Year 13 Mock Set 02
Pure Paper 1

- Advised to print in "A3-booklets", this will allow all questions to be on the left hand side.
- You can also print in A4, double-sided, and two staples on the left
- If instead you print in 2-in-1 settings, first print the second page up to the last page, then print the cover page separately (to allow all questions on the left)

This exam paper has 15 questions, for a total of 100 marks.

Question	Marks	Score
1	6	
2	8	
3	4	
4	12	
5	7	
6	7	
7	6	
8	6	
9	7	
10	9	
11	5	
12	10	
13	4	
14	5	
15	4	
Total:	100	

Andrew Chan
Last updated: 21st February 2023

1. Given that a and b are positive constants, solve the simultaneous equations

$$ab = 25 \qquad \text{(I)}$$

$$\log_4 a - \log_4 b = 3 \qquad \text{(II)}$$

Show each step of your working, giving exact values for a and b.

(6)

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Question 1 continued

(Total for Question 1 is 6 marks)

2. A sequence is defined by

$$u_1 = 3$$
$$u_{n+1} = u_n - 5, \qquad n \geq 1$$

Find the values of

(a) u_2, u_3 and u_4

(2)

(b) u_{100}

(3)

(c) $\sum_{i=1}^{100} u_i$

(3)

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Question 2 continued

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Question 2 continued

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Question 2 continued

(Total for Question 2 is 8 marks)

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3. Find the first 3 terms in ascending powers of x of

$$\left(2 - \frac{x}{2}\right)^6$$

giving each term in its simplest form.

(4)

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Question 3 continued

(Total for Question 3 is 4 marks)

4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(a) Express $2\sin x - 4\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Give the exact value of R and the value of α, in radians, to 3 significant figures.

(3)

Erling Braut Haaland is a student in Norway. He records the number of hours of daylight every day for a year. He models the number of hours of daylight, H, by the continuous function given by the formula

$$H = 12 + 4\sin\left(\frac{2\pi t}{365}\right) - 8\cos\left(\frac{2\pi t}{365}\right), \qquad 0 \leq t \leq 365$$

where t is the number of days since he began recording.

(b) Using your answer to part(a), or otherwise, find the maximum and minimum number of hours of daylight given by this formula. Give your answer to 3 significant figures.

(3)

(c) Use the formula to find the values of t when $H = 17$, giving your answers to the nearest integer.

(6)

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Question 4 continued

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Question 4 continued

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Question 4 continued

(Total for Question 4 is 12 marks)

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5. The straight line l has equation $y = mx - 2$, where m is a constant.
The curve C has equation $y = 2x^2 + x + 6$
The line l does not cross or touch the curve C.

(a) Show that m satisfies the inequality

$$m^2 - 2m - 63 < 0$$

(3)

(b) Hence find the range of possible values of m

(4)

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Question 5 continued

(Total for Question 5 is 7 marks)

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6. (a) Find $\displaystyle\int \frac{4x+3}{x}\,\mathrm{d}x, \qquad x > 0$

(2)

(b) Given that $y = 25$ at $x = 1$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(4x+3)\,y^{\frac{1}{2}}}{x} \qquad x > 0, y > 0$$

giving your answer in the form $y = \Big[g(x)\Big]^2$.

(5)

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Question 6 continued

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Question 6 continued

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Question 6 continued

(Total for Question 6 is 7 marks)

7. The point P with coordinates $\left(\frac{\pi}{2}, 1\right)$ lies on the curve with equation

$$4x \sin x = \pi y^2 + 2x, \qquad \frac{\pi}{6} \leq x \leq \frac{5\pi}{6}$$

Find an equation of the normal to the curve at P.

(6)

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Question 7 continued

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Question 7 continued

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Question 7 continued

(Total for Question 7 is 6 marks)

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8. (Note that for Edexcel questions, (i) and (ii) within a single question usually means they are two unrelated questions, they are only combined into a question since they are the same "Chapter".)

(i) Differentiate $y = 5x^2 \ln 3x, \qquad x > 0$

(2)

(ii) Given that

$$y = \frac{x}{\sin x + \cos x}, \qquad -\frac{\pi}{4} < x < \frac{3\pi}{4}$$

show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{(1+x)\sin x + (1-x)\cos x}{1 + \sin 2x}, \qquad -\frac{\pi}{4} < x < \frac{3\pi}{4}$$

(4)

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Question 8 continued

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Question 8 continued

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Question 8 continued

(Total for Question 8 is 6 marks)

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9. Use the substitution $x = 2\sin\theta$ to find the exact value of

$$\int_0^{\sqrt{3}} \frac{1}{(4-x^2)^{\frac{3}{2}}}\,\mathrm{d}x$$

(7)

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Question 9 continued

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Question 9 continued

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Question 9 continued

(Total for Question 9 is 7 marks)

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10. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(a) Prove that

$$\frac{1-\cos 2x}{1+\cos 2x} \equiv \tan^2 x \qquad x \neq (2n+1)\,90^\circ, n \in \mathbb{Z}$$

(3)

(b) Hence, or otherwise, solve, for $-90^\circ < \theta < 90^\circ$,

$$\frac{2-2\cos 2\theta}{1+\cos 2\theta} - 2 = 7\sec\theta$$

Give your answer in degrees to one decimal place.

(6)

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Question 10 continued

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Question 10 continued

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Question 10 continued

(Total for Question 10 is 9 marks)

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11.

