Year 13 Mock Pure Paper 2

18th May 2021

This exam has 14 questions, for a total of 100 marks.

- Print in "booklets" will allow all questions to be on the left hand side.
- If instead you print in 2-in-1 settings, print the second page up to the last page first, then print the first page separately.

Question	Marks	Score
1	3	
2	7	
3	7	
4	5	
5	6	
6	7	
7	8	
8	11	
9	8	
10	7	
11	12	
12	8	
13	7	
14	4	
Total:	100	

Andrew Chan

Last updated: 13th February 2023

1. O, A and B are fixed points such that

$$\overrightarrow{OA} = 4\mathbf{i} + 3\mathbf{j} \qquad \overrightarrow{OB} = 8\mathbf{i} + p\mathbf{j} \qquad |\overrightarrow{AB}| = 2\sqrt{13}$$

Find the possible values of p

(3)

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Question 1 continued

(Total for Question 1 is 3 marks)

2.

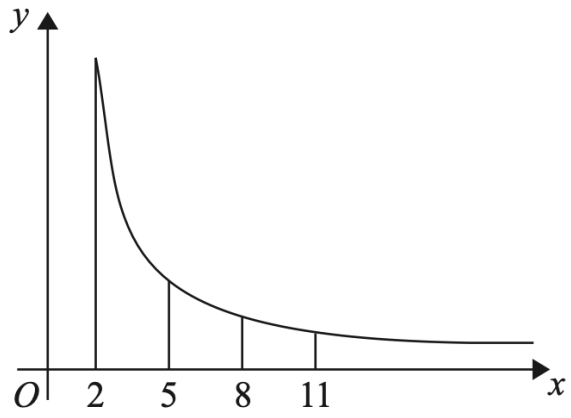

Figure 1

Figure 1 shows a sketch of part of the graph of $y = \dfrac{12}{\sqrt{x^2 - 2}}$, $x \geq 2$

The table below gives values of y rounded to 3 decimal places.

x	2	5	8	11
y	8.485	2.502	1.524	1.100

(a) Use the trapezium rule with all the values of y from the table to find an approximate value, to 2 decimal places, for

$$\int_2^{11} \frac{12}{\sqrt{x^2 - 2}} \, \mathrm{d}x$$

(4)

(b) Use your answer to part(a) to estimate a value for

$$\int_2^{11} 1 + \frac{6}{\sqrt{x^2 - 2}} \, \mathrm{d}x$$

(3)

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Question 2 continued

(Total for Question 2 is 7 marks)

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3. Given that

$$2\log_4(2x+3) = 1 + \log_4 x + \log_4(2x-1) \qquad x > \frac{1}{2}$$

(a) Show that

$$4x^2 - 16x - 9 = 0$$

(5)

(b) Hence solve the equation

$$2\log_4(2x+3) = 1 + \log_4 x + \log_4(2x-1) \qquad x > \frac{1}{2}$$

(2)

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Question 3 continued

(Total for Question 3 is 7 marks)

4. The curve with eqation $y = 6^{1-x}$ meets the curve with equation $y = 3 \times 4^x$ at the point P.

Show that the x-coordinate of P is $\dfrac{\log_{10} 2}{\log_{10} 24}$

(5)

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Question 4 continued

(Total for Question 4 is 5 marks)

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5. The temperature, θ °C, inside an oven, t minutes after the oven is switched on, is given by

$$\theta = A - 180\mathrm{e}^{-kt}$$

where A and k are constants.

Given that the temperature inside the oven is initially 18 °C,

(a) find the value of A.

(2)

The temperature inside the oven, 5 minutes after the oven is switched on, is 90 °C.

(b) Show that $k = p \ln q$, where p and q are rational numbers to be found.

(4)

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Question 5 continued

(Total for Question 5 is 6 marks)

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6. (a) Express $\dfrac{9(4+x)}{16-9x^2}$ in partial fractions.

(3)

Given that

$$\mathrm{f}(x) = \frac{9(4+x)}{16-9x^2} \qquad x \in \mathbb{R},\ -\frac{4}{3} < x < \frac{4}{3}$$

(b) express $\displaystyle\int \mathrm{f}(x)\,\mathrm{d}x$ in them form $\ln\left|\mathrm{g}(x)\right|$, where $\mathrm{g}(x)$ is a rational function.

