SOHOKMATHS

13Ma Pre January Exams Mini Test

Trigonometry (Harmonic Forms)

Implicit Differentiation

Sector, Radians, Numerical Methods

Surname ______________________________ Other names ______________________________

☐ B Mr Chan/Mr Phillips ☐ M2E Mr Chan/Ms Esteban

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill at the top of this page with your name, and tick the box with the class you belong to.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Question	Marks	Score
1	11	
2	8	
3	11	
Total:	30	

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 3 questions in this question paper.
  The total mark for this paper is 30.
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

1. (a) Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

Give the value of α correct to 2 decimal places.

(3)

(b) Solve the equation $7\cos\theta + 24\sin\theta = 18$ for $0° < \theta < 360°$.

(4)

As β varies, the greatest possible value of

$$\frac{150}{7\cos\frac{1}{2}\beta + 24\sin\frac{1}{2}\beta + 50}$$

is denoted by V,

(c) (i) find the value of V,

(ii) determine the smallest positive value of β (in degrees) for which the value of V occurs.

(4)

1
Adapted from Cambridge International Exams 9709/21
Pure Mathematics 2 May/June 2023 Q7

Marking Schemes for Question 1

Question	Answer	Marks	Guidance
7(a)	State $R = 25$	**B1**	
	Use appropriate trigonometry to find α	**M1**	Allow if found in radians .
	Obtain $\alpha = 73.74$	**A1**	or greater accuracy.
		3	

Question	Answer	Marks	Guidance
7(b)	Use correct method to find one value of θ	**M1**	
	Obtain 29.8 (or 117.7)	**A1**	or greater accuracy.
	Use correct method to find second value of θ between 0 and 360	**M1**	
	Obtain 117.7 (or 29.8)	**A1**	or greater accuracy; and no others between 0 and 360.
		4	
7(c)	State or imply expression is $\dfrac{150}{25\cos(\frac{1}{2}\beta - 73.74) + 50}$	**B1 FT**	following *their* R and α .
	Obtain $V = 6$	**B1**	
	Attempt complete method to find positive value from $\cos(\frac{1}{2}\beta - 73.74) = -1$	**M1**	for *their* α .
	Obtain 507.5	**A1**	or greater accuracy.
		4	

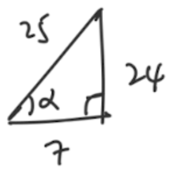
a)

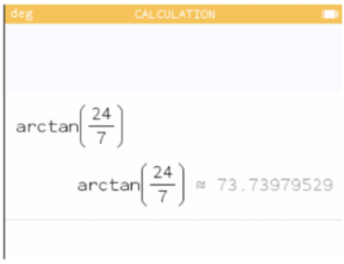
$R\cos\theta\cos\alpha + R\sin\theta\sin\alpha$

$7\cos\theta + 24\sin\theta$

$\Rightarrow R\cos\alpha = 7$

$R\sin\alpha = 24$





$\alpha = 73.74^\circ$

$\Rightarrow 7\cos\theta + 24\sin\theta = 25\cos(\theta - 73.74^\circ)$

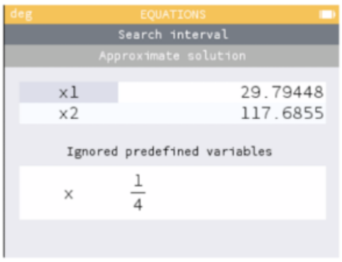
b)

$7\cos\theta + 24\sin\theta = 18$

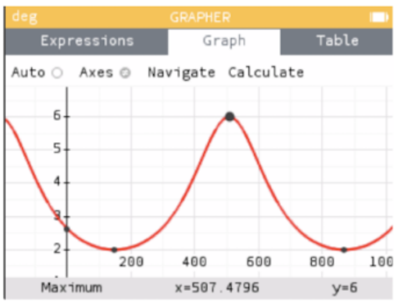
$25\cos(\theta - 73.74) = 18$

$\cos(\theta - 73.74) = 0.72$

$\theta = 29.79^\circ \quad \theta = 117.69^\circ$



c)



1) V is greatest when $\frac{150}{\text{"smallest"}} + 50 = \frac{150}{\text{"25"}} + 50$

$= \frac{150}{25}$

$= 6$

$\theta = \text{"}\frac{1}{2}\beta\text{"}$

$\Rightarrow \cos(\frac{1}{2}\beta - 73.74) = \cos(180) \quad \beta = 507.5$

(180+73.74)×2

507.48

2. The equation of a curve is $x^2y - ay^2 = 4a^3$, where a is a non-zero constant.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2xy}{2ay - x^2}$.