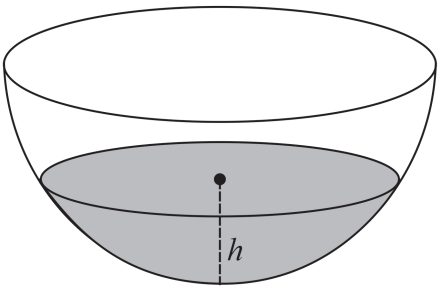

Figure 1

Figure 1 shows a mixing bowl.
Water is flowing into the bowl at a rate of 180 $\text{cm}^3\,\text{s}^{-1}$.
When the height of the water is h cm, the volume of water $V\,\text{cm}^3$ is given by

$$V = \frac{1}{3}\pi h^2(90 - h) \qquad 0 \leq h \leq 30$$

Find the rate of change of the height of water, in $\text{cm}\,\text{s}^{-1}$, when $h = 15$.
Give your answer to 2 significant figures.

(5)

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Question 11 continued

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Question 11 continued

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Question 11 continued

(Total for Question 11 is 5 marks)

12.

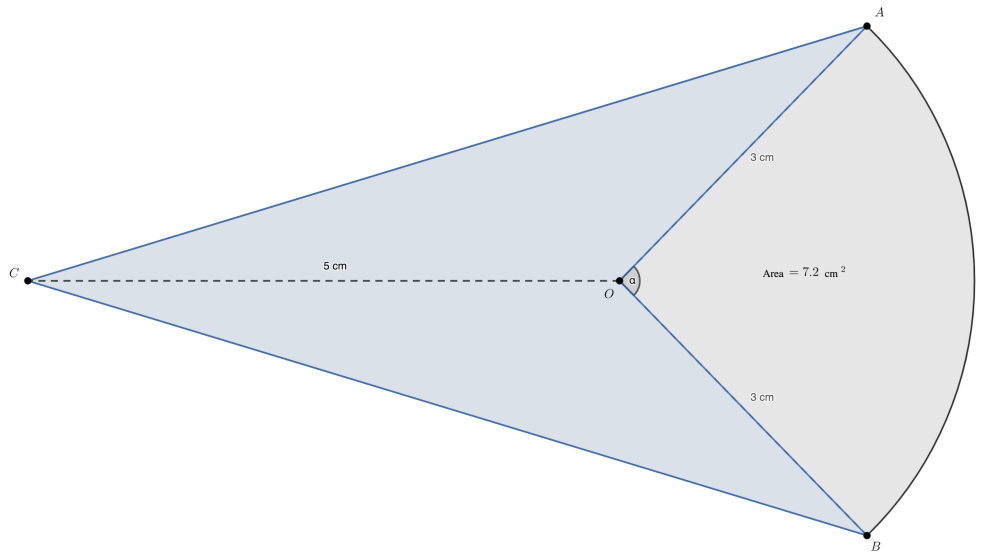

Figure 2

Figure 2 shows the design for a "Head Boy" badge.
The design consists of two congruent triangles, AOC and BOC, joined to a sector AOB of a circle with centre O.
Given that

- $\angle AOB = \alpha$
- $AO = OB = 3$ cm
- $OC = 5$ cm
- the area of sector $AOB = 7.2$ cm^2

(a) show that $\alpha = 1.6$ radians.

(2)

(b) Hence find

(i) the area of the badge, giving your answer in cm^2 to two significant figures.

(ii) the perimeter of the badge, giving your answer in cm to one decimal place.

(8)

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Question 12 continued

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Question 12 continued

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Question 12 continued

(Total for Question 12 is 10 marks)

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13. Given that n is an integer, use algebra, to prove by contradiction, that

if n^3 is even then n is even.

(4)

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Question 13 continued

(Total for Question 13 is 4 marks)

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14.

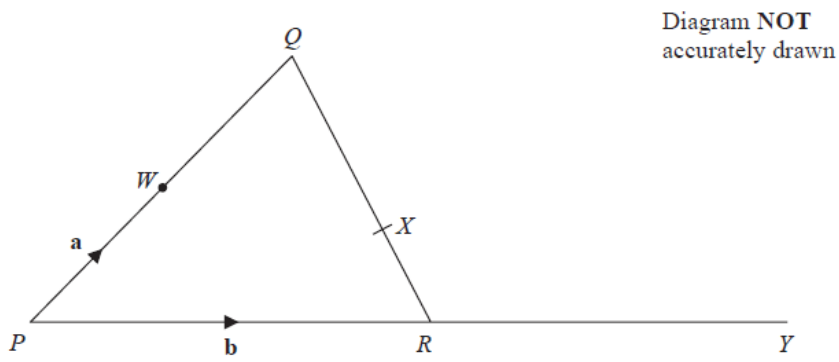

Figure 3

As shown in Figure 3,

- PQR is a triangle.
- The midpoint of PQ is W.
- X is the point on QR such that $QX : XR = 2 : 1$
- PRY is a straight line.
- $\overrightarrow{PW} = \mathbf{a}$
- $\overrightarrow{PR} = \mathbf{b}$

(a) Find, in terms of **a** and **b**

(i) $\overrightarrow{QR}$

(ii) $\overrightarrow{QX}$

(iii) $\overrightarrow{WX}$

(3)

Given that R is the midpoint of the straight line PRY

(b) Use a vector method to show that WXY is a straight line.

(2)

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Question 14 continued

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Question 14 continued

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Question 14 continued

(Total for Question 14 is 5 marks)

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15. The function f is defined by

$$\mathrm{f}(x) = x^2 + 4x + 1 \qquad x \in \mathbb{R}$$

(a) Find the range of $\mathrm{f}(x)$.

(3)

(b) Explain why the function f does not have an inverse.

(1)

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Question 15 continued

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Question 15 continued

(Total for Question 15 is 4 marks)

Total for paper is 100 marks

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