(4)

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 7 marks)

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7.

$$\mathrm{f}(x) = x\cos\left(\frac{x}{3}\right) \qquad x > 0$$

(a) Find $\mathrm{f}'(x)$

(2)

(b) Show that the equation $\mathrm{f}'(x) = 0$ can be written as

$$x = k\arctan\left(\frac{k}{x}\right)$$

where k is an integer to be found.

(2)

(c) Starting with $x_1 = 2.5$, use the iteration formula

$$x_{n+1} = k\arctan\left(\frac{k}{x_n}\right)$$

with the value of k found in part (b), to calculate the values of x_2 and x_6.
Give your answers to 3 decimal places.

(2)

(d) Using a suitable interval and a suitable function that should be stated, show that a root of $\mathrm{f}'(x) = 0$ is 2.581 correct to 3 decimal places.

(2)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 8 marks)

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8. (i) Find

$$\int \frac{(3x+2)^2}{4\sqrt{x}}\,\mathrm{d}x \qquad x > 0$$

(5)

(ii) A curve C has equation $y = \mathrm{f}(x)$

Given that

- $\mathrm{f}'(x) = x^2 + ax + b$ where a and b are constants.
- the y-intercept of C is -8
- the point $P(3, -2)$ lies on C
- the gradient of C at P is 2

find, in simplest form, $\mathrm{f}(x)$

(6)

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Question 8 continued

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 11 marks)

9. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(a) Show that

$$\frac{\cos 3\theta}{2\sin\theta} + \frac{\sin 3\theta}{2\cos\theta} \equiv \cot 2\theta \qquad \theta \neq \frac{n\pi}{2},\ n \in \mathbb{Z}$$

(4)

(b) Hence solve, for $0 < x < \frac{\pi}{2}$

$$\frac{\cos 3x}{2\sin x} + \frac{\sin 3x}{2\cos x} = 5\cos 2x$$

giving your answers to 3 decimal places where appropriate.

(4)

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Question 9 continued

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Question 9 continued

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Question 9 continued

(Total for Question 9 is 8 marks)

10.

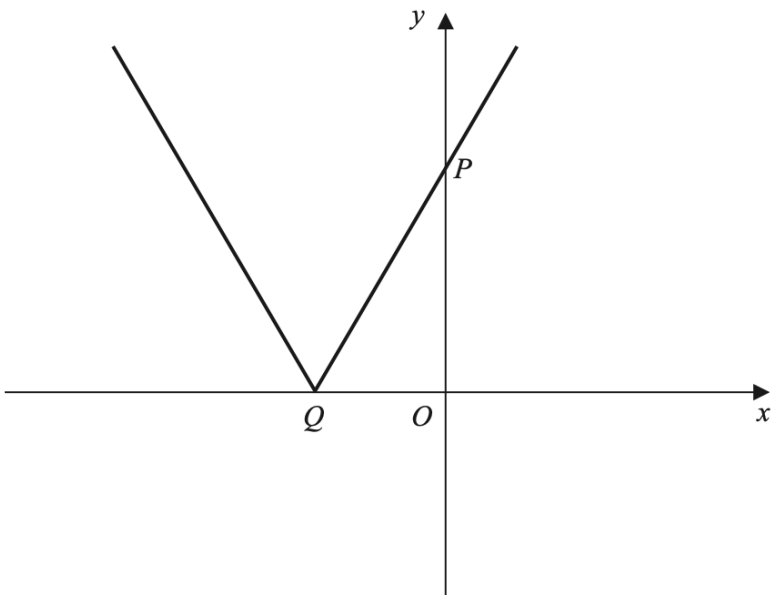

Figure 2

Figure 2 shows a sketch of the graph with equation $y = |4x + 10a|$, where a is a positive constant.

The graph cuts the y-axis at the point P and meets the x-axis at the point Q as shown.

(a) (i) State the coordinates of P

(ii) State the coordinates of Q

(2)

(b) A copy of Figure 2, labelled Diagram 1, is shown on page 29.
On this copy, sketch the graph with equation

$$y = |x| - a$$

Show on the sketch the coordinates of each point where your graph cuts or meets the coordinate axes.

(2)

(c) Hence, or otherwise, solve the equation

$$|4x + 10a| = |x| - a$$

giving your answers in terms of a.