(4)

(b) Hence find the coordinates of the points where the tangent to the curve is parallel to the y-axis.

(4)

2
Adapted from Cambridge International Exams 9709/31
Pure Mathematics 3 May/June 2023 Q5

Marking Schemes for Question 2

Question	Answer	Marks	Guidance
5(a)	State or imply $2xy + x^2\frac{dy}{dx}$ as derivative of x^2y	B1	Accept partial: $\frac{\partial}{\partial x} \to 2xy$.
	State or imply $2ay\frac{dy}{dx}$ as derivative of ay^2	B1	Accept partial: $\frac{\partial}{\partial y} \to x^2 - 2ay$.
	Equate attempted derivative to zero and solve for $\frac{dy}{dx}$	M1	
	Obtain answer $\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$ from correct working	A1	AG
		4	
5(b)	State or imply $2ay - x^2 = 0$	*M1	
	Substitute into equation of curve to obtain equation in x and a or in y and a	DM1	e.g. $2ay^2 - ay^2 = 4a^3$ or $\frac{x^4}{2a} - \frac{x^4}{4a} = 4a^3$.
	Obtain one correct point	A1	e.g. $(2a, 2a)$.
	Obtain second correct point and no others	A1	e.g. $(-2a, 2a)$.
		4	**SC**: Allow A1 A0 for $x = \pm 2a$ or for $y = 2a$.

Worked Solutions for Question 2

a) $\frac{d}{dx}(x^2y - ay^2) = \frac{d}{dx}(4a^3)$

$x^2\frac{dy}{dx} + y(2x) - 2ay\left(\frac{dy}{dx}\right) = 0$

$\frac{dy}{dx}\left[2ay - x^2\right] = 2xy$

$\frac{dy}{dx} = \frac{2xy}{2ay - x^2}$ //

b) $\frac{dy}{dx} \to \infty$ $\quad \frac{dx}{dy} = 0 \quad x^2 = 2ay$

$(2ay)(y) - ay^2 = 4a^3$

$y^2\left[2a - a\right] = 4a^3$

$y^2 = 4a^2$

$y = 2a$ or $y = -2a$ (rej) ($\because x^2 = 2ay$ means x^2 is +ve $\Rightarrow$ y is +ve)

$x = 2a$ or $x = -2a$

$(2a, 2a)$ or $(-2a, 2a)$

3.

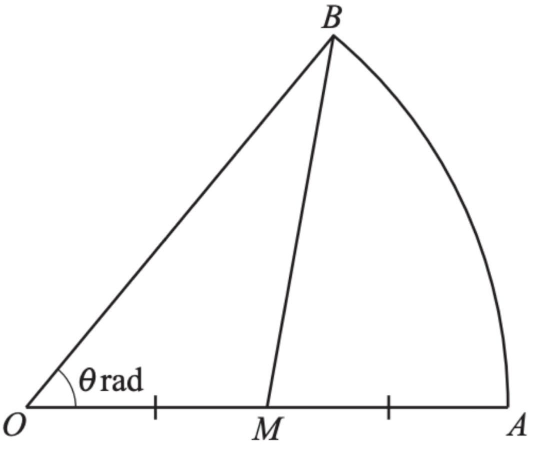


Figure 1

Figure 1 shows a sector OAB of a circle with centre O and radius OA.

The angle AOB is θ radians.

M is the mid-point of OA.

The ratio of areas $OMB : MAB$ is 2:3.

(a) Show that $\theta = 1.25 \sin \theta$.

(4)

The equation $x = 1.25 \sin x$ has only one root for $x > 0$.

This root can be found by using the iterative formula $x_{n+1} = 1.25 \sin x$.

