}h}{\mathrm{d}t} = -\,0.1\sqrt{(h-9)}$ **or** $\int \frac{\mathrm{d}h}{\sqrt{(h-9)}} = \int -0.1\,\mathrm{d}t$<br>Otherwise, those candidates who find $k = 0.1$ in part (a), should lose at least the final A1 mark in part (b). |