(3)

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Question 10 continued

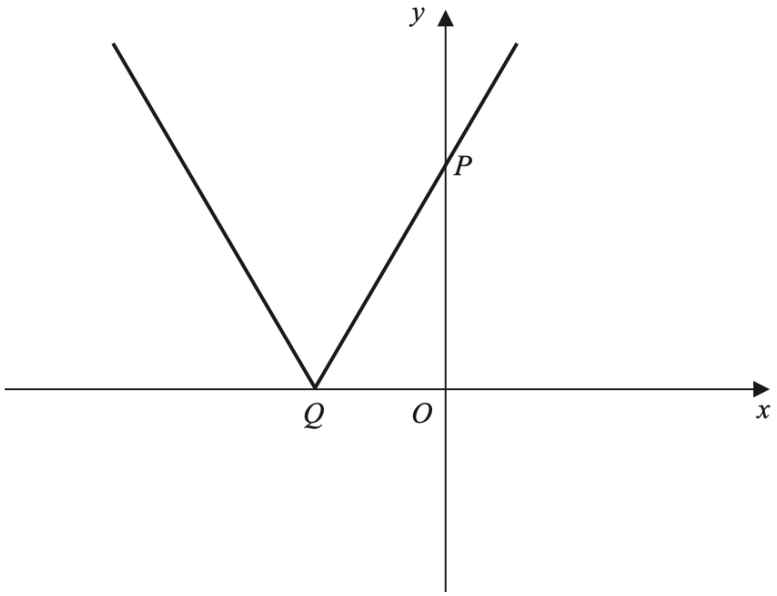

Diagram 1

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Question 10 continued

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Question 10 continued

(Total for Question 10 is 7 marks)

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11.

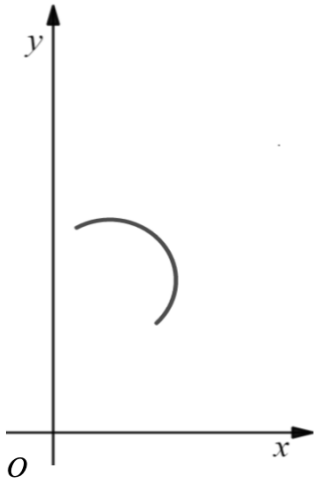

Figure 3

The curve C has parametric equations

$$x = 3 + 2\sqrt{3}\cos t \qquad y = 5\sqrt{3} + 2\sqrt{3}\sin t \qquad -\frac{\pi}{4} \leq t \leq \frac{2\pi}{3}$$

A sketch of C is shown in Figure 3.

(a) Show that all points on C satisfy the Cartesian equation

$$(x-3)^2 + (y-5\sqrt{3})^2 = 12$$

(2)

For curve C in the given domain of t,

(b) (i) state the range of x

(ii) state the range of y

(2)

The point P lies on C.
Given that the line with equation $y = mx + 12\sqrt{3}$, where m is a constant, intersects C at P,

(c) state the range of m, writing your answer using set notation.

(6)

The points $(0, 0), (0, 12\sqrt{3})$ and P form a triangle.

(d) (i) Find the largest possible area of the triangle

(ii) Find the smallest possible area of the triangle

(2)

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Question 11 continued

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Question 11 continued

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Question 11 continued

(Total for Question 11 is 12 marks)

12. The circle C has equation

$$x^2 + y^2 + 6x - 4y - 14 = 0$$

(a) (i) the coordinates of the centre of C

(ii) the exact radius of C

(3)

The line with equation $y = k$, where k is a constant, is a tangent to C.

(b) Find the range of possible values of k.

(2)

The line with equation $y = p$, where p is a negative constant, is a chord of C.
Given that the length of the chord is 4 units,

(c) find the value of p

(3)

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Question 12 continued

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Question 12 continued

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Question 12 continued

(Total for Question 12 is 8 marks)

13. (a) Prove that the sum of the first n terms of an arithmetic series is given by the formula

$$S_n = \frac{n}{2}\Big[2a + (n-1)d\Big]$$

where a is the first term of the series and d is the common difference between the terms.

(4)

(b) Find the sum of the integers which are divisible by 7 and lie between 1 and 500.

(3)

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Question 13 continued

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14. Prove by contradiction, that if a and b are positive real numbers, then $a + b \geq 2\sqrt{ab}$

(4)

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Question 14 continued

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Question 14 continued

Total for paper is 100 marks

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