(b) Using a starting value of $x_1 = 0.5$,

(i) write down the values of x_2, x_3 and x_4.

(ii) find the value of this root correct to 3 significant figures.

(iii) show, using a change in sign method, that this root is accurate to 3 significant figures.

(4)

Diagram 1 in the next page shows the graphs of $y = 1.25 \sin x$ and $y = x$ sketched on a graphical calculator.

(c) Use this diagram to show how the iterative process used in (b) converges to this root, you should state the type of convergence, and the values of x_2, x_3 and x_4 in the diagram.

(3)

Question 3 continued

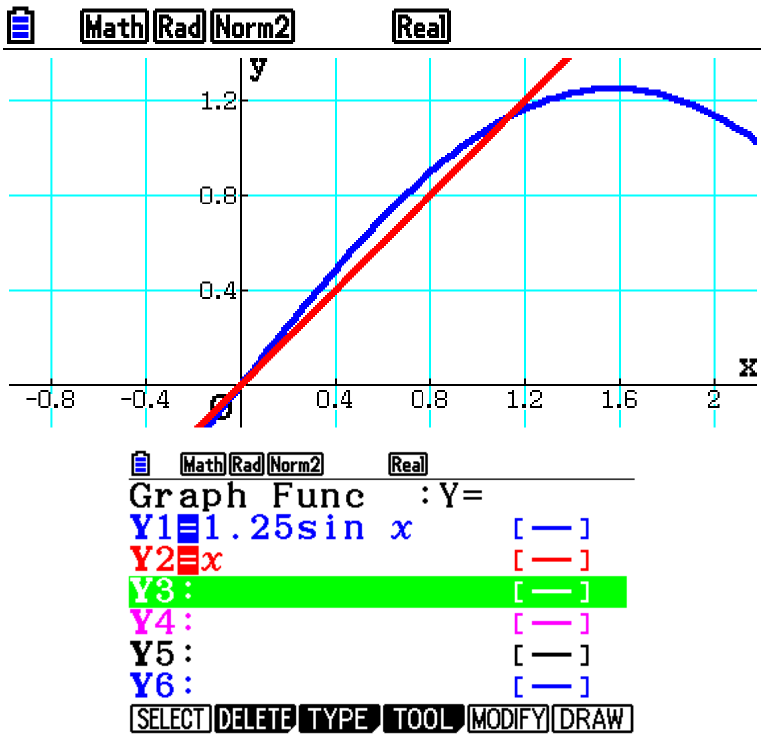
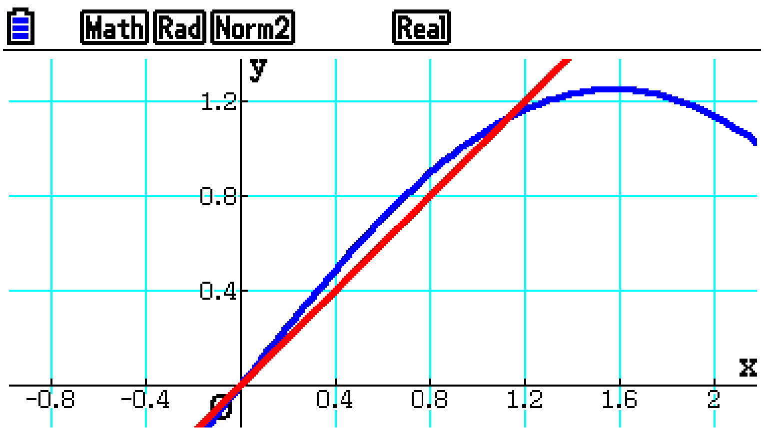


Diagram 1

A copy of Diagram 1, in case you need to redraw your sketch:



Copy of Diagram 1

3
Adapted from OCR Mathematics A (H240)
A-Level June 2022 Q10

Worked Solutions for Question 3

3.

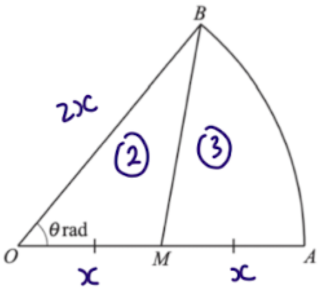


Figure 1

Let $OM = MA = x$

$OB = 2x$

Area of $\Delta OMB = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(2x)(x)\sin\theta$

Area of sector OAB

$= \frac{\theta}{2\pi}\cdot\pi r^2$

$= \frac{\theta}{2}(2x)^2$

$= 2\theta x^2$

Figure 1 shows a sector OAB of a circle with centre O and radius OA.

The angle AOB is θ radians.

M is the mid-point of OA.

The ratio of areas $OMB : MAB$ is 2:3.

(a) Show that $\theta = 1.25\sin\theta$.

$OMB = \frac{2}{5}(OAB)$

(4)

$OMB = \frac{2}{5}(OAB)$

$x^2\sin\theta = \frac{2}{5}(2\theta x^2)$

$x^2\sin\theta = \frac{4}{5}\theta x^2$

$\frac{5}{4}\sin\theta = \theta$

$\theta = 1.25\sin\theta$

The equation $x = 1.25\sin x$ has only one root for $x > 0$.

This root can be found by using the iterative formula $x_{n+1} = 1.25\sin x$.

$x_2 = 0.59928 = 0.599$

(b) Using a starting value of $x_1 = 0.5$,

(i) write down the values of x_2, x_3 and x_4.

(ii) find the value of this root correct to 3 significant figures.

(iii) show, using a change in sign method, that this root is accurate to 3 significant figures.

```
1.25sin Ans
        0.5992819233   = x_2
1.25sin Ans
        0.7050620919   = x_3
1.25sin Ans
        0.8101013978   = x_4
```

(4)

$x_2 = 0.599$
$x_3 = 0.705$
$x_4 = 0.810$ //

root ≃ 1.13 (3sf)

change in sign method ↗ 1.135 ↘ 1.125

$f(\theta) = \theta - 1.25\sin\theta$

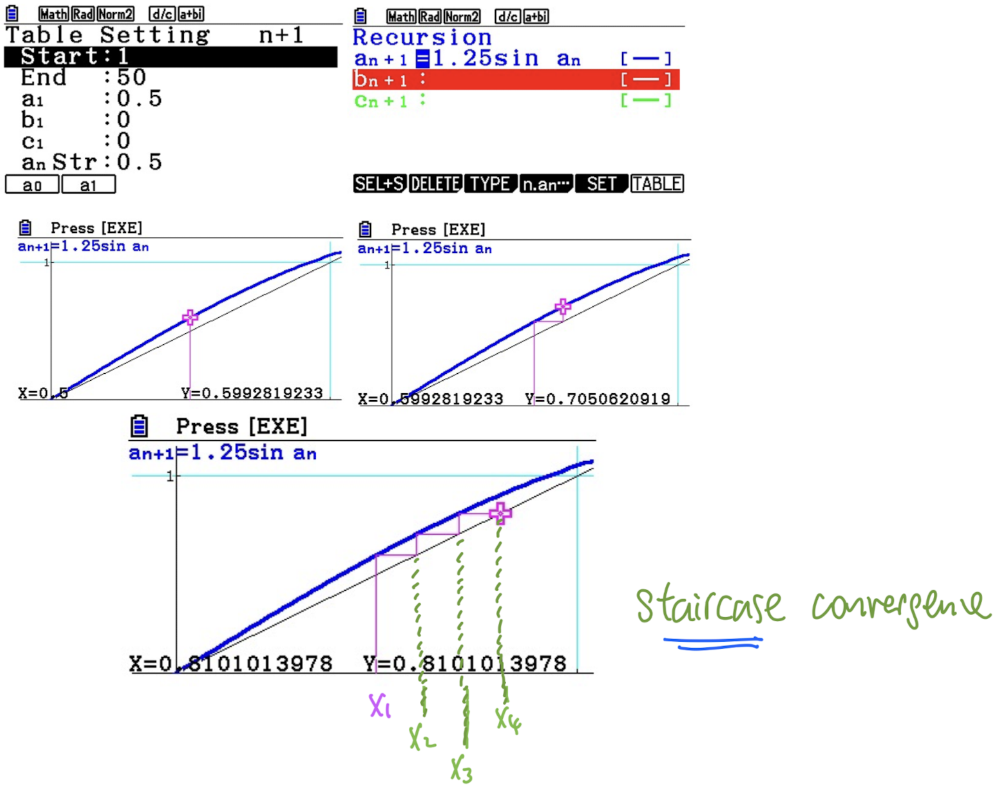
$f(1.135) > 0$

$f(1.125) < 0$

change in sign, function is continuous,

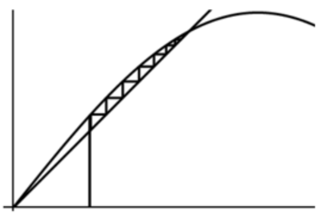
$1.125 < \text{root} < 1.135$ root ≃ 1.13

Worked Solutions for Question 3



Marking Scheme for Question 3

Question			Answer	Marks	AO	Guidance	
10	(a)		area $OMB = \frac{1}{2}\left(\frac{1}{2}r\right)r\sin\theta$	B1	1.1	Correct (possibly unsimplified) area of *OMB*	Could use other than r for the radius Could set their variable equal to *OM*, giving a radius that is double this eg $OM = x$ so area $= x^2\sin\theta$
			$2\left(\frac{1}{2}r^2\theta - \frac{1}{4}r^2\sin\theta\right) = 3\left(\frac{1}{4}r^2\sin\theta\right)$ OR $2\left(\frac{1}{2}r^2\theta\right) = 5\left(\frac{1}{4}r^2\sin\theta\right)$ OR $3\left(\frac{1}{2}r^2\theta\right) = 5\left(\frac{1}{2}r^2\theta - \frac{1}{4}r^2\sin\theta\right)$	M1	3.1a	Attempt to use ratio on two correct areas	Using two of *OMB* ($\frac{1}{4}r^2\sin\theta$), *MAB* $\left(\frac{1}{2}r^2\theta - \frac{1}{4}r^2\sin\theta\right)$ and *OAB* ($\frac{1}{2}r^2\theta$) oe with their variable Must be two correct areas Must be using the correct ratio for their two areas ie 2:3 if using *OMB* and *MAB*, 2:5 if using *OMB* and *OAB* or 3:5 if using *MAB* and *OAB* Allow ratio to be used the wrong way around eg 2*OMB* = 3*MAB*
				A1	2.1	Correct equation, in two variables (ie θ and their r)	Any correct statement linking the two areas Could use other than r for the radius Or $2x^2\theta - x^2\sin\theta$
			$\theta - \frac{1}{2}\sin\theta = \frac{3}{4}\sin\theta$ $\theta = 1.25\sin\theta$ **A.G.**	A1 **[4]**	2.1	Simplify to given answer	At least one line of working once ratio used

Question			Answer	Marks	AO	Guidance	
10	(b)		0.599	B1	1.1a	Obtain correct first iterate	3sf or better – more accurate answer is 0.599281923... Condone truncating if more sig fig given
			0.705, 0.810	M1	1.1a	Attempt correct iterative process to find at least 2 more values	M1 is for the correct process for finding θ_3 and θ_4, but these may be incorrect M0 if working in degrees
			root = 1.13	A1 **[3]**	1.1	Obtain 1.13	Possibly following B0 if first iterate is wrong but process then self corrects Must follow M1 ie a clear attempt to use the correct iterative process Must be 3sf Once M1 is awarded, allow A1 for 1.13 even if an incorrect iterate seen, as process will recover
10	(c)			B1*	3.1a	Draw $y = \theta$ on diagram	Draw straight line, starting at the origin which intersects the graph Allow point of intersection to be greater than $\theta = \frac{1}{2}\pi$ Ignore incorrect labels, such as $y = x$
				B1 dep*	2.1	Draw correct iterative process on diagram	Vertically into the curve, then horizontally into the straight line, as far as the root Initial value should be before root Needs point of intersection to be before $\theta = \frac{1}{2}\pi$
				B1 **[3]**	1.2	State 'staircase' convergence	Mark independently from other parts of question, including an incorrect diagram, as staircase can be deduced from the iterates in **(b)**

Total for paper is 30 marks